

Part I

1	2	3	4	5	6	7	8	9	10
E	D	C	D	E	D	A	C	B	B

11	12	13	14	15	16	17	18	19	20
A	C	A	A	D	E	D	A	B	C

21	22	23	24	25	26	27	28	29	30
B	B	C	E	A	?	E	AC	C	B

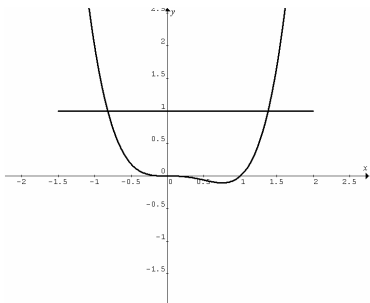
Q1 Max (or min) occurs at the vertical axis of symmetry $x = -3$,
where $\frac{(y-4)^2}{6} = 3$. $\therefore y-4 = \pm\sqrt{18}$ or $y = 4 \pm 3\sqrt{2}$.

$\therefore$ Max value $= 4 + 3\sqrt{2}$. E

Q2 No vertical asymptotes $\rightarrow$ no linear factors $\rightarrow \Delta < 0$.
 $\therefore m^2 - 4(1)(-n) < 0$, i.e. $m^2 < -4n$. D

Q3 Since $\cos^2(x) - \cot^2(x) = 1$, $\therefore x^4 - x^3 = 1$.

Graph $y = x^4 - x^3$ and $y = 1$. Only two intersections. C



Q4 At $x = \frac{\pi}{3}$, $y = 0$. Only D satisfies this requirement. Use

$\cot \theta = \frac{\cos \theta}{\sin \theta}$, not $\cot \theta = \frac{1}{\tan \theta}$, to evaluate. D

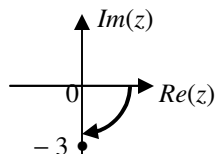
Q5 Use the chain rule. Let $u = \sqrt{3x}$, $\therefore y = \tan^{-1}(u)$.

$$\frac{du}{dx} = \frac{1}{2\sqrt{3x}} \times 3 = \frac{3}{2\sqrt{3x}}, \quad \frac{dy}{du} = \frac{1}{1+u^2} = \frac{1}{1+3x}.$$

$$\therefore \frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = \frac{3}{2\sqrt{3x}(1+3x)} = \frac{\sqrt{3}}{2\sqrt{x}(1+3x)}. \quad E$$

$$Q6 \quad z = \frac{(3-6i)(2-i)}{(2+i)(2-i)} = \frac{-15i}{5} = -3i.$$

$$\therefore |z| = 3, \quad \text{Arg}(z) = -\frac{\pi}{2}. \quad D$$



$$Q7 \quad \left[7\text{cis}\left(\frac{\pi}{4}\right) \right] [\text{acis}(b)] = 42\text{cis}\left(\frac{\pi}{20}\right),$$

$$\therefore 7\text{acis}\left(\frac{\pi}{4} + b\right) = 42\text{cis}\left(\frac{\pi}{20}\right). \text{ Hence } 7a = 42, \quad a = 6;$$

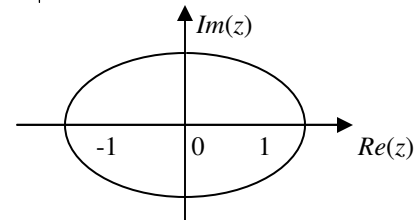
$$\frac{\pi}{4} + b = \frac{\pi}{20}, \quad b = -\frac{\pi}{5}. \quad A$$

$$Q8 \quad \Delta = (4i)^2 - 4(1+i)(-2(1-i)) = -16 + 8(1+i)(1-i) = 0 \quad C$$

$$Q9 \quad z^{\frac{1}{4}} = \sqrt{2}\text{cis}\left(\frac{\pi}{16}\right), \quad z = \left(\sqrt{2}\text{cis}\left(\frac{\pi}{16}\right)\right)^4 = (\sqrt{2})^4 \text{cis}\left(4 \times \frac{\pi}{16}\right),$$

$$\text{i.e. } z = 4\text{cis}\left(\frac{\pi}{4}\right). \text{ Hence } z^{-1} = 4^{-1}\text{cis}\left(-\frac{\pi}{4}\right) = \frac{1}{4}\text{cis}\left(-\frac{\pi}{4}\right). \quad B$$

Q10 $|z-1| + |z+1| = 3$ represents an ellipse on an Argand diagram.



B

$$Q11 \quad \int \frac{6}{\sqrt{1-4x^2}} dx = 6 \int \frac{1}{\sqrt{1-(2x)^2}} dx = \frac{6}{2} \sin^{-1}(2x) + C$$

$$= 3\sin^{-1}(2x) + C. \quad A$$

Q12 Change $\sin^2(2x)$ to $1 - \cos^2(2x)$, and let $u = \cos(2x)$, then

$$\frac{du}{dx} = -2\sin(2x) \text{ or } \sin(2x) = -\frac{1}{2} \frac{du}{dx}.$$

$$\text{When } x = \frac{\pi}{2}, \quad u = \cos\left(2 \times \frac{\pi}{2}\right) = -1.$$

$$\text{When } x = \pi, \quad u = \cos(2\pi) = 1.$$

$$\int_{\frac{\pi}{2}}^{\pi} \sin^2(2x) \sin(2x) dx = \int_{-1}^1 (1 - \cos^2(2x)) \sin(2x) dx$$

$$= -\frac{1}{2} \int_{-1}^1 (1 - u^2) \frac{du}{dx} dx = -\frac{1}{2} \int_{-1}^1 (1 - u^2) du. \quad C$$

$$Q13 \quad \int_0^2 \pi R^2 dx - \int_0^2 \pi r^2 dx = \pi \int_0^2 \left(\frac{5}{x^2 + 1} \right)^2 dx - \pi \int_0^2 1^2 dx$$

$$= \pi \int_0^2 \left(\left(\frac{5}{x^2 + 1} \right)^2 - 1 \right) dx \quad A$$

Q14 Graphics calculator: Graph $y = \frac{x+3}{2\sin(x)}$ and calc $\int dx$ from 4 to 5 to obtain -4.014 A

Q15 Linear substitution: $u = 3 - x$, $x = 3 - u$, $\frac{du}{dx} = -1$ or

$$-\frac{du}{dx} = 1.$$

$$\int (x\sqrt{3-x})dx = \int -(3-u)\sqrt{u} \frac{du}{dx} dx = \int \left(-3u^{\frac{1}{2}} + u^{\frac{3}{2}}\right) du$$

$$= -\frac{3u^{\frac{3}{2}}}{\frac{3}{2}} + \frac{u^{\frac{5}{2}}}{\frac{5}{2}} + C = -2(3-x)^{\frac{3}{2}} + \frac{2}{5}(3-x)^{\frac{5}{2}} + C \quad \text{D}$$

Q16 Substitution: $u = 2 \tan(2x)$, $\frac{du}{dx} = 4 \sec^2(2x)$ or

$$\frac{1}{4} \frac{du}{dx} = \sec^2(2x). \text{ When } x = 0, u = 2 \tan(0) = 0. \text{ When } x = \frac{\pi}{8},$$

$$u = 2 \tan\left(2 \times \frac{\pi}{8}\right) = 2.$$

$$\int_0^{\frac{\pi}{8}} \sec^2(2x) e^{2 \tan(2x)} dx = \int_0^2 \frac{1}{4} e^u \frac{du}{dx} dx = \int_0^2 \frac{1}{4} e^u du$$

$$= \left[\frac{1}{4} e^u \right]_0^2 = \frac{1}{4} e^2 - \frac{1}{4} e^0 = \frac{1}{4} (e^2 - 1). \quad \text{E}$$

Q17 $y_{\text{new}} \approx y_{\text{old}} + h y'_{\text{old}}$ where $y' = e^{-x}$ and $h = 0.1$.

$$x = 2, \quad y = 1$$

$$x = 2.1, \quad y = 1 + 0.1e^{-2} = 1.01353$$

$$x = 2.2, \quad y = 1.01353 + 0.1e^{-2.1} = 1.0258 \quad \text{D}$$

Q18 $A = \pi r^2$, $\frac{dA}{dr} = 2\pi r$. The rate of change of A is related to

$$\text{the rate of change of } r \text{ by } \frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}. \therefore 10 = 2\pi r \frac{dr}{dt},$$

$$\therefore \frac{dr}{dt} = \frac{5}{\pi r}. \quad \text{A}$$

$$\text{Q19 } \frac{dy}{dx} = y^2 + 1, \quad \frac{dx}{dy} = \frac{1}{y^2 + 1} = \frac{1}{y^2 + 1}, \quad x = \int \frac{1}{1 + y^2} dy,$$

$$x = \tan^{-1}(y) + C. \text{ At } x = 0, y = 1. \therefore 0 = \tan^{-1}(1) + C,$$

$$C = -\frac{\pi}{4}. \text{ Hence } \tan^{-1}(y) = x + \frac{\pi}{4} \text{ or } y = \tan\left(x + \frac{\pi}{4}\right). \quad \text{B}$$

$$\text{Q20 } v = \frac{2}{\sqrt{1-x^2}}, \therefore v^2 = \frac{4}{1-x^2}. \quad a = \frac{d}{dx} \left(\frac{1}{2} v^2 \right) = \frac{d}{dx} \left(\frac{2}{1-x^2} \right)$$

$$= -\frac{2}{(1-x^2)^2} \times -2x = \frac{4x}{(1-x^2)^2}. \quad \text{C}$$

$$\text{Q21 } \frac{dv}{dt} = \frac{3}{v^2 - 9}, \quad \frac{dt}{dv} = \frac{1}{\frac{dv}{dt}} = \frac{v^2 - 9}{3}. \therefore t = \int \frac{v^2 - 9}{3} dv.$$

$$\text{From } v = 2 \text{ (initial) to } v = 1 \text{ (final), } \Delta t = \int_2^1 \frac{v^2 - 9}{3} dv.$$

Check: Δt has a positive value.

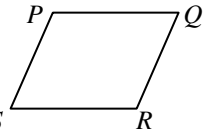
B

$$\text{Q22 } \overrightarrow{PQ} = \underline{q} - \underline{p} = \underline{i} + y\underline{j} + 3\underline{k}$$

$$\overrightarrow{SR} = \underline{r} - \underline{s} = (5-y)\underline{i} + 2x\underline{j} + 3\underline{k}$$

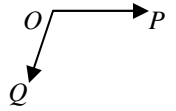
$PQRS$ is a parallelogram, $\therefore \overrightarrow{PQ} = \overrightarrow{SR}$.

$$\therefore 5-y=1, \text{ i.e. } y=4 \text{ and } 2x=y, \text{ i.e. } x=2. \quad \text{B}$$



$$\text{Q23 } \overrightarrow{OP} = 2\underline{i} + 2\underline{j} - \underline{k} \text{ and } \overrightarrow{OQ} = -4\underline{i} - 3\underline{k}.$$

$$\cos \angle POQ = \frac{\overrightarrow{OP} \cdot \overrightarrow{OQ}}{|\overrightarrow{OP}| |\overrightarrow{OQ}|} = \frac{-5}{3 \times 5} = -\frac{1}{3}. \quad \text{C}$$



Q24 Since $\sin^2 t + \cos^2 t = 1$, $\therefore$ **either** $(x+1)^2 = \sin^2 t$ and

$$y^2 = \cos^2 t \text{ **or** } (x+1)^2 = \cos^2 t \text{ and } y^2 = \sin^2 t.$$

The possibilities are:

$$x+1 = -\sin t \text{ and } y = \cos t$$

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$x+1 = \cos t$ and $y = -\sin t$. Only the second possibility leads to choice E. E

$$\text{Q25 } \underline{v} = \underline{\dot{r}} = 6t\underline{i} + 5\underline{j}. \quad \text{A}$$

Q26 Note: Since the particle moves in a straight line (given information), $\therefore$ the direction of its velocity vector must be constant until it moves backwards (if it does). None of the choices meets this requirement.

Q27 $R = ma = 5(20 - 10 \cos(2t))$, max R occurs when

$$\cos(2t) = -1, \therefore R_{\text{max}} = 5(30) = 150 \quad \text{E}$$

Q28 There are two possibilities:

$$\text{Case 1. } N + 14 \sin 30^\circ = 10g = 98, \therefore N = 91$$

$$14 \cos 30^\circ = \mu N = 91\mu, \therefore \mu = \frac{\sqrt{3}}{13}.$$

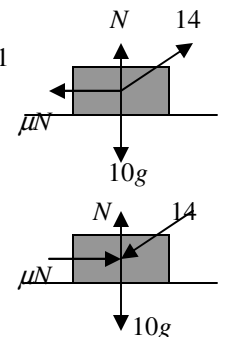
$$\text{Case 2. } N = 10g + 14 \sin 30^\circ, \therefore N = 105$$

$$14 \cos 30^\circ = \mu N = 105\mu, \therefore \mu = \frac{\sqrt{3}}{15}.$$

A, C

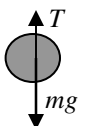
$$\text{Q29 } \underline{P} + \underline{Q} + \underline{R} = \underline{0}, \therefore \underline{R} = -\underline{P} - \underline{Q}$$

$$\therefore \underline{R} = 5\sqrt{2} \text{ SW} \quad \text{C}$$



$$\text{Q30 } a = \frac{R}{m} = \frac{mg - T}{m} = g - \frac{T}{m} = 9.8 - \frac{1000}{200} = 4.8$$

B



Part II

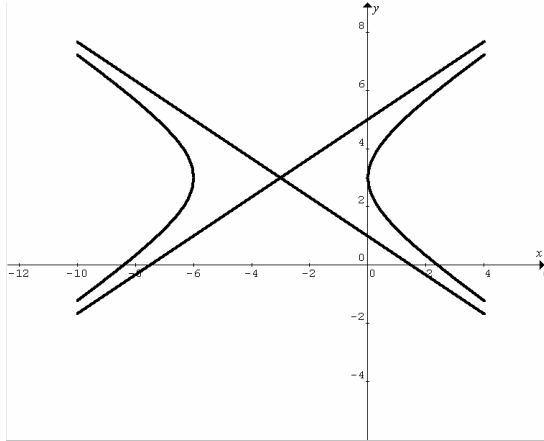
Q1a Equation of asymptote $y - k = \frac{b}{a}(x - h)$,

$$y - 3 = \frac{2}{3}(x - c),$$

$$y - 3 = \frac{2}{3}x - \frac{2c}{3}. \text{ Given } y = \frac{2}{3}x + 5, \text{ i.e. } y - 3 = \frac{2}{3}x + 2.$$

$$\therefore -\frac{2c}{3} = 2 \text{ or } c = -3.$$

Q1b



Q2 $y = e^{2x} \cos(x), \frac{dy}{dx} = 2e^{2x} \cos(x) - e^{2x} \sin(x),$

$$\frac{d^2y}{dx^2} = 2(2e^{2x} \cos(x) - e^{2x} \sin(x)) - (2e^{2x} \sin(x) + e^{2x} \cos(x))$$

$$= 3e^{2x} \cos(x) - 4e^{2x} \sin(x).$$

$$\therefore 3e^{2x} \cos(x) - 4e^{2x} \sin(x) + k(2e^{2x} \cos(x) - e^{2x} \sin(x)) + e^{2x} \cos(x)$$

$$= -2e^{2x} \sin(x).$$

$$\therefore 3 + 2k + 1 = 0 \text{ and } -4 - k = -2, \therefore k = -2.$$

Q3a The two resolutes are perpendicular,

$$\therefore (3\underline{i} - 2\underline{j} + \underline{k}) \bullet (2\underline{i} + x\underline{j} + 2\underline{k}) = 0, \therefore 6 - 2x + 2 = 0, x = 4.$$

Q3b $\underline{u} = (3\underline{i} - 2\underline{j} + \underline{k}) + (2\underline{i} + 4\underline{j} + 2\underline{k}) = 5\underline{i} + 2\underline{j} + 3\underline{k}.$

Q4a When $t = 12$ (not 8), $v = 8 \tan\left(\frac{\pi}{4}\right) = 8.$

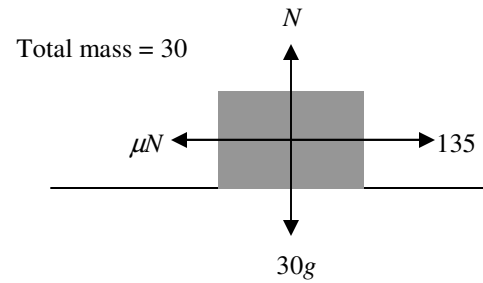
Q4b Equate displacements (area under each graph) of A and B.
Let T ($T > 12$) be the time B passes A.

$$\int_4^{12} (t - 4) \tan\left(\frac{\pi}{48}t\right) dt + 8(T - 12) = 6T. \text{ Use graphics calculator to}$$

evaluate the definite integral $= 22.89.$

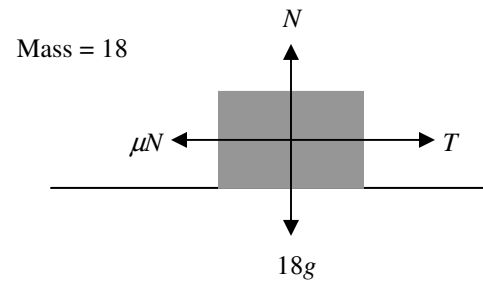
$$\therefore 22.89 + 8(T - 12) = 6T, \therefore T = 36.6 \text{ s.}$$

Q5a



$$N = 30g, R = ma, \therefore 135 - \mu(30g) = 30(0.5), \therefore \mu = 0.41.$$

Q5b



Let T be the tension in the rope.

$$N = 18g, R = ma, \therefore T - 0.41(18g) = 18(0.5),$$

$$\therefore T = 81.3 \text{ newtons.}$$

Please inform mathline@itute.com re conceptual, mathematical and/or typing errors