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**SPECIALIST MATHS  
TRIAL EXAMINATION 1  
SOLUTIONS  
2005**

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**Part I – Multiple-choice answers**

|    |   |     |   |     |   |     |   |     |   |
|----|---|-----|---|-----|---|-----|---|-----|---|
| 1. | E | 7.  | B | 13. | A | 19. | C | 25. | E |
| 2. | C | 8.  | D | 14. | E | 20. | A | 26. | D |
| 3. | E | 9.  | A | 15. | C | 21. | A | 27. | C |
| 4. | A | 10. | D | 16. | D | 22. | B | 28. | E |
| 5. | B | 11. | D | 17. | B | 23. | C | 29. | B |
| 6. | E | 12. | C | 18. | B | 24. | B | 30. | C |

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**Part I- Multiple-choice solutions**

**Question 1**

$$f(x) = 2x^2 - 5x + 3$$

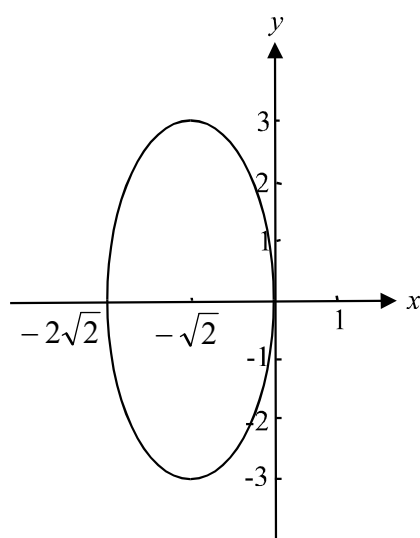
$$\frac{1}{f(x)} = \frac{1}{2x^2 - 5x + 3}$$

$$= \frac{1}{(2x-3)(x-1)}$$

The graph of  $y = \frac{1}{f(x)}$  has asymptotes at  $x = \frac{3}{2}$  and at  $x = 1$ .

The answer is E.

**Question 2**



Four obvious tangents are located at  $x = -2\sqrt{2}$ ,  $y = 3$ ,  $x = 0$  and  $y = -3$ .

The answer is C.

**Question 3**

The domain of  $y = \tan^{-1}(2x)$  is  $\mathbb{R}$  and so the domain of  $y = 2 + \tan^{-1}(2x)$  is also  $\mathbb{R}$ .  
The answer is E.

**Question 4**

$$y = \operatorname{cosec}^2\left(\frac{x}{2}\right)$$

$$= \frac{1}{\sin^2\left(\frac{x}{2}\right)}$$

Asymptotes occur when the denominator of  $\frac{1}{\sin^2\left(\frac{x}{2}\right)}$  is equal to zero.

Now,  $\sin\left(\frac{x}{2}\right) = 0$  when  $x = 0$  and then again at  $x = \pm 2\pi$ . However, we are interested in the graph from  $x = -\pi$  to  $x = \pi$ . So the only asymptote occurs at  $x = 0$ .  
The answer is A.

**Question 5**

$$\frac{1}{1+\bar{z}} = \frac{1}{1+(3-2i)}$$

$$= \frac{1}{4-2i}$$

$$= \frac{1}{4-2i} \times \frac{4+2i}{4+2i}$$

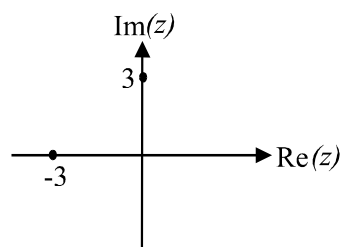
$$= \frac{4+2i}{20}$$

$$= \frac{1}{5} + \frac{1}{10}i$$

The answer is B.

**Question 6**

$z = 3i$   
So  $iz = -3$  and  $|iz| = 3$   
Also  $\operatorname{Arg}(-3) = \pi$   
(Note that  $-\pi < \operatorname{Arg} z \leq \pi$ .)  
The answer is E.



**Question 7**

$P(z)$  has real coefficients hence the roots of  $P(z)=0$  occur in conjugate pairs.

So if  $i$  is one root, then  $-i$  is another.

So  $(z-i)(z+i)=z^2+1$  is a factor

$$\begin{aligned} P(z) &= z^4 + 2z^3 + 3z^2 + 2z + 2 \\ &= (z^2 + 1)(z^2 + 2z + 2) \quad (\text{by inspection}) \\ &= (z^2 + 1)((z^2 + 2z + 1) + 1) \\ &= (z^2 + 1)((z+1)^2 - i^2) \\ &= (z^2 + 1)(z+1-i)(z+1+i) \end{aligned}$$

So all the roots are  $\pm i, -1 \pm i$ .

The answer is B.

**Question 8**

$$z^3 = 8\text{cis}\left(\frac{3\pi}{2}\right)$$

Let  $z = r\text{cis}(\theta)$

so  $z^3 = r^3\text{cis}(3\theta)$  (De Moivre's Theorem)

So  $r^3\text{cis}(3\theta) = 8\text{cis}\left(\frac{3\pi}{2}\right)$

So  $r = 2$  and  $3\theta = \frac{3\pi}{2} + 2\pi k, \quad k \in J$

$$3\theta = \frac{3\pi + 4\pi k}{2}$$

$$\theta = \frac{3\pi + 4\pi k}{6}$$

$k = 0, \quad z = 2\text{cis}\left(\frac{\pi}{2}\right)$

$k = 1, \quad z = 2\text{cis}\left(\frac{7\pi}{6}\right)$

$k = 2, \quad z = 2\text{cis}\left(\frac{11\pi}{6}\right)$

$k = 3, \quad z = 2\text{cis}\left(\frac{15\pi}{6}\right)$   
 $= 2\text{cis}\left(\frac{\pi}{2}\right)$

The solutions begin to repeat. Check that your solutions are evenly spaced around a circle with radius 2.

The answer is D.

**Question 9**

The region is described by

$$x + y \geq 4$$

$$\text{or } \operatorname{Re}(z) + \operatorname{Im}(z) \geq 4$$

$$\text{or } \operatorname{Re}(\bar{z}) + \operatorname{Im}(z) \geq 4$$

since  $z = x + yi$ ,  $\bar{z} = x - yi$

so  $\operatorname{Re}(z) = \operatorname{Re}(\bar{z})$ .

The answer is A.

**Question 10**

$$\int \sin^4\left(\frac{x}{2}\right) \cos\left(\frac{x}{2}\right) dx$$

$$= \int u^4 \times 2 \frac{du}{dx} dx$$

$$= 2 \int u^4 du$$

$$= 2 \frac{u^5}{5} + c$$

$$= \frac{2}{5} \sin^5\left(\frac{x}{2}\right) + c$$

An antiderivative is  $\frac{2}{5} \sin^5\left(\frac{x}{2}\right)$ .

The answer is D.

$$u = \sin\left(\frac{x}{2}\right)$$

$$\frac{du}{dx} = \frac{1}{2} \cos\left(\frac{x}{2}\right)$$

**Question 11**

$$\int_0^3 2x\sqrt{1+x} dx$$

$$= 2 \int_1^4 (u-1)\sqrt{u} \frac{du}{dx} dx$$

$$= 2 \int_1^4 \left( u^{\frac{3}{2}} - u^{\frac{1}{2}} \right) du$$

The answer is D.

$$u = 1 + x \text{ so } x = u - 1$$

$$\frac{du}{dx} = 1$$

$$x = 3, u = 4$$

$$x = 0, u = 1$$

**Question 12**

$$\begin{aligned}
 & \int \frac{dx}{2x\sqrt{\log_e(x)}}, \quad x > 0 \\
 &= \frac{1}{2} \int \frac{1}{\sqrt{u}} \frac{du}{dx} dx \\
 &= \frac{1}{2} \int u^{-\frac{1}{2}} du \\
 &= \frac{1}{2} \times 2u^{\frac{1}{2}} + c \\
 &= \sqrt{\log_e(x)} + c
 \end{aligned}$$

The answer is C.

$$\begin{aligned}
 u &= \log_e(x) \\
 \frac{du}{dx} &= \frac{1}{x}
 \end{aligned}$$

**Question 13**

$$\begin{aligned}
 \text{Area} &= \int_0^{\frac{\pi}{6}} \cos^2(3x) dx \\
 &= \int_0^{\frac{\pi}{6}} \frac{1}{2} (1 + \cos(6x)) dx \\
 &= \frac{1}{2} \left[ x + \frac{1}{6} \sin(6x) \right]_0^{\frac{\pi}{6}} \\
 &= \frac{1}{2} \left\{ \left( \frac{\pi}{6} + \frac{1}{6} \sin(\pi) \right) - \left( 0 + \frac{1}{6} \sin(0) \right) \right\} \\
 &= \frac{1}{2} \left( \frac{\pi}{6} + 0 \right) \\
 &= \frac{\pi}{12} \text{ units}^2
 \end{aligned}$$

The answer is A.

$$\begin{aligned}
 & \text{since } \cos(2\theta) = 2\cos^2(\theta) - 1 \\
 & \text{so } \cos^2(\theta) = \frac{1}{2}(1 + \cos(2\theta))
 \end{aligned}$$

**Question 14**

Use a calculator to find this value. It is 5.8875 correct to 4 decimal places.

The answer is E.

**Question 15**

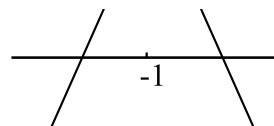
Stationary points occur when  $f'(x)=0$ , that is at  $x=-3, x=-1$  and  $x=2$ .

So options A and B are incorrect.

Now  $f'(x) > 0$  for  $-3 < x < -1$  and  $f'(x) < 0$  for  $-1 < x < 2$  as indicated in the diagram so there is a local maximum at  $x = -1$ .

Options D and E are incorrect

The answer is C.

**Question 16**

$$\begin{aligned} \int_0^4 3 \log_e(x^2 + 1) dx &\approx \frac{2}{2} [3 \log_e(1) + 6 \log_e(5) + 3 \log_e(17)] \\ &= 6 \log_e(5) + 3 \log_e(17) \\ &= \log_e(5)^6 + \log_e(17)^3 \\ &= \log_e(5^6 \times 17^3) \end{aligned}$$

The answer is D.

**Question 17**

From the formulae sheet,

$$\text{If } \frac{dy}{dx} = \frac{2x}{\sqrt{x^2 + 2}}, \quad x_0 = 1 \text{ when } y_0 = 3$$

$$\text{then } x_1 = 1.1 \text{ and } y_1 = 3 + 0.1 \times \frac{2 \times 1}{\sqrt{3}}$$

$$= 3 + \frac{0.2}{\sqrt{3}}$$

$$\text{so } x_2 = 1.2$$

$$\begin{aligned} \text{and } y_2 &= 3 + \frac{0.2}{\sqrt{3}} + 0.1 \times \frac{2 \times 1.1}{\sqrt{1.1^2 + 2}} \\ &= 3.2383 \text{ (to 4 decimal places)} \end{aligned}$$

The answer is B.

**Question 18**

$$V = 500 \sin^{-1} \left( \frac{x}{10} \right)$$

$$\frac{dV}{dx} = \frac{500}{\sqrt{100 - x^2}}$$

Now  $\frac{dx}{dt} = \frac{dx}{dV} \cdot \frac{dV}{dt}$  (Chain rule)

$$= \frac{\sqrt{100 - x^2}}{500} \times 10$$

$$= \frac{\sqrt{100 - x^2}}{50}$$

When  $x = 5$ ,

$$\frac{dx}{dt} = \frac{\sqrt{75}}{50}$$

$$= \frac{\sqrt{3}}{10} \text{ cm/sec}$$

The answer is B.

**Question 19**

Firstly, only options A and C have a magnitude of 2.

Secondly,  $\left( \underline{i} + \sqrt{2} \underline{j} + \underline{k} \right) \cdot \left( \underline{i} + 2 \underline{j} - 2\sqrt{5} \underline{k} \right)$

$$= 1 + 2\sqrt{2} - 2\sqrt{5}$$

$$\neq 0$$

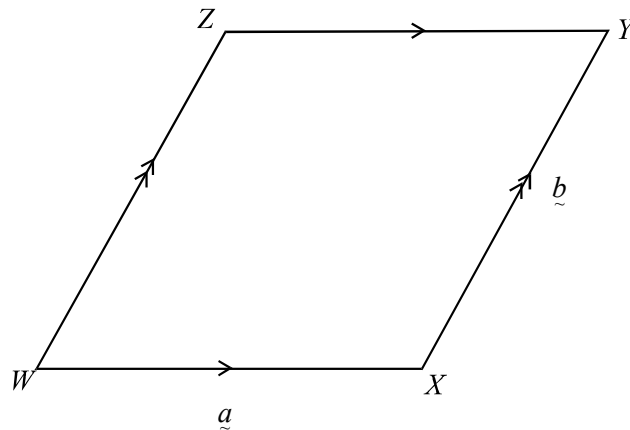
and

$$\frac{2}{5} \left( 2 \underline{i} + 4 \underline{j} + \sqrt{5} \underline{k} \right) \cdot \left( \underline{i} + 2 \underline{j} - 2\sqrt{5} \underline{k} \right)$$

$$= \frac{2}{5} \times (2 + 8 - 10)$$

$$= 0$$

The answer is C.

**Question 20**

Now,  $\vec{WY} = \vec{a} + \vec{b}$  so C and D are incorrect.

Also,  $|\vec{WY}| = |\vec{a} + \vec{b}|$ .

Note that  $|\vec{a}| + |\vec{b}| > |\vec{WY}|$

The answer is A.

**Question 21**

The scalar resolute of  $2\vec{i} + \vec{j} - \vec{k}$  in the direction of  $\vec{i} - \vec{j} + 3\vec{k}$  is given by

$$\begin{aligned} & \left( 2\vec{i} + \vec{j} - \vec{k} \right) \cdot \frac{1}{\sqrt{1+1+9}} (\vec{i} - \vec{j} + 3\vec{k}) \\ &= \frac{1}{\sqrt{11}} (2 - 1 - 3) \\ &= -\frac{2}{\sqrt{11}} \end{aligned}$$

The answer is A.

**Question 22**

The ball hits the ground when  $v = 0$ ; that is at  $t = 1$ . The displacement of the ball in the first second is  $\frac{1}{2} \times 2 \times 1 = 1\text{m}$ .

The answer is B.

**Question 23**

$$\underline{r}(t) = \sin(t)\underline{i} + \cos(2t)\underline{j}, \quad 0 \leq t \leq \frac{\pi}{2}$$

$$x = \sin(t) \quad y = \cos(2t) \\ = 1 - 2\sin^2(t)$$

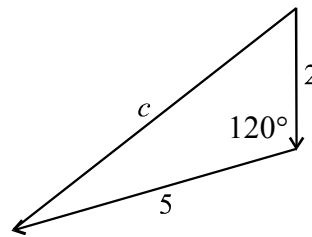
$$\text{So,} \quad = 1 - 2x^2 \quad 0 \leq x \leq 1$$

The answer is C.

**Question 24**

$$c^2 = 4 + 25 - 20 \cos(120^\circ) \\ = 29 + 20 \times \frac{1}{2}$$

$$c^2 = 39 \\ c = \sqrt{39}$$



The answer is B.

**Question 25**

Initial momentum is  $3 \times 5 = 15 \text{ kg m/s}$ .

Momentum after 2 seconds is  $3 \times 8 = 24 \text{ kg m/s}$ .

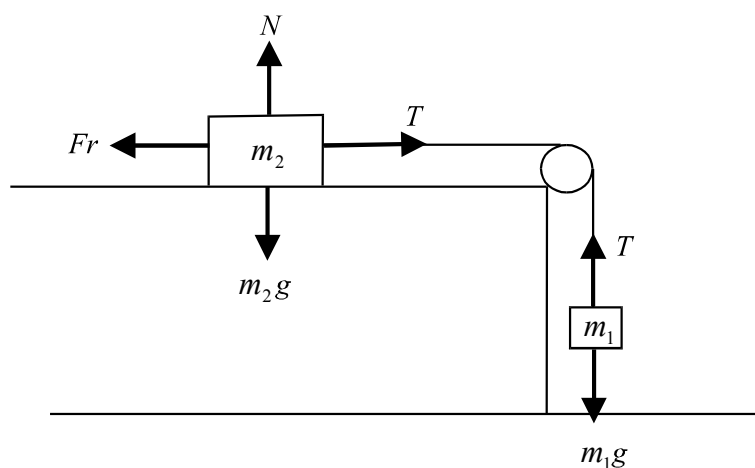
Change in momentum is  $24 - 15 = 9 \text{ kg m/s}$ .

The answer is E.

**Question 26**

All the normal forces and gravitational forces are correct. All the diagrams that include  $P$  are possible. Option D is not possible because if there is no  $P$  force, then the tendency of the mass is to slip down the plane and hence the friction force opposes this and should be directed up the plane.

The answer is D.

**Question 27**

Since  $m_2$  is at the point of moving,  $Fr = \mu N$ .

So,  $Fr = T$  and  $T = m_1g$

So,  $Fr = m_1g$

and  $\mu N = m_1g$

$$\mu = \frac{m_1g}{N}$$

Since  $N = m_2g$ ,

$$\begin{aligned}\mu &= \frac{m_1g}{m_2g} \\ &= \frac{m_1}{m_2}\end{aligned}$$

The answer is C.

**Question 28**

The amount of substance left to react is  $S_0 - S$ .

So  $\frac{dS}{dt} = k(S_0 - S)$ .

The answer is E.

**Question 29**

Since the acceleration is constant we can use the constant acceleration formulae.

Now,  $v = u + at$  gives us  $v = t$  if  $u = 0$  since  $a = 1$  so option A is possible.

Using the same formula, option B is not possible.

Using  $x = ut + \frac{1}{2}at^2$  gives us  $2x = t^2$  if  $u = 0$  since  $a = 1$  so option C is possible.

Using  $v^2 = u^2 + 2ax$  gives us  $2x = v^2$  if  $u = 0$  since  $a = 1$  so option D is possible.

Using  $x = ut + \frac{1}{2}at^2$  gives us  $x = \frac{t^2 + 2t}{2}$  if  $u = 1$  since  $a = 1$  so option E is possible.

So B is not true.

The answer is B.

**Question 30**

Between  $t = 0$  and  $t = 2$  the gradient of the  $v/t$  graph is positive and increasing from zero.

This rules out options B, D and E.

Between  $t = 2$  and  $t = 5$  the gradient of the  $v/t$  graph is zero and hence the acceleration is zero.

This rules out options A and E.

Between  $t = 5$  and  $t = 7$  the gradient is positive but decreasing to zero.

This rules out options B, D and E.

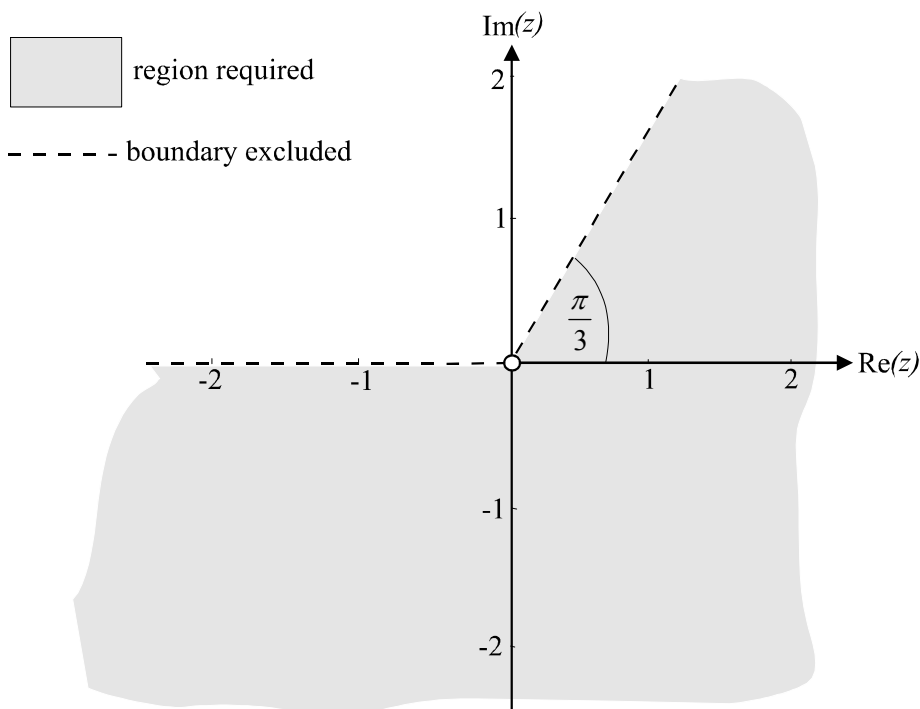
Only option C describes all these three features.

The answer is C.

**PART II****Question 1**

Now  $-\pi < \text{Arg } z \leq \pi$

We require  $\left\{ z : \text{Arg } z < \frac{\pi}{3} \right\}$ .



**(1 mark)** correct region

**(1 mark)** correct marking of boundaries

**Question 2**

$$z^6 = 729$$

$$z^6 - 729 = 0$$

$$(z^3 - 27)(z^3 + 27) = 0 \quad \textbf{(1 mark)}$$

$$(z - 3)(z^2 + 3z + 9)(z + 3)(z^2 - 3z + 9) = 0 \quad \textbf{(1 mark)}$$

$$(z - 3) \left( \left( z^2 + 3z + \frac{9}{4} \right) - \frac{9}{4} + 9 \right) (z + 3) \left( \left( z^2 - 3z + \frac{9}{4} \right) - \frac{9}{4} + 9 \right) = 0$$

$$(z - 3) \left( \left( z + \frac{3}{2} \right)^2 - \frac{27i^2}{4} \right) (z + 3) \left( \left( z - \frac{3}{2} \right)^2 - \frac{27i^2}{4} \right) = 0$$

$$(z - 3) \left( z + \frac{3}{2} - \frac{\sqrt{27}i}{2} \right) \left( z + \frac{3}{2} + \frac{\sqrt{27}i}{2} \right) (z + 3) \left( z - \frac{3}{2} - \frac{\sqrt{27}i}{2} \right) \left( z - \frac{3}{2} + \frac{\sqrt{27}i}{2} \right) = 0$$

So,  $z = \pm 3, -\frac{3}{2} \pm \frac{3\sqrt{3}i}{2}, \frac{3}{2} \pm \frac{3\sqrt{3}i}{2}$

**(1 mark)**

**Question 3**

$$\frac{(x-1)^2}{4} - \frac{(y+2)^2}{9} = 1$$

The centre is at  $(1, -2)$ .

The asymptotes are  $y + 2 = \pm \frac{3}{2}(x - 1)$

$$y + 2 = \frac{3}{2}(x - 1) \text{ or } y + 2 = -\frac{3}{2}(x - 1)$$

$$y = \frac{3}{2}x - \frac{7}{2} \quad y = -\frac{3}{2}x - \frac{1}{2}$$

x-intercepts

$$y = 0 \quad \frac{(x-1)^2}{4} - \frac{4}{9} = 1$$

$$\frac{(x-1)^2}{4} = \frac{13}{9}$$

$$(x-1)^2 = \frac{52}{9}$$

$$x - 1 = \pm \sqrt{\frac{52}{9}}$$

$$x = 1 \pm \frac{2\sqrt{13}}{3}$$

$$(x = 3.4 \text{ or } -1.4)$$

y-intercepts

$$x = 0 \quad \frac{1}{4} - \frac{(y+2)^2}{9} = 1$$

$$-\frac{(y+2)^2}{9} = \frac{3}{4}$$

$$-(y+2)^2 = \frac{27}{4}$$

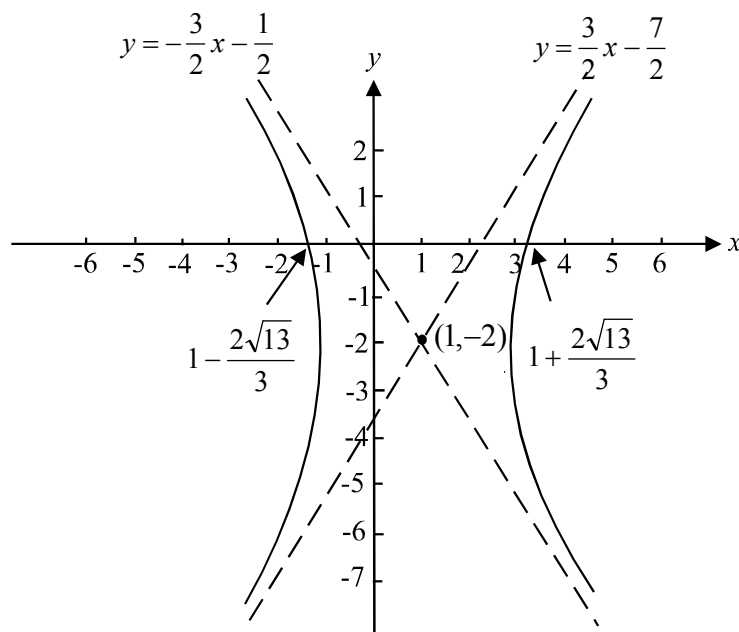
$$(y+2)^2 = -\frac{27}{4}$$

$$y^2 + 4y + \frac{43}{4} = 0$$

$$\Delta = 16 - 4 \times 1 \times \frac{43}{4}$$

$$< 0$$

There are no solutions and hence no y-intercepts.



(1 mark) – centre  
 (1 mark) – asymptotes  
 (1 mark) – intercepts  
 (1 mark) – shape of graph

**Question 4**

$$y = 4x^2 \tan^{-1}\left(\frac{x}{a}\right)$$

$$\begin{aligned} \frac{dy}{dx} &= 8x \tan^{-1}\left(\frac{x}{a}\right) + 4x^2 \times \frac{a}{x^2 + a^2} \\ &= 8x \tan^{-1}\left(\frac{x}{a}\right) + \frac{4ax^2}{x^2 + a^2} \end{aligned} \quad (1 \text{ mark})$$

$$\begin{aligned} \text{LS} &= \frac{dy}{dx} - \frac{2y}{x} \\ &= 8x \tan^{-1}\left(\frac{x}{a}\right) + \frac{4ax^2}{x^2 + a^2} - \frac{8x^2 \tan^{-1}\left(\frac{x}{a}\right)}{x} \\ &= \frac{4ax^2}{x^2 + a^2} \end{aligned} \quad (1 \text{ mark})$$

$$\begin{aligned} \text{RS} &= 12 - \frac{108}{x^2 + 9} \\ &= \frac{12(x^2 + 9) - 108}{x^2 + 9} \\ &= \frac{12x^2}{x^2 + 9} \end{aligned}$$

Now LS = RS

$$\text{So } 4a = 12 \text{ and } a^2 = 9$$

$$a = 3 \quad a = \pm 3$$

So  $a = 3$  (Note that  $a = -3$  does not satisfy both criteria.)

(1 mark)

**Question 5**

$$\begin{aligned} \text{a. } \vec{AB} &= \vec{AO} + \vec{OB} \\ &= -\vec{OA} + \vec{OB} \\ &= -(2\hat{i} + 5\hat{j}) + 4\hat{i} + 3\hat{j} \\ &= 2\hat{i} - 2\hat{j} \end{aligned} \quad (1 \text{ mark})$$

$$\begin{aligned} \text{b. } \vec{AC} &= \vec{AB} - \left( \vec{AB} \cdot \vec{OB} \right) \frac{\vec{OB}}{|\vec{OB}|^2} \\ &= 2\hat{i} - 2\hat{j} - \left( (2\hat{i} - 2\hat{j}) \cdot (4\hat{i} + 3\hat{j}) \right) \frac{1}{5} (4\hat{i} + 3\hat{j}) \\ &= 2\hat{i} - 2\hat{j} - \frac{2}{25} (4\hat{i} + 3\hat{j}) \\ &= \frac{1}{25} (42\hat{i} - 56\hat{j}) \end{aligned} \quad (1 \text{ mark})$$

(1 mark)

c. Find  $\vec{OC}$ .

$$\vec{OC} = \vec{OA} + \vec{AC}$$

$$= 2\vec{i} + 5\vec{j} + \frac{1}{25}(42\vec{i} - 56\vec{j})$$

$$= \frac{1}{25}(92\vec{i} + 69\vec{j}) \quad (1 \text{ mark})$$

$$C \text{ is the point } \left(\frac{92}{25}, \frac{69}{25}\right). \quad (1 \text{ mark})$$

### Question 6

$$\text{Let } \frac{1}{x(x-2)} \equiv \frac{A}{x} + \frac{B}{x-2}$$

$$\frac{1}{x(x-2)} \equiv \frac{A(x-2) + Bx}{x(x-2)}$$

$$\text{True iff } 1 \equiv A(x-2) + Bx$$

$$\text{Put } x=2, \quad 1=2B \quad B=\frac{1}{2}$$

$$\text{Put } x=0, \quad 1=-2A \quad A=-\frac{1}{2}$$

$$\text{So } \frac{1}{x(x-2)} = \frac{-1}{2x} + \frac{1}{2(x-2)} \quad (1 \text{ mark})$$

If you have time, check your answer

$$\text{i.e. } \frac{-1}{2x} + \frac{1}{2(x-2)} = \frac{-(x-2) + x}{2x(x-2)}$$

$$= \frac{1}{x(x-2)}$$

$$\text{So } \int_e^5 \frac{1}{x(x-2)} dx = \int_e^5 \left( \frac{-1}{2x} + \frac{1}{2(x-2)} \right) dx$$

$$= \frac{1}{2} \int_e^5 \left( -\frac{1}{x} + \frac{1}{x-2} \right) dx \quad (1 \text{ mark})$$

$$= \frac{1}{2} [-\log_e(x) + \log_e(x-2)]_e^5$$

$$= \frac{1}{2} \left[ \log_e \left( \frac{x-2}{x} \right) \right]_e^5$$

$$= \frac{1}{2} \left\{ \log_e \left( \frac{3}{5} \right) - \log_e \left( \frac{e-2}{e} \right) \right\}$$

$$= \frac{1}{2} \log_e \left( \frac{3}{5} \times \frac{e}{e-2} \right)$$

$$= \frac{1}{2} \log_e \left( \frac{3e}{5(e-2)} \right)$$

(1 mark)

**Total 20 marks**