

Student Name: _____

Specialist Mathematics

Written examination 2



2005 Trial Examination

Reading Time: 15 minutes

Writing Time: 1 Hour and 30 minutes

QUESTION BOOK

Structure of Book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
5	5	60

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, and rulers, up to 4 pages (2 A4 sheets) of pre written notes and an approved graphics calculator and/or scientific calculator.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question book of 14 pages.
- Formula Sheet.

Instructions

- Print your name in the space provided on the top of this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other electronic communication devices into the examination room.

Specific Instructions

A decimal approximation will not be accepted if the question specifically asks for an exact answer. In questions worth more than one mark, appropriate working must be shown. The diagrams are not drawn to scale. Marks are given as specified for each question.

Question 1

a. Use the fact that $\frac{5\pi}{12} = \frac{\pi}{6} + \frac{\pi}{4}$, show that

i. $\sin\left(\frac{5\pi}{12}\right) = \frac{\sqrt{2}(\sqrt{3}+1)}{4}$

ii. $\cos\left(\frac{5\pi}{12}\right) = \frac{\sqrt{2}(\sqrt{3}-1)}{4}$

2 + 2 = 4 marks

Let $u = \text{cis}\left(\frac{5\pi}{12}\right) = \cos\left(\frac{5\pi}{12}\right) + i \sin\left(\frac{5\pi}{12}\right)$, and let v be the complex conjugate of u .

b. Find the **exact** values of $\text{Arg}(u - v)$ and $\text{Arg}(u + v)$.

3 marks

c. Find the **exact** value of $\text{Arg}(uv)$ and $\text{Arg}\left(\frac{u}{v}\right)$.

2 marks

d. Find a quadratic polynomial in the complex number z whose roots are u and v .

3 marks
Total 12 marks

Question 2

Consider the function $f : [0,1] \rightarrow \mathbb{R}$ where $f(x) = \sin\left(\frac{\pi x}{2}\right)$.

- a. Show that $x = 0$ and $x = 1$ are solutions to the equation $f(x) = x$.

2 marks

- b. State the domain and range of f .

1 mark

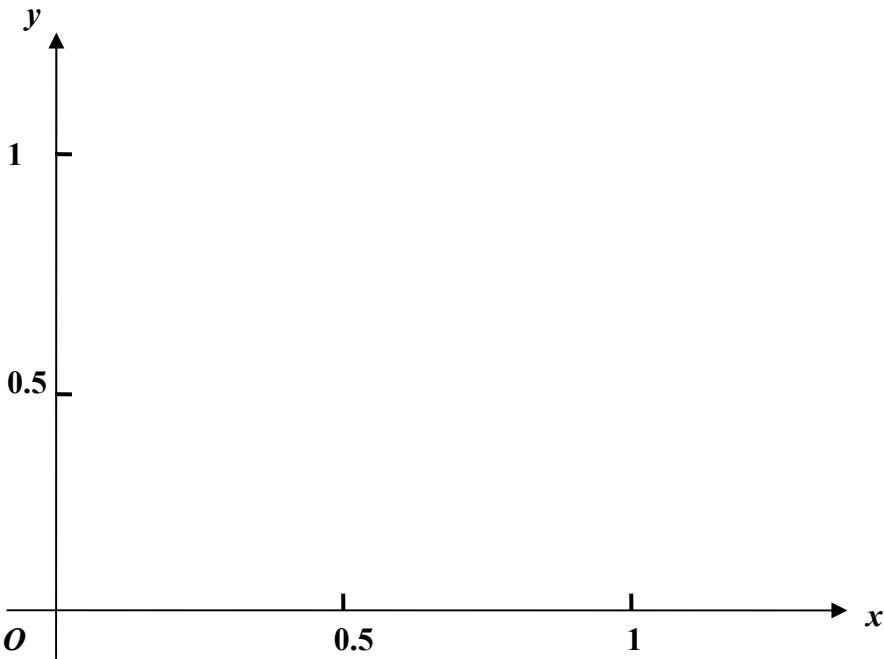
- c. Does f^{-1} , the inverse function of f , exist? Explain.

2 marks

- d. Find the rule for f^{-1} . State the domain and range of f^{-1} .

2 marks

e. On the axes below, sketch the graphs of f and f^{-1} , showing their points of intersection.



2 marks

f. Find the exact value of the area between the graphs of f and f^{-1} .

3 marks

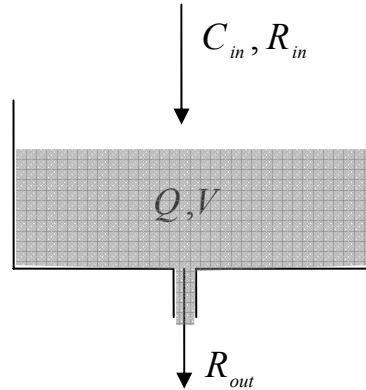
- g. Find the exact value of the volume of the solid resulting from rotating the area between the graph of f^{-1} , the y -axis and the line $y = 1$, about the y -axis.

3 marks

Total 15 marks

Question 3

A solution of salt of concentration C_{in} gram per litre is poured at a rate of R_{in} litre per minute into a tank which contains Q_0 grams of salt dissolved in V_0 litres of water. The mixture is kept uniform in the tank by stirring and then allowed to flow out at a rate of R_{out} litre per minute. Let Q grams and V litres be the amount of salt and the volume of the mixture in the tank t minutes after the solution is poured into the tank.



- a. Express the volume V in terms of V_0, R_{in}, R_{out} and t .

1 mark

- b. Express the concentration of the out-flow C_{out} in terms of Q, V_0, R_{in}, R_{out} and t .

1 mark

- c. Find the differential equation relating Q to V_0, R_{in}, R_{out} and t . State the initial condition.

2 marks

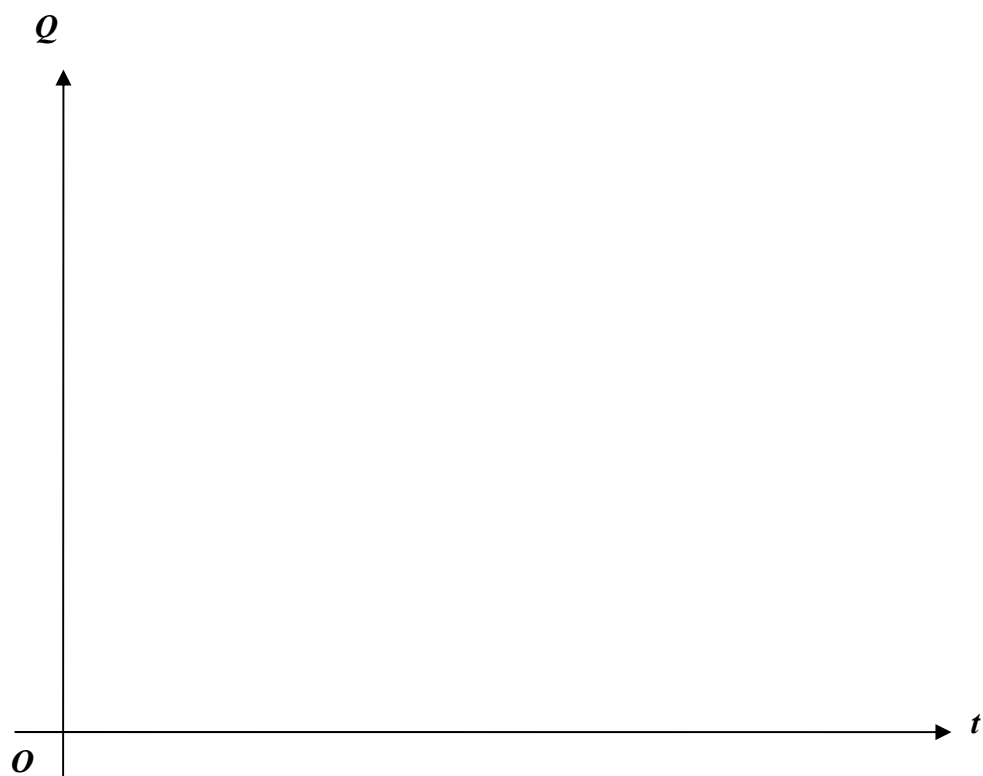
- d.** Solve the differential equation obtained in **part c** under the condition $R_{out} = R_{in}$ express Q as a function of t .

[illegible]

4 marks

- e. Now, consider the special case when $Q_0 = 200, V_0 = 10, C_{in} = 5$ and $R_{out} = R_{in} = 5$.
- Find the **exact** time for Q to be half the initial amount.

- ii. Sketch the graph of Q on the axes below, showing any intercepts and asymptotes.



$2 + 2 = 4$ marks
Total 12 marks

Question 4

A particle of mass m kg is projected vertically upwards with speed u m/s. The air resistance is kmv N, where k is a constant and v is the speed of the particle after t seconds from projection. Let x m be the height of the particle from the point of projection at time t seconds.

- a. Draw a diagram showing all forces acting on the particle during the upward motion.

1 mark

- b. Write down the equation for the upward motion.

1 mark

- c. Find the time for the particle to reach its maximum height in terms of u , k , and g .

3 mark

- d. Show that $\frac{dx}{dv} = -\frac{v}{g + kv}$.

1 mark

Question 5

The motion of a particle is described by $\vec{r}(t) = a \cos\left(\frac{\pi t}{3}\right)\vec{i} + b \sin\left(\frac{\pi t}{3}\right)\vec{j}$ metres at time t seconds, where a and b are constants such that $a > b > 0$.

- a. Find the initial position of the particle. When does the particle return to its initial position for the first time?

2 mark

- b. Determine velocity at any time t . Find the time(s) when the velocity is perpendicular to the position vector.

3 marks

- c. Find the acceleration at any time t . Find the maximum magnitude of the acceleration and when it occurs

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4 marks

Total 9 marks

END OF EXAMINATION 2