



Victorian Certificate of Education

2004

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

STUDENT NUMBER

Letter

Figures

Words

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SPECIALIST MATHEMATICS

Written examination 2

(Analysis task)

Wednesday 3 November 2004

Reading time: 11.45 am to 12.00 noon (15 minutes)

Writing time: 12.00 noon to 1.30 pm (1 hour 30 minutes)

QUESTION AND ANSWER BOOK

Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
5	5	60

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, up to four pages (two A4 sheets) of pre-written notes (typed or handwritten) and an approved scientific and/or graphics calculator (memory may be retained).
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 15 pages with a detachable sheet of miscellaneous formulas in the centrefold.
- Working space is provided throughout the book.

Instructions

- Detach the formula sheet from the centre of this book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other electronic communication devices into the examination room.

Instructions

Answer **all** questions in the spaces provided.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working must be shown.

Where an instruction to **use calculus** is stated for a question, you must show an appropriate derivative or antiderivative.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude $g \text{ m/s}^2$, where $g = 9.8$.

Working space

Question 1

The velocity-time graph below shows the velocity of a lift as it travels from the first floor to the twelfth floor of a tall building during the T seconds of its motion.



The velocity v m/s at time t s for $0 \leq t \leq 2$ is given by $v = t^2(3 - t)$. After the first two seconds, the lift moves with a constant velocity of 4 m/s for a time, and then decelerates to rest in the final two seconds.

The acceleration of the lift is a m/s² at time t s, and the velocity-time graph is symmetrical about $t = \frac{1}{2}T$.

- a. i. Express a in terms of t for the first two seconds of the motion of the lift.

1 mark

- ii. Hence find the maximum acceleration of the lift during the first two seconds of its motion.

2 marks

- b.** Given that the total distance travelled by the lift during its journey is 41 metres, use calculus to find the exact value of T .

3 marks

A girl of mass 29 kg is standing in the lift during its journey. The variable reaction force on the girl due to the lift is R newtons.

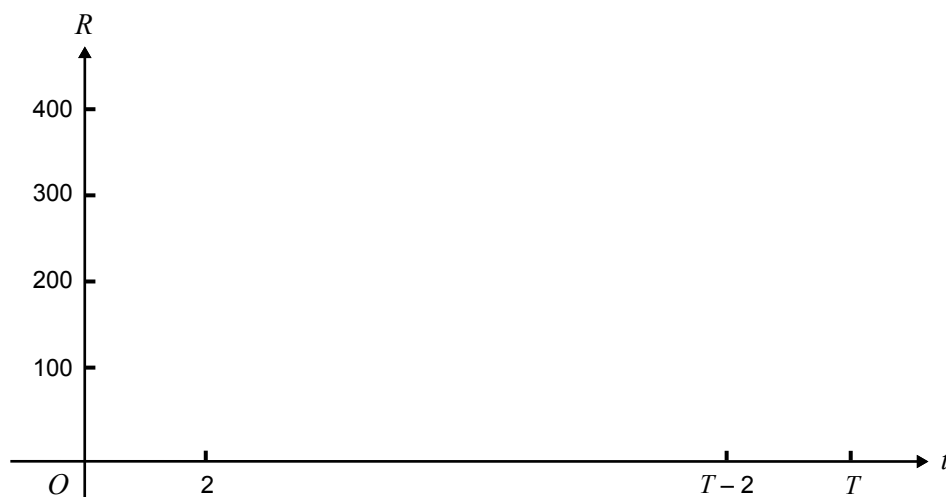
- c.** Show that $R = 29(a + g)$.

1 mark

- d.** Hence find the maximum value of R correct to the nearest integer.

1 mark

- e. Sketch the graph of R versus t on the axes below.



3 marks

- f. A second lift travels from the twelfth floor to the first floor of the building. It begins its journey at exactly the same time as the first lift. At all times during their motions, both lifts move with the same speed. Exactly one second after the two lifts begin to move, the reaction force on a boy standing in the second lift is equal in magnitude to the reaction force on the girl in the first lift.
- i. Find the mass of the boy in kg, correct to one decimal place.

3 marks

- ii. Will there be another instant when the reaction forces on the girl and the boy are equal in magnitude? Explain your answer.

1 mark

Total 15 marks

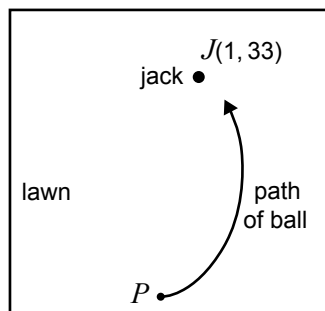
TURN OVER

Question 2

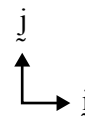
The game of lawn bowls is played on a horizontal lawn. The aim is to roll a ball (usually called a ‘bowl’) to come to rest as close as possible to a target ball called the ‘jack’.



Bowler



View from above



Let $\hat{i}$ be a unit vector to the right and $\hat{j}$ be a unit vector in the forward direction as shown. Displacements are measured in metres.

At one stage during a game, the jack is at the point $J(1, 33)$. The path of a particular ball in this game is modelled by

$$\underline{r}(t) = 2 \sin\left(\frac{2}{15}t\right) \hat{i} + \left(2 + \frac{5}{3}t - \frac{5}{3} \sin\left(\frac{1}{3}t\right)\right) \hat{j}, \quad 0 \leq t \leq \frac{15}{2}\pi$$

where t is the time in seconds after the ball is released from the point P .

- a. Write down the coordinates of P .

1 mark

- b. Find an expression for the velocity, in metres per second, of the ball at time t seconds after the ball is released.

2 marks

- c. At the instant the ball is released, what angle does its path make with the forward direction? Give your answer correct to the nearest tenth of a degree.

3 marks

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[illegible]

Total 12 marks

- c. i. Write a definite integral, the value of which gives the time it takes in minutes for the oil depth in the tank to reach $\sqrt{3}$ metres.

1 mark

- ii. Show that the exact time taken for the oil depth to reach a depth of $\sqrt{3}$ metres is $\frac{4\pi}{\sqrt{3}}$ minutes.

2 marks

Total 8 marks

TURN OVER

Question 4

Consider the cubic equation $z^3 + az^2 + bz + c = 0$, where a , b and c are real numbers. Two of the roots of this equation are 4 and $-1 - 2i$.

- a. i.** State the third root.

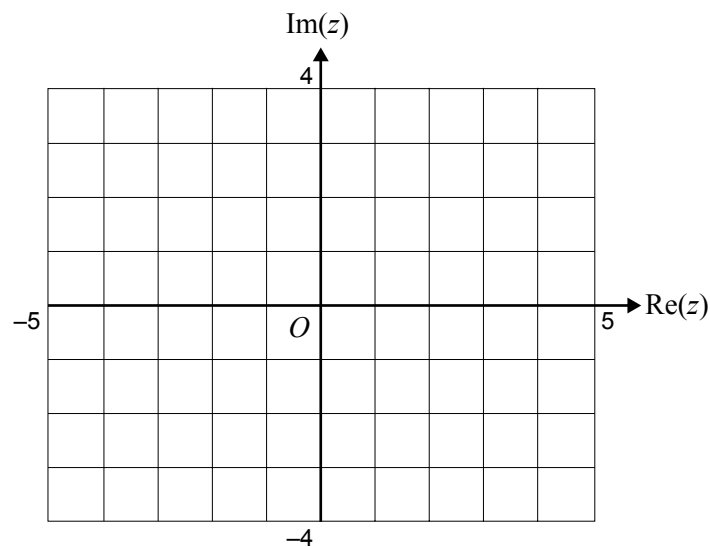
1 mark

- ii.** Find the values of a , b and c .

2 marks

- b.** Plot the three roots on the Argand diagram below.

Label the real root M , and the complex roots P and Q where P lies above the real axis.



1 mark

Let $\hat{i}$ be a unit vector in the direction of the real axis and $\hat{j}$ be a unit vector in the direction of the imaginary axis.

- c. Express the vector $\vec{MQ}$ in terms of $\hat{i}$ and $\hat{j}$.

1 mark

A circle K passes through P , Q , M and $D(d, 0)$, where $d < 0$.

- d. i. Express the vector $\vec{DQ}$ in terms of d , $\hat{i}$ and $\hat{j}$.

1 mark

- ii. Use a scalar product to find d .

2 marks

The straight line that passes through Q and O can be described as the subset S of the complex plane, where $S = \{z : wz + \bar{w}\bar{z} = 0, z \in \mathbb{C}\}$ and w is a complex constant.

- e.** Find a possible w .

3 marks

The chord of the circle K that passes through Q and O can be described as the intersection $S \cap T$, where S is the subset defined above, and T is another subset of the complex plane.

- f.** Find a possible T .

3 marks

Total 14 marks

Working space

TURN OVER

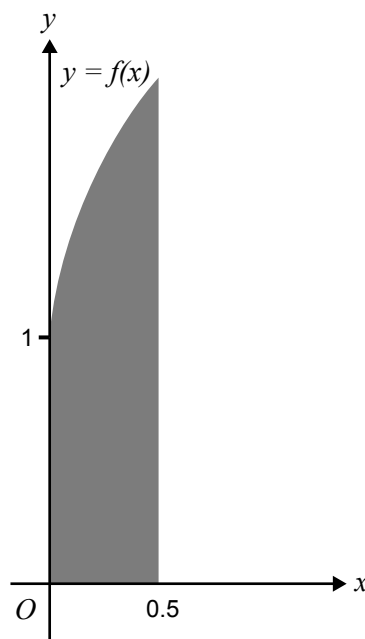
Question 5

Consider the function f with rule $f(x) = 2x^{\frac{1}{2}}(1-x^2)^{\frac{1}{4}} + \frac{1}{(1-x^2)^{\frac{1}{4}}}$.

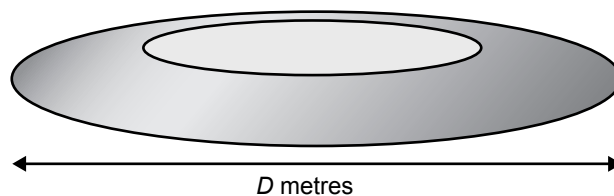
- a. State the largest domain for which f is defined.

1 mark

A solid platform for a statue is constructed by rotating, about the x -axis, the region enclosed by the curve $y = f(x)$, the line $x = 0.5$, and the coordinate axes. Lengths are measured in metres.



The platform is laid flat on a horizontal surface as shown below (diagram not to scale).



- b. Let D metres be the diameter of the base of the platform. Find D correct to three significant figures.

1 mark

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[illegible]

END OF QUESTION AND ANSWER BOOK

SPECIALIST MATHEMATICS

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

Specialist Mathematics Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder:	$2\pi rh$
volume of a cylinder:	$\pi r^2 h$
volume of a cone:	$\frac{1}{3}\pi r^2 h$
volume of a pyramid:	$\frac{1}{3}Ah$
volume of a sphere:	$\frac{4}{3}\pi r^3$
area of a triangle:	$\frac{1}{2}bc \sin A$
sine rule:	$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
cosine rule:	$c^2 = a^2 + b^2 - 2ab \cos C$

Coordinate geometry

ellipse:	$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$	hyperbola:	$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$
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Circular (trigometric) functions

$$\cos^2(x) + \sin^2(x) = 1$$

$$1 + \tan^2(x) = \sec^2(x)$$

$$\sin(x+y) = \sin(x) \cos(y) + \cos(x) \sin(y)$$

$$\cos(x+y) = \cos(x) \cos(y) - \sin(x) \sin(y)$$

$$\tan(x+y) = \frac{\tan(x) + \tan(y)}{1 - \tan(x) \tan(y)}$$

$$\cos(2x) = \cos^2(x) - \sin^2(x) = 2 \cos^2(x) - 1 = 1 - 2 \sin^2(x)$$

$$\sin(2x) = 2 \sin(x) \cos(x)$$

$$\cot^2(x) + 1 = \operatorname{cosec}^2(x)$$

$$\sin(x-y) = \sin(x) \cos(y) - \cos(x) \sin(y)$$

$$\cos(x-y) = \cos(x) \cos(y) + \sin(x) \sin(y)$$

$$\tan(x-y) = \frac{\tan(x) - \tan(y)}{1 + \tan(x) \tan(y)}$$

$$\tan(2x) = \frac{2 \tan(x)}{1 - \tan^2(x)}$$

function	$\sin^{-1}$	$\cos^{-1}$	$\tan^{-1}$
domain	$[-1, 1]$	$[-1, 1]$	R
range	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Algebra (Complex numbers)

$$z = x + yi = r(\cos \theta + i \sin \theta) = r \operatorname{cis} \theta$$

$$|z| = \sqrt{x^2 + y^2} = r$$

$$z_1 z_2 = r_1 r_2 \operatorname{cis}(\theta_1 + \theta_2)$$

$$z^n = r^n \operatorname{cis}(n\theta) \text{ (de Moivre's theorem)}$$

$$-\pi < \operatorname{Arg} z \leq \pi$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} \operatorname{cis}(\theta_1 - \theta_2)$$

Calculus

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\int x^n dx = \frac{1}{n+1} x^{n+1} + c, n \neq -1$$

$$\frac{d}{dx}(e^{ax}) = ae^{ax}$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax} + c$$

$$\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$$

$$\int \frac{1}{x} dx = \log_e(x) + c, \text{ for } x > 0$$

$$\frac{d}{dx}(\sin(ax)) = a \cos(ax)$$

$$\int \sin(ax) dx = -\frac{1}{a} \cos(ax) + c$$

$$\frac{d}{dx}(\cos(ax)) = -a \sin(ax)$$

$$\int \cos(ax) dx = \frac{1}{a} \sin(ax) + c$$

$$\frac{d}{dx}(\tan(ax)) = a \sec^2(ax)$$

$$\int \sec^2(ax) dx = \frac{1}{a} \tan(ax) + c$$

$$\frac{d}{dx}(\sin^{-1}(x)) = \frac{1}{\sqrt{1-x^2}}$$

$$\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c, a > 0$$

$$\frac{d}{dx}(\cos^{-1}(x)) = \frac{-1}{\sqrt{1-x^2}}$$

$$\int \frac{-1}{\sqrt{a^2-x^2}} dx = \cos^{-1}\left(\frac{x}{a}\right) + c, a > 0$$

$$\frac{d}{dx}(\tan^{-1}(x)) = \frac{1}{1+x^2}$$

$$\int \frac{a}{a^2+x^2} dx = \tan^{-1}\left(\frac{x}{a}\right) + c$$

product rule:

$$\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$$

quotient rule:

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

chain rule:

$$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$$

mid-point rule:

$$\int_a^b f(x) dx \approx (b-a) f\left(\frac{a+b}{2}\right)$$

trapezoidal rule:

$$\int_a^b f(x) dx \approx \frac{1}{2}(b-a)(f(a) + f(b))$$

Euler's method:

$$\text{If } \frac{dy}{dx} = f(x), x_0 = a \text{ and } y_0 = b, \text{ then } x_{n+1} = x_n + h \text{ and } y_{n+1} = y_n + hf(x_n)$$

acceleration:

$$a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v \frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

$$\text{constant (uniform) acceleration: } v = u + at \quad s = ut + \frac{1}{2}at^2 \quad v^2 = u^2 + 2as \quad s = \frac{1}{2}(u+v)t$$

TURN OVER

Vectors in two and three dimensions

$$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$$

$$|\vec{r}| = \sqrt{x^2 + y^2 + z^2} = r$$

$$\vec{r}_1 \cdot \vec{r}_2 = r_1 r_2 \cos \theta = x_1 x_2 + y_1 y_2 + z_1 z_2$$

$$\dot{\vec{r}} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\vec{i} + \frac{dy}{dt}\vec{j} + \frac{dz}{dt}\vec{k}$$

Mechanics

momentum:

$$\vec{p} = m\vec{v}$$

equation of motion:

$$\vec{R} = m\vec{a}$$

friction:

$$F \leq \mu N$$