

**SPECIALIST MATHS
TRIAL EXAMINATION 2
SOLUTIONS
2006**

Section 1 – Multiple-choice answers

1.	E	7.	B	13.	D	19.	E
2.	D	8.	C	14.	D	20.	C
3.	E	9.	A	15.	E	21.	B
4.	D	10.	B	16.	C	22.	C
5.	B	11.	C	17.	D		
6.	A	12.	A	18.	E		

Section 1- Multiple-choice solutions

Question 1

The equation of the asymptote through $(-3, 0)$ is $y = \frac{2}{3}x + 2$.

For the hyperbola $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ the asymptotes are given by $y - k = \pm \frac{b}{a}(x - h)$.

We require $b = 2$ and $a = 3$.

The required equation is $\frac{x^2}{9} - \frac{(y-2)^2}{4} = 1$

The answer is E.

Question 2

	function	asymptotes
A.	$y = \frac{1}{(x-1)(x+1)}$	$y = 0, x = 1, x = -1$
B.	$y = \frac{x^2 + 1}{x}$ $= x + \frac{1}{x}$	$y = x, x = 0$
C.	$y = \frac{x^3 + 1}{x^2}$ $= x + \frac{1}{x^2}$	$y = x, x = 0$
D.	$y = \frac{x^4 + 1}{x^2}$ $= x^2 + \frac{1}{x^2}$	$y = x^2, x = 0$
E.	$y = x^2 + \frac{1}{x^2 + 1}$	$y = x^2$

The answer is D.

Question 3

$$f(x) = \sec(ax)$$

$$= \frac{1}{\cos(ax)}$$

The graph has asymptotes for
 $\cos(ax) = 0$

$$ax = \dots -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \dots$$

$$x = \dots -\frac{3\pi}{2a}, -\frac{\pi}{2a}, \frac{\pi}{2a}, \frac{3\pi}{2a}, \dots$$

The answer is E.

Question 4

$$\sin^2(\theta) = \frac{1}{2}(1 - \cos(2\theta))$$

$$\sin(\theta) = \sqrt{\frac{1}{2}(1 - \cos(2\theta))} \quad (\text{option A})$$

$$\sin^2(\theta) + \cos^2(\theta) = 1$$

$$\sin(\theta) = \sqrt{1 - \cos^2(\theta)} \quad (\text{option B})$$

$$\sin(\theta) = 2 \sin\left(\frac{\theta}{2}\right) \cos\left(\frac{\theta}{2}\right) \quad (\text{option C, double angle formula})$$

$$\begin{aligned} \sin(\theta) &= \sin(3\theta - 2\theta) \\ &= \sin(3\theta)\cos(2\theta) - \cos(3\theta)\sin(2\theta) \quad (\text{option D) is incorrect} \end{aligned}$$

$$\begin{aligned} \sin(\theta) &= \sin\left(\frac{\theta}{3} + \frac{2\theta}{3}\right) \\ &= \sin\left(\frac{\theta}{3}\right)\cos\left(\frac{2\theta}{3}\right) + \cos\left(\frac{\theta}{3}\right)\sin\left(\frac{2\theta}{3}\right) \quad (\text{option E}) \end{aligned}$$

The answer is D.

Question 5

$$\begin{aligned} w &= a \operatorname{cis}(60^\circ) \\ &= a(\cos(60^\circ) + i \sin(60^\circ)) \\ &= a\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) \\ &= \frac{a}{2}(1 + \sqrt{3}i) \end{aligned}$$

The answer is B.

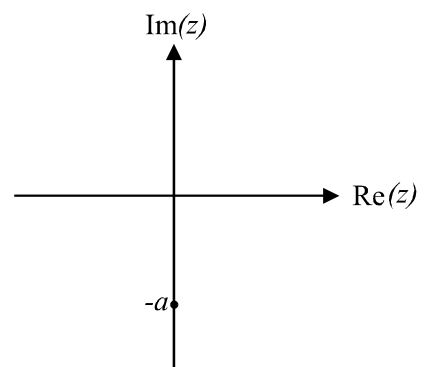
Question 6

$$\begin{aligned} -i(\bar{w} + w) &= -i\left(\frac{a}{2}(1 - \sqrt{3}i) + \frac{a}{2}(1 + \sqrt{3}i)\right) \\ &= -i\left(\frac{a}{2} + \frac{a}{2}\right) \\ &= -ai \end{aligned}$$

This complex number is located on the negative branch of the imaginary axis, a units from the origin.

Hence in polar form it can be expressed as $a \operatorname{cis}(-90^\circ)$.

The answer is A.



Question 7

$$\text{Let } z^3 = w$$

$$(r\text{cis}(\theta))^3 = w$$

$$r^3\text{cis}(3\theta) = a\text{cis}(60^\circ) \quad (\text{De Moivre})$$

$$r^3 = a \quad 3\theta = 60^\circ + 360^\circ k$$

$$r = \sqrt[3]{a} \quad \theta = 20^\circ + 120^\circ k \quad k \in \mathbb{Z}$$

$$\theta = 20^\circ, 140^\circ, 260^\circ \dots$$

The three cube roots are $\sqrt[3]{a}\text{cis}(20^\circ)$, $\sqrt[3]{a}\text{cis}(140^\circ)$ and $\sqrt[3]{a}\text{cis}(260^\circ)$.

If $a = 1$, the cube roots lie on the circle with radius a units. If $a > 1$, the cube roots lie on a circle with radius $\sqrt[3]{a}$ units which is inside the circle with radius a units. So option A is correct.

Option B is incorrect.

Options C, D and E are correct.

The answer is B.

Question 8

$$y = \sin^{-1}\left(\frac{x}{\sqrt{a}}\right)$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{a-x^2}}$$

$$= (a-x^2)^{-\frac{1}{2}}$$

$$\frac{d^2y}{dx^2} = -\frac{1}{2}(a-x^2)^{-\frac{3}{2}} \times -2x$$

$$= \frac{x}{(a-x^2)^{\frac{3}{2}}}$$

The answer is C.

Question 9

$$xy^2 = 1$$

$$\frac{d}{dx}(xy^2) = \frac{d}{dx}(1)$$

$$1 \times y^2 + x \times 2y \frac{dy}{dx} = 0$$

$$2xy \frac{dy}{dx} = -y^2$$

$$\frac{dy}{dx} = \frac{-y}{2x} \quad x \neq 0$$

$$\text{At } x=4, y^2 = \frac{1}{4}, y = \pm \frac{1}{2}$$

$$\text{In the fourth quadrant } y = -\frac{1}{2}$$

$$\text{So } \frac{dy}{dx} = \frac{1}{2} \div 8$$

$$= \frac{1}{16}$$

The answer is A.

Question 10

$$\int_0^{\pi} (\sin(x)\cos(x))^3 dx$$

$$= \int_0^{\pi} \sin^3(x)\cos^3(x) dx$$

$$= \int_0^{\pi} \sin(x)(1 - \cos^2(x))(\cos^3(x) dx)$$

$$= \int_1^{-1} -\frac{du}{dx} (1 - u^2) u^3 dx$$

$$= \int_{-1}^1 (u^3 - u^5) du$$

$$\text{where } u = \cos(x)$$

$$\frac{du}{dx} = -\sin(x)$$

$$x = \pi, u = -1$$

$$x = 0, u = 1$$

The answer is B.

Question 11

Stationary points on the graph of $y = F(x)$ occur at $x = -5$, $x = 0$, $x = 3$ and $x = 7$.

The gradient (i.e. the value of $y = f(x)$) of the graph of $y = F(x)$ is different i.e. positive or negative on either side of each of these stationary points and hence there is no stationary point of inflection.

Since $f(x) > 0$ for $x \in (-5, 0) \cup (3, 7)$, the gradient of the graph of $y = F(x)$ is positive for these values.

At $x = 0$, there is a maximum turning point since the gradient is positive to the left of $x = 0$ and negative to the right. Therefore, option C is incorrect.

Options D and E are both correct.

The answer is C.

Question 12

$$\begin{aligned} \text{volume} &= \pi \int_0^{\frac{\pi}{2}} x^2 dy \\ &= \pi \int_0^{\frac{\pi}{2}} \cos^2(y) dy \\ &= \pi \int_0^{\frac{\pi}{2}} \frac{1}{2} (\cos(2y) + 1) dy \\ &= \frac{\pi}{2} \int_0^{\frac{\pi}{2}} (\cos(2y) + 1) dy \end{aligned}$$

Now,

$$y = \cos^{-1}(x)$$

$$x = \cos(y)$$

$$x^2 = \cos^2(y)$$

The answer is A.

Question 13

A. $\underline{a} \cdot \underline{a} = \underline{c} \cdot \underline{c}$

$$|\underline{a}| |\underline{a}| \cos(0) = |\underline{c}| |\underline{c}| \cos(0)$$

$$|\underline{a}|^2 = |\underline{c}|^2 \text{ may or may not be true}$$

B. $\underline{a} \cdot \underline{b} = \underline{b} \cdot \underline{c}$ may or may not be true

C. $(\underline{a} \cdot \underline{c}) \underline{b} = 0 \times \underline{b}$
 $= \underline{0}$
 $\neq \underline{b}$

D. $\underline{a} \cdot (\underline{a} + \underline{c})$
 $= \underline{a} \cdot \underline{a} + \underline{a} \cdot \underline{c}$
 $= \underline{a} \cdot \underline{a}$ since $\underline{a} \cdot \underline{c} = 0$
 $= |\underline{a}|^2$

E. $(\underline{a} + \underline{c}) \cdot (\underline{a} - \underline{c}) = \underline{a} \cdot \underline{a} - \underline{a} \cdot \underline{c} + \underline{a} \cdot \underline{c} - \underline{c} \cdot \underline{c}$
 $= \underline{a} \cdot \underline{a} - \underline{c} \cdot \underline{c}$
 $= |\underline{a}| |\underline{a}| \cos(0) - |\underline{c}| |\underline{c}| \cos(0)$
 $= |\underline{a}|^2 - |\underline{c}|^2$

Now $|\underline{a}| - |\underline{c}| = |\underline{a}|^2 - |\underline{c}|^2$ may or may not be true.

The answer is D.

Question 14

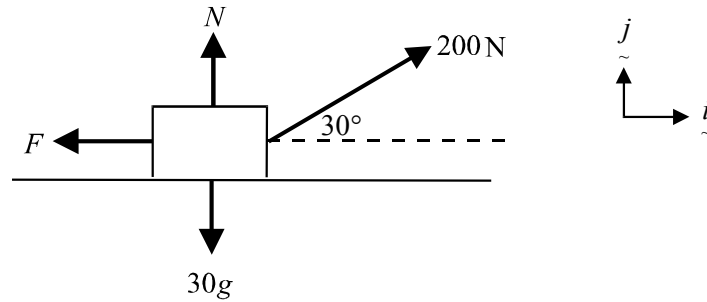
$\underline{c}$ is the component of $\underline{a}$ in the direction of $\underline{b}$.

Hence $\underline{c} = (\underline{a} \cdot \hat{\underline{b}}) \hat{\underline{b}}$.

The answer is D.

Question 15

Draw a diagram.



$$\underline{R} = m \underline{a}$$

$$(200 \cos(30^\circ) - F)\underline{i} + (N + 200 \sin(30^\circ) - 30g)\underline{j} = 30a \underline{i}$$

$$100\sqrt{3} - F = 30a \quad \text{and} \quad N + 100 - 30g = 0$$

$$100\sqrt{3} - 0.5(30g - 100) = 30a$$

$$N = 30g - 100$$

$$100\sqrt{3} - 15g + 50 = 30a$$

$$a = \frac{10\sqrt{3} - 1.5g + 5}{3}$$

The answer is E.

Question 16

Resolving

$$1 = Q \sin(\theta) \quad - (1) \quad \text{and} \quad \sqrt{3} = Q \cos(\theta) \quad - (2)$$

$$(1) \div (2) \text{ gives } \frac{\sin(\theta)}{\cos(\theta)} = \frac{1}{\sqrt{3}}$$

$$\tan(\theta) = \frac{1}{\sqrt{3}}$$

$$\theta = 30^\circ$$

$$\sin(\theta) = \frac{1}{2}$$

$$\text{So } Q = 2$$

$$\left(\text{Check } \cos(\theta) = \frac{\sqrt{3}}{2}, Q = 2 \right).$$

 Q has a magnitude of 2N and acts in a direction of $N30^\circ E$.

The answer is C.

Question 17

Let $\underline{v}$ be the velocity of the particle. So $\underline{v} = 6\underline{i} + 8\underline{j}$ where $\underline{i}$ runs in the horizontal direction and $\underline{j}$ runs in the vertical direction.

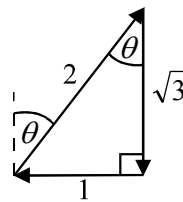
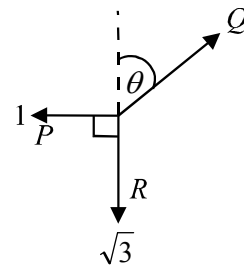
$$\underline{P} = m \underline{v}$$

$$= 3(6\underline{i} + 8\underline{j})$$

$$|\underline{P}| = 3\sqrt{36 + 64}$$

$$= 30 \text{ kg m/s}$$

The answer is D.



Question 18

$$\frac{dy}{dx} = a + by \quad \text{where } a \text{ and } b \text{ are constants}$$

$$\frac{dx}{dy} = \frac{1}{a + by}$$

$$x = \int \frac{1}{a + by} dy \quad (\text{option A})$$

$$= \frac{1}{b} \int \frac{b}{a + by} dy$$

$$x = \frac{1}{b} \log_e |a + by| + c$$

$$y(0) = 0$$

$$0 = \frac{1}{b} \log_e |a| + c$$

$$c = -\frac{1}{b} \log_e |a|$$

$$= \log_e |a|^{\frac{1}{b}} \quad (\text{option B})$$

$$x = \frac{1}{b} \log_e |a + by| - \frac{1}{b} \log_e |a|$$

$$= \frac{1}{b} \log_e \left| \frac{a + by}{a} \right| \quad (\text{option C})$$

$$bx = \log_e \left| \frac{a + by}{a} \right|$$

$$e^{bx} = \frac{a + by}{a} \quad (\text{option D})$$

$$ae^{bx} = a + by$$

$$by = a(e^{bx} - 1)$$

$$y = \frac{a}{b}(e^{bx} - 1) \quad (\text{making option E not true})$$

The answer is E.

Question 19

$$\begin{aligned} \frac{dy}{dx} &= \frac{1}{x+1}, & x_0 &= 0, & y_0 &= 0, & h &= 0.1 \\ x_{n+1} &= x_n + h & y_{n+1} &= y_n + h f(x_n) \\ \text{So, } x_1 &= 0 + 0.1 & y_1 &= 0 + 0.1 \times \frac{1}{1} \\ &= 0.1 & &= 0.1 \\ x_2 &= 0.1 + 0.1 & y_2 &= 0.1 + 0.1 \times \frac{1}{1.1} \\ &= 0.2 & &= \frac{1}{10} + \frac{1}{11} \\ & & &= \frac{21}{110} \end{aligned}$$

The answer is E.

Question 20

$$\begin{aligned} \frac{dS}{dt} &= \text{rate of inflow} - \text{rate of outflow} \\ &= \frac{dS}{dl} \cdot \frac{dl}{dt} (\text{inflow}) - \frac{dS}{dl} \cdot \frac{dl}{dt} (\text{outflow}) \\ &= 0 \times 2 - \frac{S}{300-3t} \times 5 \\ &= \frac{-5S}{3(100-t)} \end{aligned}$$

Note that the amount of sugar per litre in the tank is $\frac{S}{300-3t}$ since every second the volume of solution decreases by 3L.

The answer is C.

Question 21

For $y = \sqrt{x}$, $x > 0$ therefore the slope diagram is not appropriate to this function since gradients are assigned for $x \leq 0$.

A similar argument is true for $y = \log_e(x)$.

Note that for $x = 2$, $\frac{dy}{dx} = 0$.

This is true for $y = -x(x-4)$ since $\frac{dy}{dx} = -2x + 4$ and when $x = 2$, $\frac{dy}{dx} = 0$.

For $y = 2x - x^2$, $\frac{dy}{dx} = 2 - 2x$, so at $x = 2$, $\frac{dy}{dx} = -2$ so the slope diagram is not suitable for this function.

Similarly for $y = 1 - e^{-x}$, $\frac{dy}{dx} = e^{-x}$.

At $x = 2$ $\frac{dy}{dx} = e^{-2} = 0.135$
 $\neq 0$

The answer is B.

Question 22

$$v^2 = -2 \int 5 \, dx$$

$$\frac{1}{2} v^2 = - \int 5 \, dx$$

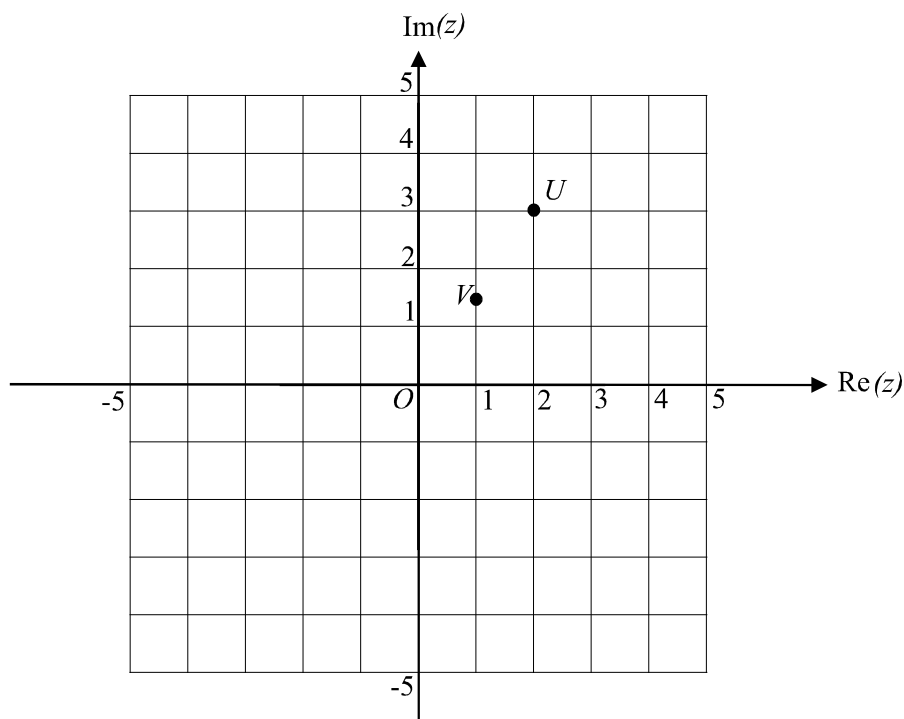
$$\frac{d}{dx} \left(\frac{1}{2} v^2 \right) = -5$$

Since $a = \frac{d}{dx} \left(\frac{1}{2} v^2 \right)$ we have $a = -5$.

Since the particle is moving with constant, negative acceleration, the velocity will be decreasing.
The answer is C.

Section 2

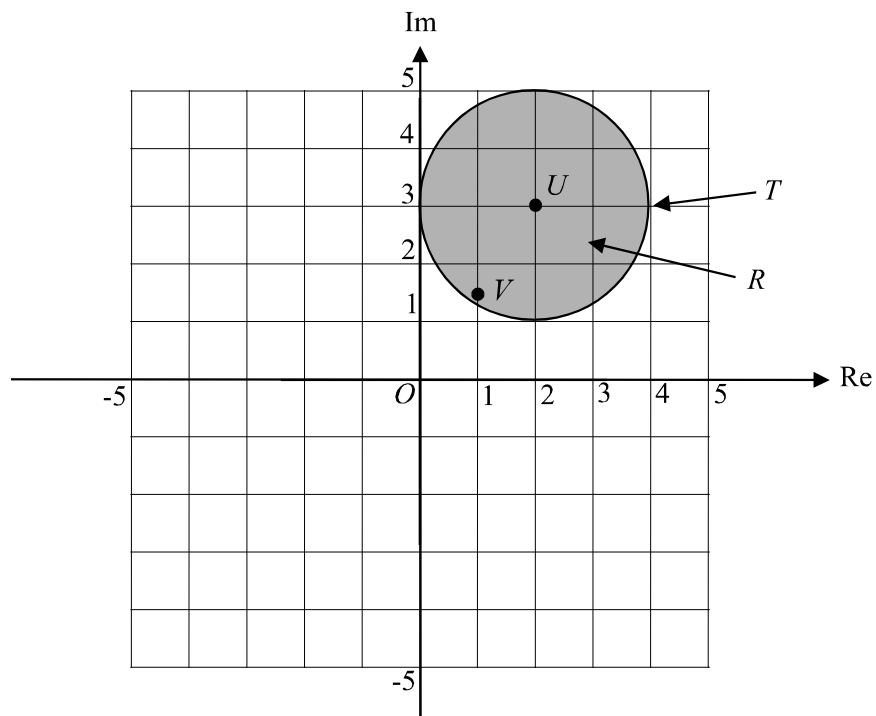
Question 1



- a. (1 mark) correctly showing U and V .
- b. T is a circle with centre at U and radius of 2 units.
 Since w lies on the imaginary axis, the circle touches that axis at $0 + 3i$ only.
 So $b = 3$.

(1 mark)

c. i.



(1 mark)

ii. $v \in R$ if $|v - u| \leq 2$

$$LS = \left| 1 + \frac{3i}{2} - 2 - 3i \right|$$

$$= \left| -1 - \frac{3}{2}i \right|$$

$$= \sqrt{1 + \frac{9}{4}}$$

$$= \frac{\sqrt{13}}{2}$$

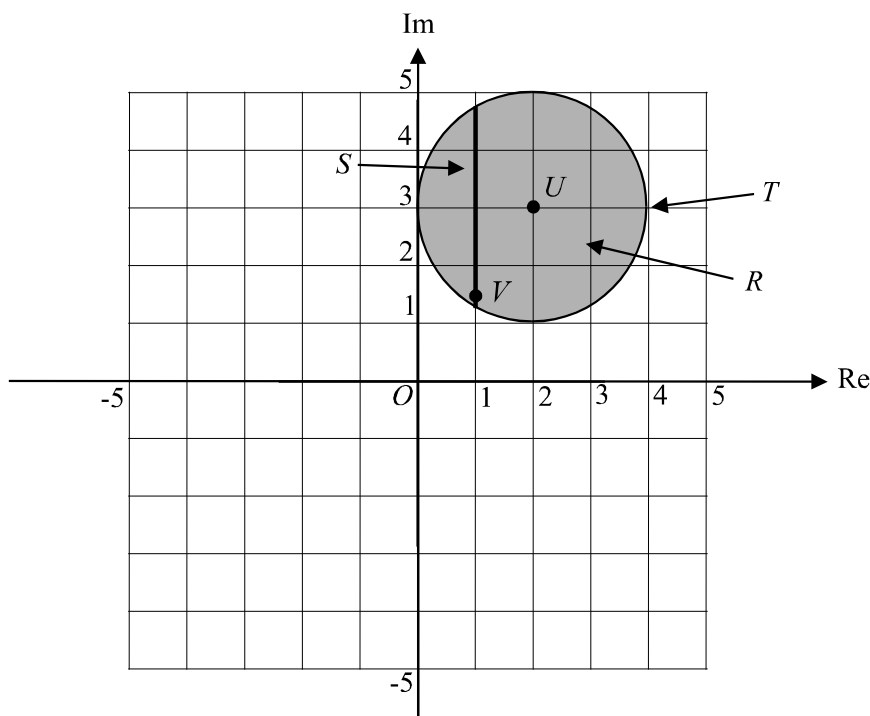
$$< 2$$

So $v \in R$

(1 mark)

(1 mark)

d. i.

**(1 mark)**

- ii. Since $p \in S$ and $p = x + yi$, then $x = 1$ and the maximum and minimum values of y lie on the circle defined by

$$|z - u| = 2$$

$$|1 + yi - 2 - 3i| = 2$$

$$|-1 + (y - 3)i| = 2$$

$$\sqrt{1 + (y - 3)^2} = 2$$

$$1 + (y - 3)^2 = 4$$

$$y^2 - 6y + 6 = 0$$

$$y = \frac{6 \pm \sqrt{36 - 4 \times 1 \times 6}}{2}$$

(1 mark)

$$= \frac{6 \pm \sqrt{12}}{2}$$

So the maximum and minimum values of y are given respectively by

$$y = 3 + \sqrt{3} \text{ and } y = 3 - \sqrt{3}$$

(1 mark)

e. Method 1

The vectors $\vec{OV}$ and $\vec{VU}$ are parallel if

$\vec{OV} = k\vec{VU}$ where k is a constant.

Now, $\vec{OV} = \hat{i} + \frac{3}{2}\hat{j}$

$$\begin{aligned}\vec{VU} &= \vec{VO} + \vec{OU} \\ &= -\vec{OV} + \vec{OU} \\ &= -\hat{i} - \frac{3}{2}\hat{j} + 2\hat{i} + 3\hat{j} \\ &= \hat{i} + \frac{3}{2}\hat{j} \quad \text{(1 mark)}\end{aligned}$$

So $\hat{i} + \frac{3}{2}\hat{j} = k\left(\hat{i} + \frac{3}{2}\hat{j}\right)$ where $k = 1$ (1 mark)

So $\vec{OV}$ and $\vec{VU}$ are parallel and since they share the point V , they must be collinear.

(1 mark)

Method 2

$$\vec{OU} = 2\hat{i} + 3\hat{j}$$

$$= 2\left(\hat{i} + \frac{3}{2}\hat{j}\right) \quad \text{(1 mark)}$$

$$= 2\vec{OV} \quad \text{(1 mark)}$$

So $\vec{OV}$ and $\vec{VU}$ are parallel and since they share the point O , they must be collinear.

(1 mark)

Total 11 marks

Question 2

a. i. $\underline{a} = \sqrt{2} \sin(t) \underline{i} + \sqrt{2} \cos(t) \underline{j}$

$$x = \sqrt{2} \sin(t) \quad y = \sqrt{2} \cos(t)$$

$$x^2 = 2 \sin^2(t) \quad y^2 = 2 \cos^2(t)$$

$$x^2 + y^2 = 2 \sin^2(t) + 2 \cos^2(t)$$

$$x^2 + y^2 = 2(\sin^2(t) + \cos^2(t))$$

$$x^2 + y^2 = 2$$

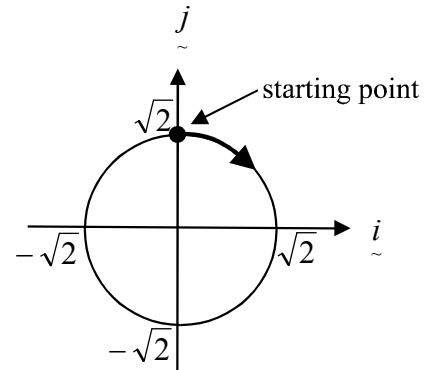
(1 mark)

ii. At $t = 0$, $\underline{a} = 0 \underline{i} + \sqrt{2} \underline{j}$

Particle A starts at the point $(0, \sqrt{2})$ in the $\underline{i} - \underline{j}$ plane.

It moves in a clockwise direction on a circular path with centre at $(0, 0)$ and radius $\sqrt{2}$.

(We know it moves in a clockwise direction since as t increases from $t = 0$, $\sin(t)$ increases and $\cos(t)$ decreases so the $\underline{i}$ component increases and the $\underline{j}$ component decreases.)



(1 mark) stating starting point
(1 mark) clockwise direction around a circle with correct centre and radius

b. i. $\underline{b} = \sqrt{2} \cos(t) \underline{i} + \underline{j}$

The Cartesian equation of the path of particle B is $y = 1$.

(1 mark)

ii. Since $x = \sqrt{2} \cos(t)$, the domain of the Cartesian equation that describes the path of particle B is $x \in [-\sqrt{2}, \sqrt{2}]$.

(1 mark)

c. $\underline{a} \cdot \underline{b}$

$$= \left(\sqrt{2} \sin(t) \underline{i} + \sqrt{2} \cos(t) \underline{j} \right) \cdot \left(\sqrt{2} \cos(t) \underline{i} + \underline{j} \right)$$

$$= 2 \sin(t) \cos(t) + \sqrt{2} \cos(t) \quad (1 \text{ mark})$$

When A and B are at right angles, $\underline{a} \cdot \underline{b} = 0$ (1 mark)

$$2 \sin(t) \cos(t) + \sqrt{2} \cos(t) = 0$$

$$\sqrt{2} \cos(t) (\sqrt{2} \sin(t) + 1) = 0$$

$$\sqrt{2} \cos(t) = 0 \quad \text{or} \quad \sqrt{2} \sin(t) + 1 = 0$$

$$t = \frac{\pi}{2}, \frac{3\pi}{2} \dots \quad \sin(t) = -\frac{1}{\sqrt{2}}$$

$$t = \frac{5\pi}{4}, \frac{7\pi}{4} \dots$$

(1 mark)

S	A
T	C

A and B are first at right angles at $t = \frac{\pi}{2}$ seconds.

(1 mark)

d.

$$\underline{a} = \sqrt{2} \sin(t) \underline{i} + \sqrt{2} \cos(t) \underline{j}$$

$$\underline{\dot{a}} = \sqrt{2} \cos(t) \underline{i} - \sqrt{2} \sin(t) \underline{j}$$

$$|\underline{\dot{a}}| = \sqrt{2 \cos^2(t) + 2 \sin^2(t)} \quad (1 \text{ mark})$$

$$= \sqrt{2(\cos^2(t) + \sin^2(t))}$$

$$= \sqrt{2}$$

So particle A moves with a constant speed of $\sqrt{2}$ units.

(1 mark)

e.

i. $\underline{a} = \sqrt{2} \sin(t) \underline{i} + \sqrt{2} \cos(t) \underline{j}$

$$\underline{b} = \sqrt{2} \cos(t) \underline{i} + \underline{j}$$

 A and B will collide iff

$$\sqrt{2} \sin(t) = \sqrt{2} \cos(t) \text{ AND } \sqrt{2} \cos(t) = 1$$

(1 mark)

$$\frac{\sin(t)}{\cos(t)} = 1$$

$$\cos(t) = \frac{1}{\sqrt{2}}$$

$$\tan(t) = 1$$

$$t = \frac{\pi}{4}, \frac{7\pi}{4}, \dots$$

S	A
T	C

$$t = \frac{\pi}{4}, \frac{5\pi}{4}, \dots$$

Since A and B will be at the same spot at the same time, that is at $t = \frac{\pi}{4}$, they will collide.

(1 mark)

- ii. At $t = 0$, A is at the point $(0, \sqrt{2})$ on the $\underline{i} - \underline{j}$ plane.

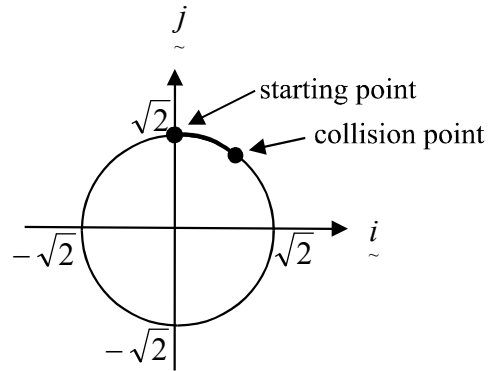
At $t = \frac{\pi}{4}$, when A collides with B,

$$\begin{aligned} \underline{a} &= \sqrt{2} \sin\left(\frac{\pi}{4}\right) \underline{i} + \sqrt{2} \cos\left(\frac{\pi}{4}\right) \underline{j} \\ &= \sqrt{2} \cdot \frac{1}{\sqrt{2}} \underline{i} + \sqrt{2} \cdot \frac{1}{\sqrt{2}} \underline{j} \\ &= \underline{i} + \underline{j} \end{aligned}$$

So A is at the point $(1, 1)$ when the collision occurs.

Particle A has travelled $\frac{1}{8}$ of the way around the circle with perimeter $2\sqrt{2}\pi$.

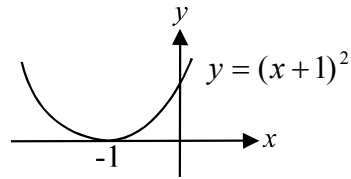
So particle A has travelled $\frac{\sqrt{2}}{4}\pi$ units.



(1 mark)
Total 14 marks

Question 3

- a. For $g(x) = \log_e(x^2 + 2x + 1)$,
 $x^2 + 2x + 1 > 0$
 $(x+1)^2 > 0$
 $x \in \mathbb{R} \setminus \{-1\}$
 $d_g = \mathbb{R} \setminus \{-1\} \text{ or } (-\infty, -1) \cup (-1, \infty)$

**(1 mark)**

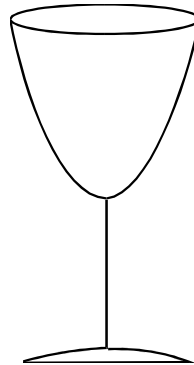
- b. i. Let $y = \log_e(x^2 + 2x + 1)$
 Swap x and y
 $x = \log_e(y^2 + 2y + 1)$
 $e^x = (y+1)^2$
 $\pm \sqrt{e^x} = y+1$
 $y = -1 \pm \sqrt{e^x}$
- } These steps must be shown

(1 mark)

- ii. $r_{g^{-1}} = d_g$
 $= \mathbb{R} \setminus \{-1\} \text{ or } (-\infty, -1) \cup (-1, \infty)$

(1 mark)

- c. First find $g(10)$.
 $g(10) = \log_e(10^2 + 20 + 1)$
 $= \log_e(121)$
 The diameter of the rim is $2 \log_e(121)$ cm.

**(1 mark)**

d. i. volume = $\pi \int_0^{10} y^2 dx$
 $= \pi \int_0^{10} (\log_e(x^2 + 2x + 1))^2 dx$

(1 mark) correct integrand
 (1 mark) correct terminals

ii. Using a calculator,

$$\pi \int_0^{10} (\log_e(x^2 + 2x + 1))^2 dx = 121.98089\pi \text{ cm}^3 \quad (1 \text{ mark})$$

$$= 1.2198089\pi \times 100 \text{ cm}^3$$

$$= 1.2198089\pi \text{ standard drinks}$$

$$= 3.8 \text{ standard drinks (correct to 1 decimal place)}$$

(1 mark)

- e. Using a calculator, try different values of a where a is an integer. We require the largest value of a so that the value of y when $x = 10$ is less than 5.

For $y = \ln(x^2 + ax + 1)$

when $x = 10$ and $a = 2$, $y = 4.795...$

For $y = \ln(x^2 + ax + 1)$

(1 mark) for trying some values of a

when $x = 10$, and $a = 3$, $y = 4.875...$

For $y = \ln(x^2 + ax + 1)$

when $x = 10$, and $a = 4$, $y = 4.948...$

For $y = \ln(x^2 + ax + 1)$

when $x = 10$, and $a = 5$, $y = 5.017...$

So the maximum value of a is 4.

(1 mark)

- f. For the function
 $f(x) = \log_e(x^2 + ax + 1)$, $a > 0$ and $0 \leq x \leq 10$

$$f'(x) = \frac{2x + a}{x^2 + ax + 1}, \quad a > 0 \text{ and } 0 \leq x \leq 10$$

so $f'(x) > 0$ since $a > 0$ and $x \geq 0$

so $f'(x) \neq 0$ for all $0 \leq x \leq 10$

(1 mark)

So for $x \in [0, 10]$ there is no horizontal slope. Hence when the mould is placed upright there is no point where the slope is vertical.

(1 mark)

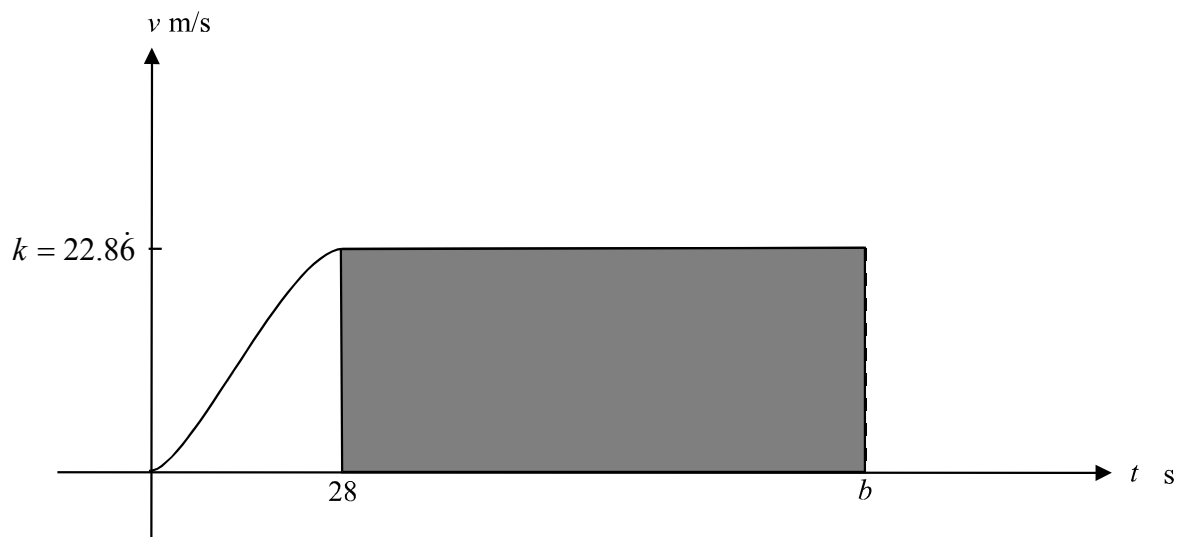
Total 12 marks

Question 4

- a. The maximum speed of the train is given by $\frac{-28^3}{480} + \frac{28 \times 28^2}{320} = 22.86 \text{ m/s}.$

(1 mark)

b.



The shaded area on the graph above represents the distance travelled by the train whilst it is travelling at maximum speed.

$$\text{So } (b - 28) \times 22.86 = 2\,058$$

$$b = 118$$

(1 mark)

- c. i. average acceleration = $\frac{v(50) - v(10)}{50 - 10}$
- $$= \left(22.86 - \left(\frac{-10^3}{480} + \frac{28 \times 100}{320} \right) \right) \div 40$$
- $$= 0.41 \text{ m/s}^2 \text{ (to 2 decimal places)}$$

(1 mark)

- ii. For $t = 10$, $v(t) = \frac{-t^3}{480} + \frac{28t^2}{320}$
- $$a(t) = \frac{-3t^2}{480} + \frac{56t}{320}$$
- $$a(10) = \frac{-300}{480} + \frac{560}{320}$$
- $$= 1.125 \text{ m/s}^2$$

(1 mark)

d. Method 1

Use a graphics calculator to solve

$$0 = \frac{-23 \cdot 6}{\pi} \tan^{-1}(t - 130) + 11 \cdot 691$$

$t = 198 \cdot 91$ seconds (to 2 decimal places)

So $c = 198.91$ (to 2 decimal places).

(1 mark)**Method 2**

By hand.

$$0 = \frac{-23 \cdot 6}{\pi} \tan^{-1}(t - 130) + 11 \cdot 691$$

$$\frac{-11 \cdot 691 \times \pi}{-23 \cdot 6} = \tan^{-1}(t - 130)$$

$$1 \cdot 556... = \tan^{-1}(t - 130)$$

$$\tan(1 \cdot 556...) = t - 130$$

$$t = 198 \cdot 9136...$$

$$= 198 \cdot 91 \text{ seconds (to two decimal places)}$$

So $c = 198.91$ (to 2 decimal places).

(1 mark)

e.
$$\text{distance} = \int_0^{28} \left(\frac{-t^3}{480} + \frac{28t^2}{320} \right) dt + \int_{118}^c g(t) dt + 2058$$

(1 mark) 1 correct integral**(1 mark)** 2nd correct integral and including 2 058**f.** Sketch the function $y = g(t)$ for $118 \leq t \leq 198 \cdot 91$.

There is a rapid decrease in the velocity of the train between approximately

$t = 120$ and $t = 140$ and then there is virtually zero velocity between $t = 140$ and $t = 198 \cdot 91$ when the train finally comes to rest.

This suggests that the train decelerates quickly but then crawls along for approximately one minute; maybe having to wait for another train already in the station.

(1 mark)

g.
$$g'(t) = \frac{-23 \cdot 6}{\pi(1 + (t - 130)^2)}$$

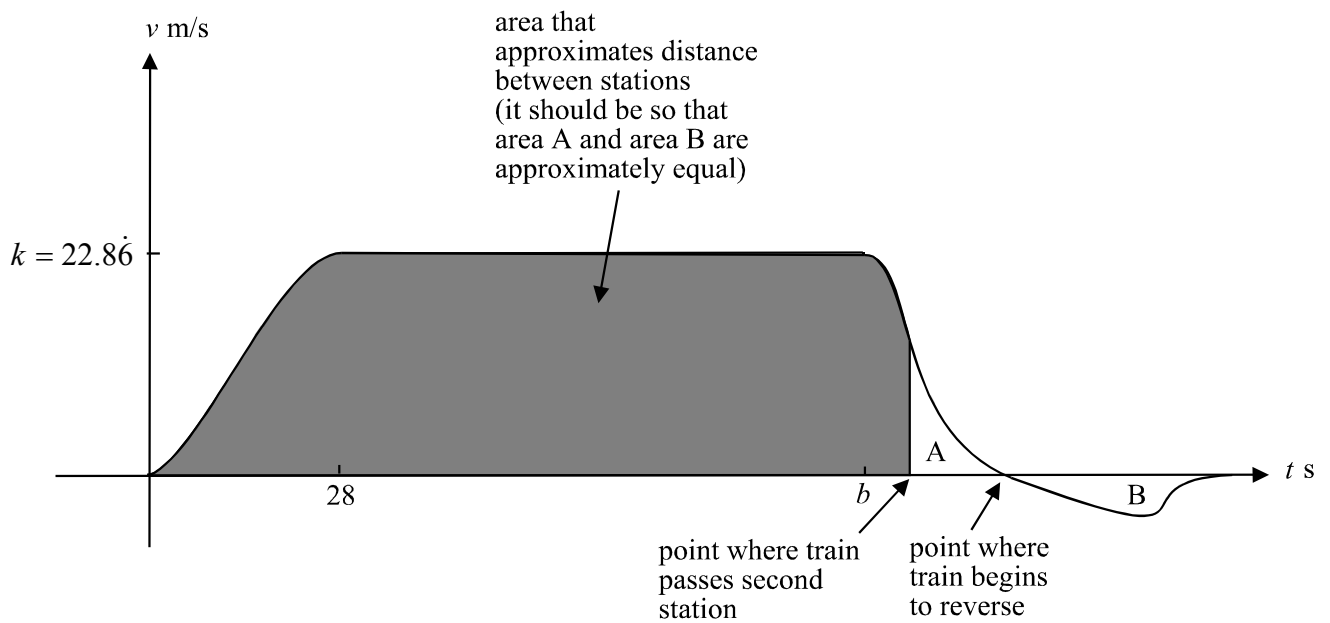
The velocity is constant when $g'(t) = 0$.

Now, $\frac{-23 \cdot 6}{\pi(1 + (t - 130)^2)} = 0$ has no real solution since $-23 \cdot 6 \neq 0$.

So the velocity of the train is never constant for $b < t \leq c$.

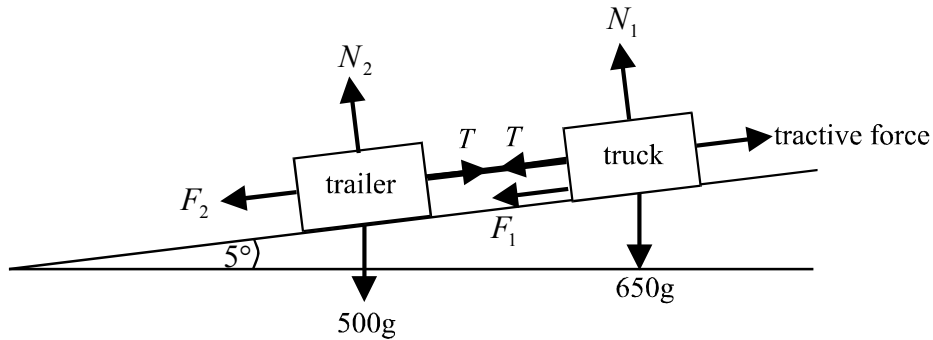
(1 mark)

h.

**(1 mark)** correct graph**(1 mark)** showing area**(1 mark)** showing reverse point**Total 12 marks**

Question 5

- a. Draw a force diagram.



(1 mark) diagram showing forces around the trailer

Around the trailer

$$T - 500g \sin(5^\circ) - F_2 = ma \quad (1 \text{ mark}) \quad \text{and } N_2 = 500g \cos(5^\circ) \quad (1 \text{ mark})$$

$$2000 - 427 \cdot 0631 - \mu N_2 = 500 \times 0 \cdot 15$$

$$\mu \times 500g \cos(5^\circ) = 1497 \cdot 93 \dots$$

$$\mu = 0 \cdot 307 \text{ (to 3 decimal places)}$$

(1 mark)

- b.

$$s = \frac{1}{2}(u + v)t$$

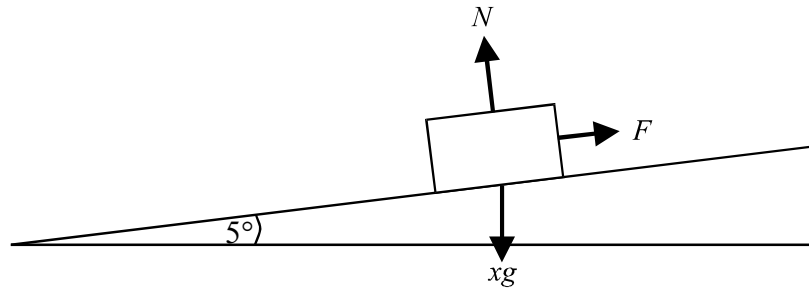
(1 mark)

$$150 = \frac{1}{2}(4 + 0)t$$

$$t = 75 \text{ secs}$$

(1 mark)

c. i.



Draw a force diagram.

The mass of the trailer is unknown since it is being loaded with timber. Let the mass be x kg.

$$\begin{aligned}\text{So, } F &= xg \sin(5^\circ) \\ &= 0.0872xg \text{ (correct to 4 decimal places)}\end{aligned}$$

(1 mark)

$$\text{Now } F \leq \mu N$$

When $F = \mu N$, the trailer is on the point of rolling down the track.

$$\begin{aligned}\text{Now, } \mu N &= 0.307 \times xg \cos(5^\circ) \\ &= 0.3058xg \text{ (correct to 4 decimal places)}\end{aligned}$$

Since $0.0872xg < 0.3058xg$, the trailer is not at the point of rolling down the track.

(1 mark)

ii. $F \leq \mu N$

$$\begin{aligned}\mu &\geq \frac{F}{N} \\ &\geq \frac{xg \sin(5^\circ)}{xg \cos(5^\circ)} \\ &\geq 0.0874\ldots\end{aligned}$$

So $\mu = 0.087$ (to 3 decimal places).

(1 mark)**Total 9 marks****Total for Section 2 – 58 marks**