



Victorian Certificate of Education 2005

SPECIALIST MATHEMATICS

Written examination 1 (Facts, skills and applications)

Monday 31 October 2005

Reading time: 11.45 am to 12.00 noon (15 minutes)

Writing time: 12.00 noon to 1.30 pm (1 hour 30 minutes)

PART I MULTIPLE-CHOICE QUESTION BOOK

This examination has two parts: Part I (multiple-choice questions) and Part II (short-answer questions). Part I consists of this question book and must be answered on the answer sheet provided for multiple-choice questions.

Part II consists of a separate question and answer book.

You must complete **both** parts in the time allotted. When you have completed one part continue immediately to the other part.

Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
30	30	30

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, up to four pages (two A4 sheets) of pre-written notes (typed or handwritten), one approved graphics calculator (memory DOES NOT need to be cleared) and, if desired, one scientific calculator.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question book of 15 pages, with a detachable sheet of miscellaneous formulas in the centrefold and a blank page for rough working.
- Answer sheet for multiple-choice questions.

Instructions

- Detach the formula sheet from the centre of this book during reading time.
- Check that your **name** and **student number** as printed on your answer sheet for multiple-choice questions are correct, **and** sign your name in the space provided to verify this.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

At the end of the examination

- Place the answer sheet for multiple-choice questions (Part I) inside the front cover of the question and answer book (Part II).
- You may retain this question book.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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Instructions for Part I

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.

Choose the response that is **correct** for the question.

A correct answer scores 1, an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Take the **acceleration due to gravity** to have magnitude $g \text{ m/s}^2$, where $g = 9.8$.

Question 1

The maximum value of y reached by the ellipse with equation

$$\frac{3(x+3)^2}{5} + \frac{(y-4)^2}{6} = 3$$

is

- A. $-4 + 3\sqrt{2}$
- B. $4 + \sqrt{5}$
- C. $3\sqrt{2}$
- D. $4 + \sqrt{6}$
- E. $4 + 3\sqrt{2}$

Question 2

The graph of $f(x) = \frac{1}{x^2 + mx - n}$, where m and n are real constants, has no vertical asymptotes if

- A. $m^2 < 4n$
- B. $m^2 > 4n$
- C. $m^2 = -4n$
- D. $m^2 < -4n$
- E. $m^2 > -4n$

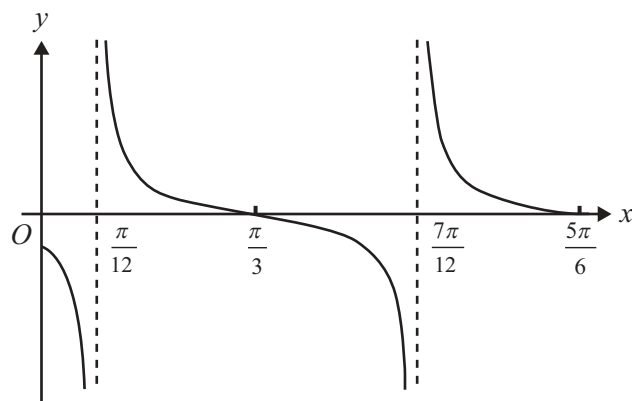
Question 3

The number of real solutions to $x^4 - x^3 = \operatorname{cosec}^2(x) - \cot^2(x)$ is

- A. 0
- B. 1
- C. 2
- D. 3
- E. 4

Question 4

Part of the graph of $y = f(x)$ is shown below.



$f(x)$ could be

- A. $y = -\tan\left(2x - \frac{\pi}{6}\right)$
- B. $y = -\tan\left(2x - \frac{\pi}{3}\right)$
- C. $y = \cot\left(2x - \frac{\pi}{12}\right)$
- D. $y = \cot\left(2x - \frac{\pi}{6}\right)$
- E. $y = \cot\left(2x + \frac{\pi}{6}\right)$

Question 5

If $y = \tan^{-1}(\sqrt{3x})$, then $\frac{dy}{dx}$ is equal to

- A. $\frac{1}{1+3x}$
- B. $\frac{\sqrt{3}}{1+3x}$
- C. $\frac{\sqrt{3}}{1+3x^2}$
- D. $\frac{1}{2\sqrt{3x}(1+3x)}$
- E. $\frac{\sqrt{3}}{2\sqrt{x}(1+3x)}$

Question 6

If $z = \frac{3-6i}{2+i}$, then $|z|$ and $\text{Arg}(z)$ are, respectively

- A. -3 and $\frac{\pi}{2}$
- B. 3 and $\frac{\pi}{2}$
- C. -3 and $\frac{3\pi}{2}$
- D. 3 and $-\frac{\pi}{2}$
- E. -3 and $-\frac{\pi}{2}$

Question 7

Let $u = 7 \text{cis}\left(\frac{\pi}{4}\right)$ and $v = a \text{cis}(b)$, where a and b are real constants.

If $uv = 42 \text{cis}\left(\frac{\pi}{20}\right)$, then

- A. $a = 6$ and $b = -\frac{\pi}{5}$
- B. $a = 35$ and $b = -\frac{\pi}{5}$
- C. $a = 6$ and $b = \frac{\pi}{5}$
- D. $a = 35$ and $b = \frac{1}{5}$
- E. $a = 6$ and $b = \frac{1}{5}$

Question 8

The value of the discriminant for the quadratic equation $(1+i)z^2 + 4iz - 2(1-i) = 0$ is

- A. -32
- B. -16
- C. 0
- D. 16
- E. 32

Question 9

$\sqrt{2} \operatorname{cis}\left(\frac{\pi}{16}\right)$ is one of the fourth roots of a complex number, z .

z^{-1} is equal to

- A. $\frac{1}{4} \operatorname{cis}\left(\frac{4}{\pi}\right)$
- B. $\frac{1}{4} \operatorname{cis}\left(-\frac{\pi}{4}\right)$
- C. $\frac{1}{4\sqrt{2}} \operatorname{cis}\left(-\frac{\pi}{4}\right)$
- D. $2^8 \operatorname{cis}\left(\frac{64}{\pi}\right)$
- E. $2^8 \operatorname{cis}\left(-\frac{\pi}{64}\right)$

Question 10

If $z \in \mathbb{C}$, which one of the following relations does **not** represent a circle on an Argand diagram?

- A. $z\bar{z} = 4$
- B. $|z - 1| + |z + 1| = 3$
- C. $(z - 3 + i)(\bar{z} - 3 - i) = 5$
- D. $3|z - 2 + i| = 7$
- E. $|z - 3| = 2$

Question 11

Which one of the following is an antiderivative of $\frac{6}{\sqrt{1-4x^2}}$ for $-\frac{1}{2} < x < \frac{1}{2}$?

- A. $3 \sin^{-1}(2x)$
- B. $6 \sin^{-1}(2x)$
- C. $3 \sin^{-1}\left(\frac{x}{2}\right)$
- D. $6 \sin^{-1}\left(\frac{x}{2}\right)$
- E. $12 \sin^{-1}\left(\frac{x}{2}\right)$

Question 12

With a suitable substitution, $\int_{\frac{\pi}{2}}^{\pi} \sin^2(2x) \sin(2x) dx$ can be expressed as

A. $\frac{1}{2} \int_{\frac{\pi}{2}}^{\pi} (1-u^2) du$

B. $\frac{1}{2} \int_{-1}^1 (1-u^2) du$

C. $-\frac{1}{2} \int_{-1}^1 (1-u^2) du$

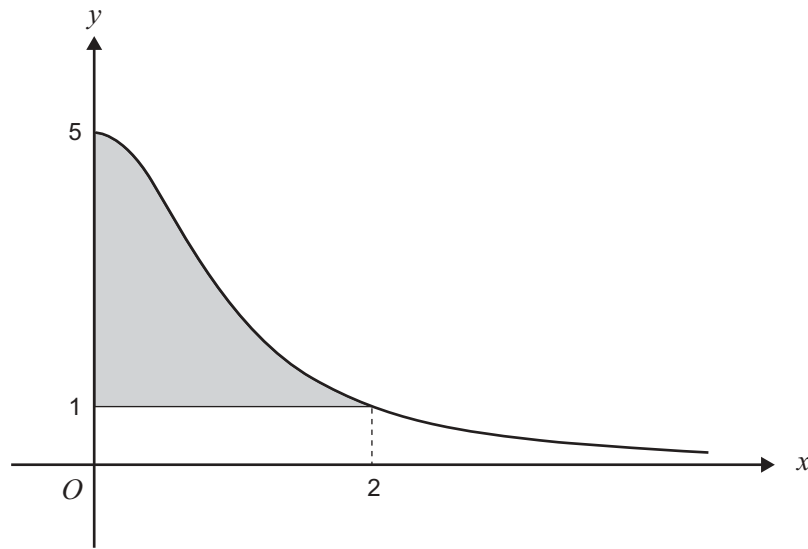
D. $-2 \int_{-1}^1 (1-u^2) du$

E. $-\frac{1}{2} \int_{\frac{\pi}{2}}^{\pi} (1-u^2) du$

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Question 13

The graph of $f: [0, \infty) \rightarrow \mathbb{R}$, where $f(x) = \frac{5}{x^2 + 1}$, is shown below. The shaded region is bounded by the graph of f , the y -axis and the line with equation $y = 1$.



The shaded region is rotated about the x -axis to form a solid of revolution.

The volume of the solid, in cubic units, is given by

- A. $\pi \int_0^2 \left(\left(\frac{5}{x^2 + 1} \right)^2 - 1 \right) dx$
- B. $\pi \int_0^2 \left(\frac{5}{x^2 + 1} - 1 \right)^2 dx$
- C. $\pi \int_0^2 \left(\frac{5}{x^2 + 1} - 1 \right) dx$
- D. $\pi \int_0^2 \left(\frac{5}{x^2 + 1} \right)^2 dx$
- E. $\pi \int_0^2 \left(\frac{4}{x^2 + 1} \right)^2 dx$

Question 14

The value, correct to three decimal places, of $\int_4^5 \left(\frac{x+3}{2\sin(x)} \right) dx$ is

- A. -4.014
- B. -3.523
- C. 3.094
- D. 3.523
- E. 4.014

Question 15

An antiderivative of $x\sqrt{3-x}$, for $x < 3$, is

- A. $-\frac{2x}{3}(3-x)^{\frac{3}{2}}$
- B. $-\frac{x^2(3-x)^{\frac{3}{2}}}{3}$
- C. $2(3-x)^{\frac{3}{2}} - \frac{2}{5}(3-x)^{\frac{5}{2}}$
- D. $-2(3-x)^{\frac{3}{2}} + \frac{2}{5}(3-x)^{\frac{5}{2}}$
- E. $-2(3-x)^{\frac{3}{2}} - \frac{2}{5}(3-x)^{\frac{5}{2}}$

Question 16

$\int_0^{\frac{\pi}{8}} \sec^2(2x)e^{2\tan(2x)} dx$ is equal to

- A. $\frac{1}{4}\left(e^{\frac{\pi}{4}} - 1\right)$
- B. $\frac{1}{2}\left(e^{\frac{\pi}{4}} - 1\right)$
- C. $\frac{1}{4}\left(e^{\frac{\pi}{8}} - 1\right)$
- D. $\frac{1}{2}(e^2 - 1)$
- E. $\frac{1}{4}(e^2 - 1)$

Question 17

Euler's method, with a step size of 0.1, is used to solve the differential equation

$$\frac{dy}{dx} = e^{-x}, \text{ with initial condition } y = 1 \text{ at } x = 2.$$

When $x = 2.2$, the value obtained for y , correct to four decimal places, is

- A. 1.0122
- B. 1.0222
- C. 1.0233
- D. 1.0258
- E. 1.0271

Question 18

An oil slick is in the shape of a circle. Its surface area is increasing at a rate of $10 \text{ m}^2/\text{s}$.

Let r metres be the radius of the oil slick at time t seconds.

The rate of increase of r , in m/s , is given by

A. $\frac{5}{\pi r}$

B. $\frac{20}{\pi r}$

C. $\frac{10}{\pi r^2}$

D. $\frac{1}{20\pi r}$

E. $20\pi r$

Question 19

Given that $\frac{dy}{dx} = y^2 + 1$, and that $y = 1$ at $x = 0$, then

A. $y = \tan\left(x - \frac{\pi}{4}\right)$

B. $y = \tan\left(x + \frac{\pi}{4}\right)$

C. $x = \log_e\left(\frac{y^2 + 1}{2}\right)$

D. $y = \frac{1}{3}y^3 + y - \frac{1}{3}$

E. $y = y^2x + x + 1$

Question 20

A particle travels in a straight line with velocity v at time t .

If the velocity of the particle is given by $v = \frac{2}{\sqrt{1-x^2}}$, for $0 < x < 1$, then the acceleration is given by

- A. $2 \sin^{-1}(x)$
- B. $\frac{4 \sin^{-1}(x)}{\sqrt{1-x^2}}$
- C. $\frac{4x}{(1-x^2)^2}$
- D. $\frac{2x}{(1-x^2)^2}$
- E. $\frac{2x}{(1-x^2)^{\frac{3}{2}}}$

Question 21

A particle travelling in a straight line has velocity v m/s at time t s. Its acceleration is given by $\frac{dv}{dt} = \frac{3}{v^2 - 9}$. The time taken, in seconds, for the velocity to decrease from 2 m/s to 1 m/s is given by

- A. $\int_1^2 \frac{v^2 - 9}{3} dv$
- B. $\int_2^1 \frac{v^2 - 9}{3} dv$
- C. $\int_2^1 \frac{3}{v^2 - 9} dt$
- D. $\int_2^1 \frac{3}{v^2 - 9} dv$
- E. $\int_1^2 \frac{3}{v^2 - 9} dv$

Question 22

$PQRS$ is a parallelogram. The position vectors of P , Q , R and S are, respectively,

$$\underline{p} = -3\underline{k}, \quad \underline{q} = \underline{i} + y\underline{j}, \quad \underline{r} = 5\underline{i} + 2x\underline{j} + \underline{k} \quad \text{and} \quad \underline{s} = y\underline{i} - 2\underline{k}.$$

The values of x and y are

- A. $x = 0, \quad y = 5$
- B. $x = 2, \quad y = 4$
- C. $x = 3, \quad y = 6$
- D. $x = 8, \quad y = 4$
- E. $x = 12, \quad y = 6$

Question 23

Point P has coordinates $(2, 2, -1)$ and point Q has coordinates $(-4, 0, -3)$.

The cosine of angle POQ is equal to

- A. $-\frac{1}{5}$
- B. $\frac{1}{3}$
- C. $-\frac{1}{3}$
- D. $\frac{11}{15}$
- E. $-\frac{11}{15}$

Question 24

The path of a particle satisfies the Cartesian equation $(x+1)^2 + y^2 = 1$.

The position vector of the particle at time t , $t \geq 0$, could be given by

- A. $\cos(t)\mathbf{i} + \sin(t)\mathbf{j}$
- B. $\cos(t)\mathbf{i} + (\sin(t) - 1)\mathbf{j}$
- C. $(\cos(t) + 1)\mathbf{i} + \sin(t)\mathbf{j}$
- D. $(\sin(t) + 1)\mathbf{i} + \cos(t)\mathbf{j}$
- E. $(\sin(t) - 1)\mathbf{i} + \cos(t)\mathbf{j}$

Question 25

The position vector of a particle at time t is given by

$$\mathbf{r}(t) = (3t^2 - 2)\mathbf{i} - (7 - 5t)\mathbf{j} - 4t\mathbf{k}.$$

The velocity vector of the particle at time t is given by

- A. $6t\mathbf{i} + 5\mathbf{j}$
- B. $6t\mathbf{i} - 5\mathbf{j}$
- C. $6t\mathbf{i} - 5\mathbf{j} - 4\mathbf{k}$
- D. $(6t - 2)\mathbf{i} - 2\mathbf{j} - 4\mathbf{k}$
- E. $(t^3 - 2t)\mathbf{i} - (7t - \frac{5}{2}t^2)\mathbf{j} - 4t\mathbf{k}$

Question 26

A particle initially at the origin starts from rest at $t = 0$. The particle moves in a straight line in such a way that its acceleration at time t is given by $e^{-0.1t} \mathbf{i} + (6t) \mathbf{j}$.

The velocity of the particle at time t is given by

- A. $-(0.1e^{-0.1t}) \mathbf{i} + 6 \mathbf{j}$
- B. $-(10e^{-0.1t}) \mathbf{i} + (3t^2) \mathbf{j}$
- C. $10(1 - e^{-0.1t}) \mathbf{i} + (3t^2) \mathbf{j}$
- D. $0.1(1 - e^{-0.1t}) \mathbf{i} + (3t^2) \mathbf{j}$
- E. $10(10 - t - 10e^{-0.1t}) \mathbf{i} + (t^3) \mathbf{j}$

Question 27

A body of mass 5 kg is acted on by a force of variable magnitude. The acceleration, $a \text{ m/s}^2$, of the body at time $t \text{ s}$, $t \geq 0$, is given by $a = 20 - 10 \cos(2t)$.

The maximum value of the magnitude of the force, measured in newtons, is

- A. 20
- B. 30
- C. 50
- D. 100
- E. 150

Question 28

A body of mass 10 kg, at rest on a rough horizontal surface, is acted on by a force of magnitude 14 N acting at an angle of 30° to the horizontal.

If the body is on the point of moving, the coefficient of friction between the body and the surface is equal to

- A. $\frac{\sqrt{3}}{13}$
- B. $\frac{\sqrt{3}}{14}$
- C. $\frac{\sqrt{3}}{15}$
- D. $\frac{7}{\sqrt{3}}$
- E. $\frac{1}{14 - \sqrt{3}}$

Question 29

A particle is held in equilibrium by three concurrent coplanar forces $\underline{P}$, $\underline{Q}$ and $\underline{R}$.

$\underline{P}$ has magnitude 5 newtons and acts in the east direction. $\underline{Q}$ has magnitude 5 newtons and acts in the north direction.

The magnitude and direction of $\underline{R}$ are, respectively

- A. 5 newtons, southeast
- B. $5\sqrt{2}$ newtons, northeast
- C. $5\sqrt{2}$ newtons, southwest
- D. 10 newtons, southwest
- E. 10 newtons, northeast

Question 30

A body of mass 200 kg is lowered vertically by a crane. The crane cable exerts a constant force of 1000 N vertically upwards on the body.

The magnitude of the acceleration of the body, in m/s^2 , is

- A. 0.49
- B. 4.8
- C. 5.0
- D. 9.8
- E. 14.8



Victorian Certificate of Education

2005

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

STUDENT NUMBER

Letter

Figures

Words

SPECIALIST MATHEMATICS

Written examination 1 (Facts, skills and applications)

Monday 31 October 2005

Reading time: 11.45 am to 12.00 noon (15 minutes)

Writing time: 12.00 noon to 1.30 pm (1 hour 30 minutes)

PART II QUESTION AND ANSWER BOOK

This examination has two parts: Part I (multiple-choice questions) and Part II (short-answer questions). Part I consists of a separate question book and must be answered on the answer sheet provided for multiple-choice questions.

Part II consists of this question and answer book.

You must complete **both** parts in the time allotted. When you have completed one part continue immediately to the other part.

Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
5	5	20

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, up to four pages (two A4 sheets) of pre-written notes (typed or handwritten), one approved graphics calculator (memory DOES NOT need to be cleared) and, if desired, one scientific calculator.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 9 pages.

Instructions

- Detach the formula sheet from the centre of the Part I book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

At the end of the examination

- Place the answer sheet for multiple-choice questions (Part I) inside the front cover of this question and answer book (Part II).

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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Instructions for Part II

Answer **all** questions in the spaces provided.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working must be shown.

Where an instruction to **use calculus** is stated for a question, you must show an appropriate derivative or antiderivative.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude $g \text{ m/s}^2$, where $g = 9.8$.

Question 1

Consider the hyperbola with equation

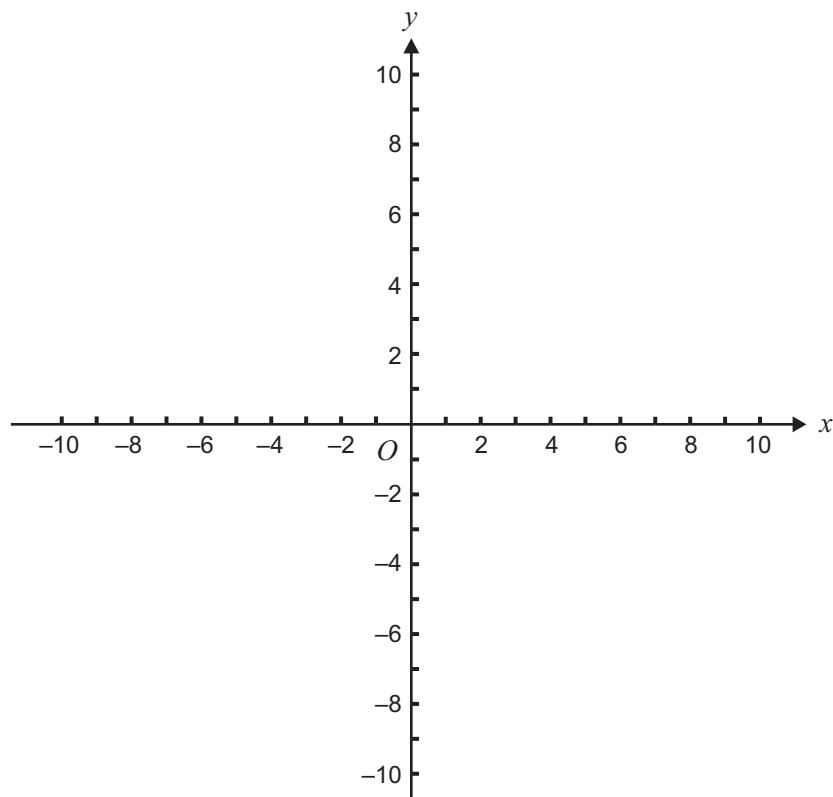
$$\frac{(x-c)^2}{9} - \frac{(y-3)^2}{4} = 1, \text{ where } c \text{ is a real constant.}$$

The equation of one of the asymptotes of this hyperbola is $y = \frac{2}{3}x + 5$.

- a. Show that $c = -3$.

1 mark

- b. Sketch the hyperbola on the following set of axes, clearly showing the asymptotes.



3 marks

$y = e^{2x}\cos(x)$ is a solution of the differential equation

where $k \in R$.

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PART II – continued
TURN OVER

Question 3

The vector resolute of $\underline{u}$ in the direction of $\underline{v}$ is $3\underline{i} - 2\underline{j} + \underline{k}$.

The vector resolute of $\underline{u}$ perpendicular to $\underline{v}$ is $2\underline{i} + x\underline{j} + 2\underline{k}$.

a. Show that $x = 4$.

1 mark

b. Hence find $\underline{u}$.

2 marks

Question 4

At time $t = 0$, cyclist A, travelling at a speed of 6 m/s along a straight bicycle path, passes cyclist B who is stationary.

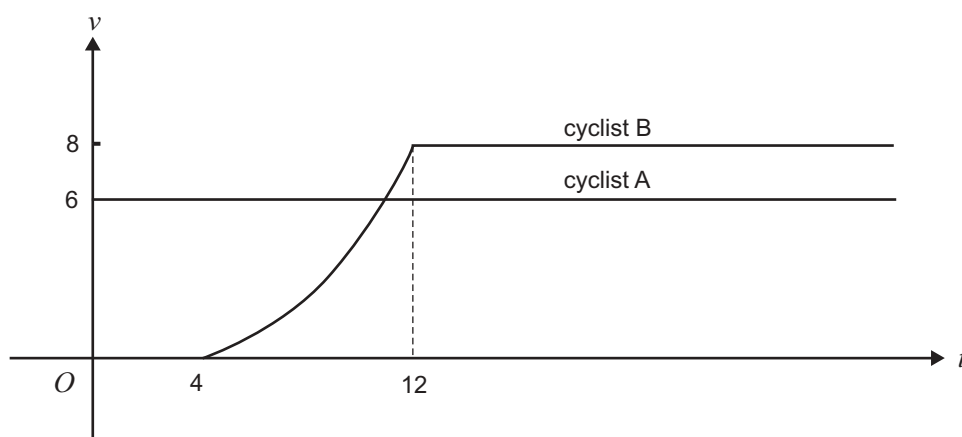
Four seconds later, at $t = 4$, cyclist B accelerates in the direction of cyclist A for 8 seconds in such a way that

her speed, v m/s, is given by $v = (t - 4) \tan\left(\frac{\pi}{48}t\right)$.

- a. Show that cyclist B accelerates to a speed of 8 m/s.

1 mark

Cyclist B then maintains her speed of 8 m/s. The velocity-time graph that represents this situation is shown below.



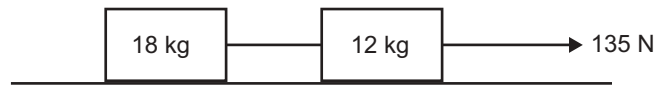
- b. Find the time at which cyclist B passes cyclist A, correct to the nearest tenth of a second.

3 marks

Question 5

Two boxes made of the same material have masses 12 kg and 18 kg. They are joined by a light rope as shown in the diagram.

The boxes are pulled along a rough horizontal plane with an acceleration of 0.5 m/s^2 by a horizontal force of magnitude 135 N.



- a. Show that the coefficient of friction between the boxes and the plane is 0.41, correct to two decimal places.

3 marks

- b.** Find the magnitude of the tension in the light rope that joins the two boxes.

2 marks

SPECIALIST MATHEMATICS

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

Specialist Mathematics Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder:	$2\pi rh$
volume of a cylinder:	$\pi r^2 h$
volume of a cone:	$\frac{1}{3}\pi r^2 h$
volume of a pyramid:	$\frac{1}{3}Ah$
volume of a sphere:	$\frac{4}{3}\pi r^3$
area of a triangle:	$\frac{1}{2}bc \sin A$
sine rule:	$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
cosine rule:	$c^2 = a^2 + b^2 - 2ab \cos C$

Coordinate geometry

ellipse:	$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$	hyperbola:	$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$
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Circular (trigometric) functions

$$\cos^2(x) + \sin^2(x) = 1$$

$$1 + \tan^2(x) = \sec^2(x)$$

$$\sin(x+y) = \sin(x) \cos(y) + \cos(x) \sin(y)$$

$$\cos(x+y) = \cos(x) \cos(y) - \sin(x) \sin(y)$$

$$\tan(x+y) = \frac{\tan(x) + \tan(y)}{1 - \tan(x) \tan(y)}$$

$$\cos(2x) = \cos^2(x) - \sin^2(x) = 2 \cos^2(x) - 1 = 1 - 2 \sin^2(x)$$

$$\sin(2x) = 2 \sin(x) \cos(x)$$

$$\cot^2(x) + 1 = \operatorname{cosec}^2(x)$$

$$\sin(x-y) = \sin(x) \cos(y) - \cos(x) \sin(y)$$

$$\cos(x-y) = \cos(x) \cos(y) + \sin(x) \sin(y)$$

$$\tan(x-y) = \frac{\tan(x) - \tan(y)}{1 + \tan(x) \tan(y)}$$

$$\tan(2x) = \frac{2 \tan(x)}{1 - \tan^2(x)}$$

function	Sin^{-1}	Cos^{-1}	Tan^{-1}
domain	$[-1, 1]$	$[-1, 1]$	R
range	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Algebra (Complex numbers)

$$z = x + yi = r(\cos \theta + i \sin \theta) = r \operatorname{cis} \theta$$

$$|z| = \sqrt{x^2 + y^2} = r$$

$$z_1 z_2 = r_1 r_2 \operatorname{cis}(\theta_1 + \theta_2)$$

$$z^n = r^n \operatorname{cis}(n\theta) \text{ (de Moivre's theorem)}$$

$$-\pi < \operatorname{Arg} z \leq \pi$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} \operatorname{cis}(\theta_1 - \theta_2)$$

Calculus

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\int x^n dx = \frac{1}{n+1} x^{n+1} + c, n \neq -1$$

$$\frac{d}{dx}(e^{ax}) = ae^{ax}$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax} + c$$

$$\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$$

$$\int \frac{1}{x} dx = \log_e(x) + c, \text{ for } x > 0$$

$$\frac{d}{dx}(\sin(ax)) = a \cos(ax)$$

$$\int \sin(ax) dx = -\frac{1}{a} \cos(ax) + c$$

$$\frac{d}{dx}(\cos(ax)) = -a \sin(ax)$$

$$\int \cos(ax) dx = \frac{1}{a} \sin(ax) + c$$

$$\frac{d}{dx}(\tan(ax)) = a \sec^2(ax)$$

$$\int \sec^2(ax) dx = \frac{1}{a} \tan(ax) + c$$

$$\frac{d}{dx}(\sin^{-1}(x)) = \frac{1}{\sqrt{1-x^2}}$$

$$\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c, a > 0$$

$$\frac{d}{dx}(\cos^{-1}(x)) = \frac{-1}{\sqrt{1-x^2}}$$

$$\int \frac{-1}{\sqrt{a^2-x^2}} dx = \cos^{-1}\left(\frac{x}{a}\right) + c, a > 0$$

$$\frac{d}{dx}(\tan^{-1}(x)) = \frac{1}{1+x^2}$$

$$\int \frac{a}{a^2+x^2} dx = \tan^{-1}\left(\frac{x}{a}\right) + c$$

product rule:

$$\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$$

quotient rule:

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

chain rule:

$$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$$

mid-point rule:

$$\int_a^b f(x) dx \approx (b-a) f\left(\frac{a+b}{2}\right)$$

trapezoidal rule:

$$\int_a^b f(x) dx \approx \frac{1}{2}(b-a)(f(a) + f(b))$$

Euler's method:

$$\text{If } \frac{dy}{dx} = f(x), x_0 = a \text{ and } y_0 = b, \text{ then } x_{n+1} = x_n + h \text{ and } y_{n+1} = y_n + hf(x_n)$$

acceleration:

$$a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v \frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

$$\text{constant (uniform) acceleration: } v = u + at \quad s = ut + \frac{1}{2}at^2 \quad v^2 = u^2 + 2as \quad s = \frac{1}{2}(u+v)t$$

TURN OVER

Vectors in two and three dimensions

$$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$$

$$|\vec{r}| = \sqrt{x^2 + y^2 + z^2} = r$$

$$\dot{\vec{r}} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\vec{i} + \frac{dy}{dt}\vec{j} + \frac{dz}{dt}\vec{k}$$

$$\vec{r}_1 \cdot \vec{r}_2 = r_1 r_2 \cos \theta = x_1 x_2 + y_1 y_2 + z_1 z_2$$

Mechanics

momentum:

$$\vec{p} = m\vec{v}$$

equation of motion:

$$\vec{R} = m\vec{a}$$

friction:

$$F \leq \mu N$$