



Victorian Certificate of Education 2004

SPECIALIST MATHEMATICS

Written examination 1 (Facts, skills and applications)

Monday 1 November 2004

Reading time: 11.45 am to 12.00 noon (15 minutes)

Writing time: 12.00 noon to 1.30 pm (1 hour 30 minutes)

PART I MULTIPLE-CHOICE QUESTION BOOK

This examination has two parts: Part I (multiple-choice questions) and Part II (short-answer questions). Part I consists of this question book and must be answered on the answer sheet provided for multiple-choice questions.

Part II consists of a separate question and answer book.

You must complete **both** parts in the time allotted. When you have completed one part continue immediately to the other part.

Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
30	30	30

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, up to four pages (two A4 sheets) of pre-written notes (typed or handwritten) and an approved scientific and/or graphics calculator (memory may be retained).
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question book of 18 pages, with a detachable sheet of miscellaneous formulas in the centrefold and a blank page for rough working.
- Answer sheet for multiple-choice questions.

Instructions

- Detach the formula sheet from the centre of this book during reading time.
- Check that your **name** and **student number** as printed on your answer sheet for multiple-choice questions are correct, **and** sign your name in the space provided to verify this.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

At the end of the examination

- Place the answer sheet for multiple-choice questions (Part I) inside the front cover of the question and answer book (Part II).
- You may retain this question book.

Students are NOT permitted to bring mobile phones and/or any other electronic communication devices into the examination room.

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Instructions for Part I

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.

Choose the response that is **correct** for the question.

A correct answer scores 1, an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Take the **acceleration due to gravity** to have magnitude $g \text{ m/s}^2$, where $g = 9.8$.

Question 1

The graph of $y = \frac{-x^2 + 1}{2x}$ has

- A. no straight line asymptotes.
- B. $y = 2x$ as its only straight line asymptote.
- C. $x = 0$ as its only straight line asymptote.
- D. $y = 0$ and $y = -\frac{1}{2}x$ as its only straight line asymptotes.
- E. $x = 0$ and $y = -\frac{1}{2}x$ as its only straight line asymptotes.

Question 2

The x -axis is tangent to an ellipse at the point $(1, 0)$ and the y -axis is tangent to the same ellipse at the point $(0, -2)$.

Which one of the following could be the equation of this ellipse?

- A. $\frac{(x-1)^2}{4} + (y+2)^2 = 1$
- B. $\frac{(x+1)^2}{4} + (y-2)^2 = 1$
- C. $(x-1)^2 + \frac{(y+2)^2}{4} = 1$
- D. $(x+1)^2 + \frac{(y-2)^2}{4} = 1$
- E. $(x-2)^2 + \frac{(y+1)^2}{4} = 1$

Question 3

Which one of the following is **not** equal to $\tan\left(\frac{\pi}{5}\right)$?

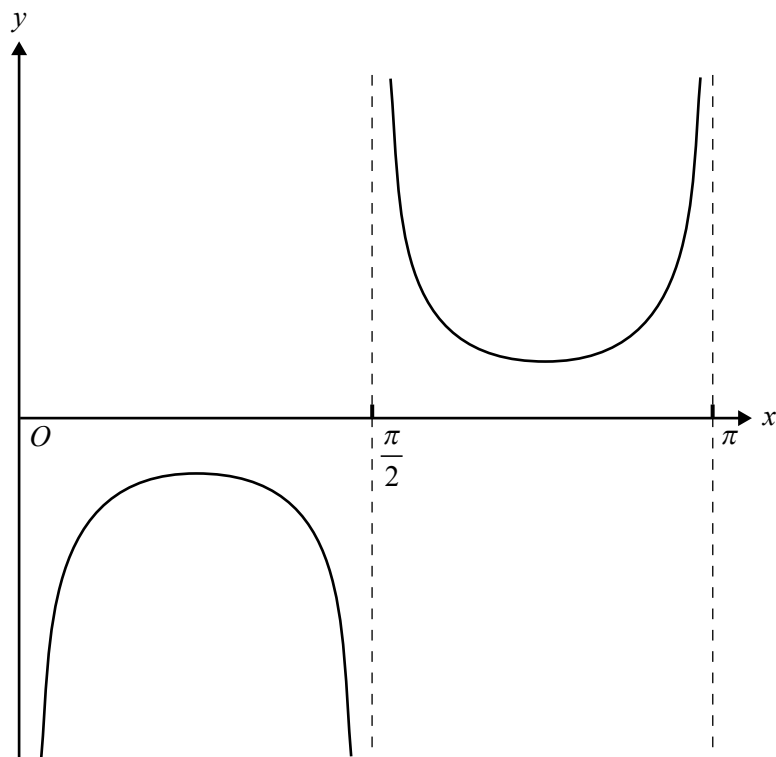
A. $\frac{\sin\left(\frac{\pi}{5}\right)}{\cos\left(\frac{\pi}{5}\right)}$

B. $\frac{1}{\cot\left(\frac{\pi}{5}\right)}$

C. $\cot\left(\frac{3\pi}{10}\right)$

D. $\frac{2\tan\left(\frac{\pi}{10}\right)}{1-\tan^2\left(\frac{\pi}{10}\right)}$

E. $\frac{2\tan\left(\frac{2\pi}{5}\right)}{1-\tan^2\left(\frac{2\pi}{5}\right)}$

Question 4

The graph of $y = -\sec(a(x - b))$ is shown above for $0 \leq x \leq \pi$.

The values of a and b could be

- A. $a = 1, b = \frac{\pi}{2}$
- B. $a = 1, b = \frac{\pi}{4}$
- C. $a = 2, b = \frac{\pi}{2}$
- D. $a = 2, b = \frac{\pi}{4}$
- E. $a = 2, b = -\frac{\pi}{4}$

Question 5

If $z = x + yi$, where x and y are non-zero real numbers, which one of the following is a real number?

- A. $\frac{1}{z}$
- B. $\frac{1}{\bar{z}}$
- C. $\frac{1}{z - \bar{z}}$
- D. $\frac{1}{z} - \frac{1}{\bar{z}}$
- E. $\frac{1}{z} + \frac{1}{\bar{z}}$

Question 6

If $\text{Arg}(1 + ai) = -\frac{\pi}{3}$, then the real number a is

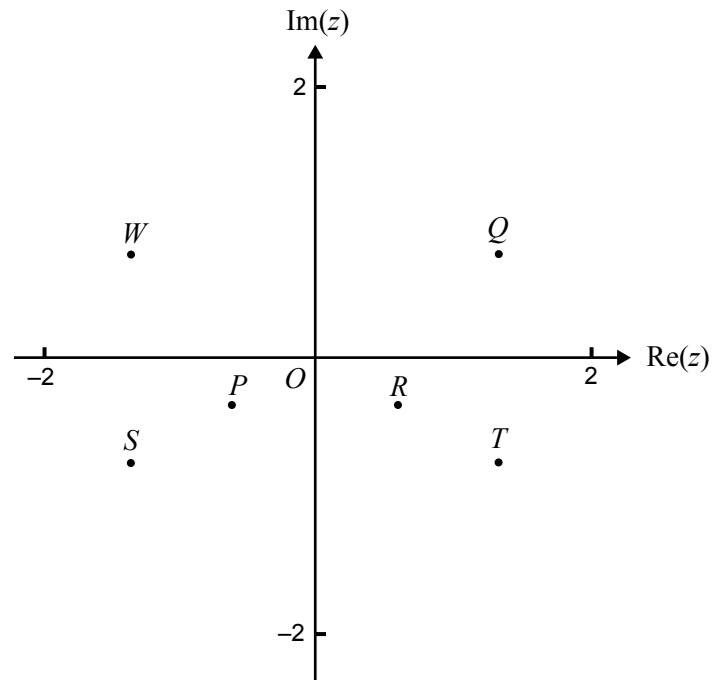
- A. $-\frac{\pi}{\sqrt{3}}$
- B. $-\frac{\sqrt{3}}{2}$
- C. $-\sqrt{3}$
- D. $\frac{1}{\sqrt{3}}$
- E. $\sqrt{3}$

Question 7

$P(z)$ is a polynomial in z of degree 4 with real coefficients.

Which one of the following statements **must** be **false**?

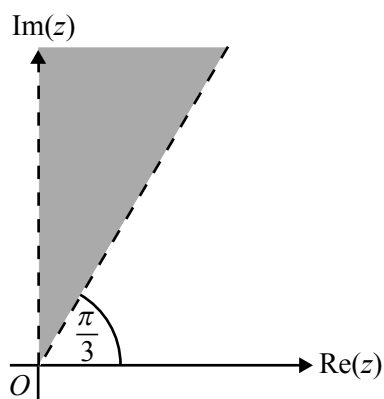
- A. $P(z) = 0$ has no real roots.
- B. $P(z) = 0$ has one real root and three non-real roots.
- C. $P(z) = 0$ has one (repeated) real root and two non-real roots.
- D. $P(z) = 0$ has two real roots and two non-real roots.
- E. $P(z) = 0$ has four real roots.

Question 8

The point W on the Argand diagram above represents a complex number w where $|w| = 1.5$.

The complex number w^{-1} is best represented by the point

- A. P
- B. Q
- C. R
- D. S
- E. T

Question 9

The shaded region (with boundaries excluded) of the complex plane shown above is best described by

- A. $\left\{ z: \text{Arg}(z) > \frac{\pi}{3} \right\}$
- B. $\left\{ z: \text{Arg}(z) > \frac{\pi}{3} \right\} \cup \left\{ z: \text{Arg}(z) < \frac{\pi}{2} \right\}$
- C. $\left\{ z: \text{Arg}(z) > \frac{\pi}{3} \right\} \cap \left\{ z: \text{Arg}(z) < \frac{\pi}{2} \right\}$
- D. $\left\{ z: \text{Arg}(z) > \frac{\pi}{2} \right\} \cup \left\{ z: \text{Arg}(z) < \frac{\pi}{3} \right\}$
- E. $\left\{ z: \text{Arg}(z) > \frac{\pi}{2} \right\} \cap \left\{ z: \text{Arg}(z) < \frac{\pi}{3} \right\}$

Question 10

Which one of the following is an antiderivative of $\frac{1}{x^2 + 16}$?

- A. $\log_e(x^2 + 16)$
- B. $\frac{1}{2x} \log_e(x^2 + 16)$
- C. $\tan^{-1}\left(\frac{x}{4}\right)$
- D. $\frac{1}{4} \tan^{-1}\left(\frac{x}{4}\right)$
- E. $4 \tan^{-1}\left(\frac{x}{4}\right)$

Question 11

$\int_0^a \left(\sin^2\left(\frac{3x}{2}\right) - \cos^2\left(\frac{3x}{2}\right) \right) dx$ is equal to

- A. $-\frac{4}{3} \sin\left(\frac{3a}{4}\right)$
- B. $-\frac{1}{3} \sin(3a)$
- C. $\frac{1}{3} \sin(3a)$
- D. $\frac{1}{3} (1 - \sin(3a))$
- E. $-\frac{1}{3} (\cos(3a) - 1)$

Question 12

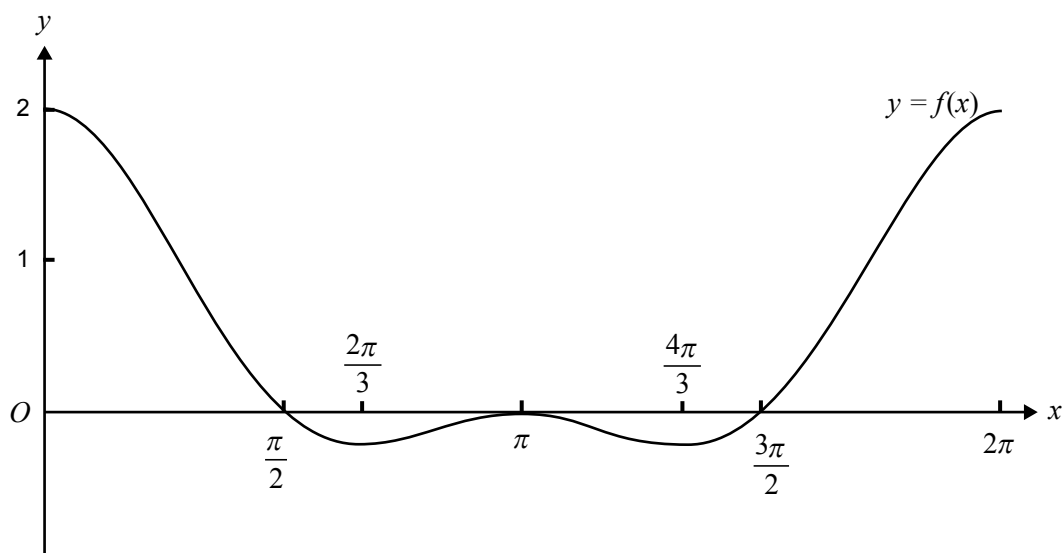
With a suitable substitution, $\int_0^{\frac{\pi}{3}} \cos^2(x) \sin^3(x) dx$ can be expressed as

- A. $\int_{\frac{1}{2}}^1 u^2 (1 - u^2) du$
- B. $\int_1^{\frac{1}{2}} u^2 (1 - u^2) du$
- C. $\int_0^{\frac{\pi}{3}} u^2 (1 - u^2) du$
- D. $-\int_0^{\frac{\pi}{3}} u^2 (1 - u^2) du$
- E. $-\int_0^{\frac{\sqrt{3}}{2}} u^2 (1 - u^2) du$

Question 13

An antiderivative of $\frac{2}{(3-x)^2} - \frac{1}{3-x}$, for $x < 3$, is

- A. $\log_e(x-3) - \frac{2}{x-3}$
 B. $\log_e(x-3) + \frac{2}{x-3}$
 C. $\log_e(3-x) - \frac{2}{3-x}$
 D. $\log_e(3-x) + \frac{2}{3-x}$
 E. $-\log_e(3-x) + \frac{2}{3-x}$

Question 14

The graph of $y = f(x)$ is shown above.

Let $F(x)$ be an antiderivative of $f(x)$.

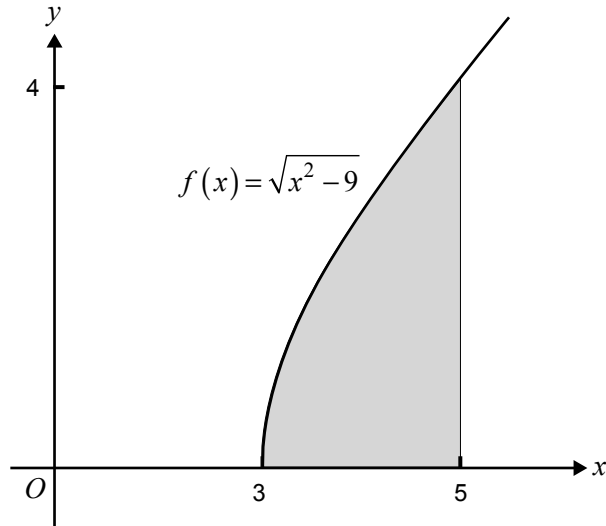
The stationary points of the graph of $y = F(x)$ could be

- A. local maximums at $x = 0, \pi$ and 2π , and local minimums at $x = \frac{2\pi}{3}$ and $\frac{4\pi}{3}$
 B. stationary points of inflexion at $x = 0, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}$ and 2π , a local maximum at $x = \frac{\pi}{2}$, and a local minimum at $x = \frac{3\pi}{2}$
 C. stationary points of inflexion at $x = 0, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}$ and 2π , a local minimum at $x = \frac{\pi}{2}$, and a local maximum at $x = \frac{3\pi}{2}$
 D. a stationary point of inflexion at $x = \pi$, a local maximum at $x = \frac{\pi}{2}$, and a local minimum at $x = \frac{3\pi}{2}$
 E. a stationary point of inflexion at $x = \pi$, a local minimum at $x = \frac{\pi}{2}$, and a local maximum at $x = \frac{3\pi}{2}$

The following information relates to Questions 15 and 16.

The graph of $f : [3, \infty) \rightarrow \mathbb{R}$, where $f(x) = \sqrt{x^2 - 9}$, is shown below.

The shaded region is bounded by this graph, the x -axis, and the line with equation $x = 5$.



Question 15

The midpoint rule with **two** equal intervals is used to estimate the area of the shaded region.

The value obtained, calculated correct to two decimal places, is

- A. 4.65
- B. 5.06
- C. 5.16
- D. 5.29
- E. 5.80

Question 16

The shaded region is rotated about the y -axis to form a solid of revolution.

The volume of this solid, in cubic units, is given by

- A. $\pi \int_0^4 (y^2 - 9) dy$
- B. $\pi \int_0^4 (34 - y^2) dy$
- C. $\pi \int_0^4 (y^2 + 9) dy$
- D. $\pi \int_0^4 (16 - y^2) dy$
- E. $\pi \int_0^4 \left(5 - \sqrt{y^2 + 9}\right)^2 dy$

Question 17

The vectors $\underline{p}$ and $\underline{q}$ are given by $\underline{p} = 2\underline{i} + x\underline{j} + 3\underline{k}$ and $\underline{q} = -4\underline{i} + y\underline{j} - 6\underline{k}$, where x and y are real numbers. The magnitude of vector $\underline{p}$ is 4 units, and $\underline{p}$ and $\underline{q}$ are parallel.

The values of x and y could be

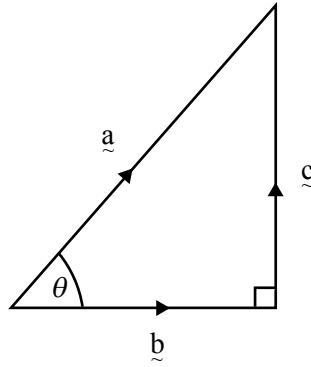
- A. $x = \sqrt{3}$, $y = -2\sqrt{3}$
- B. $x = 3$, $y = -6$
- C. $x = \sqrt{3}$, $y = 2\sqrt{3}$
- D. $x = \sqrt{29}$, $y = -\sqrt{29}$
- E. $x = -\sqrt{3}$, $y = -2\sqrt{3}$

Question 18

The vectors $\underline{u}$ and $\underline{v}$ are given by $\underline{u} = 3\underline{i} - 4\underline{j} + 5\underline{k}$ and $\underline{v} = 2\underline{i} + 3\underline{j} - \underline{k}$.

$\underline{u} \cdot (\underline{u} - 2\underline{v})$ is equal to

- A. 20
- B. 45
- C. 52
- D. 72
- E. 78

Question 19

The right-angled triangle shown above has sides represented by the vectors $\underline{a}$, $\underline{b}$ and $\underline{c}$. Which one of the following statements is **false**?

- A. $|\underline{b}|^2 + |\underline{c}|^2 = |\underline{a}|^2$
- B. $\underline{b} \cdot (\underline{a} - \underline{c}) = |\underline{b}|^2$
- C. $\underline{b} \cdot (\underline{a} - \underline{b}) = |\underline{b}| |\underline{c}|$
- D. $\underline{a} \cdot \underline{b} = |\underline{a}| |\underline{b}| \cos(\theta)$
- E. $\underline{a} \cdot \underline{c} = |\underline{a}| |\underline{c}| \sin(\theta)$

Question 20

The position vector of a particle at time t is given by $\underline{r}(t) = 2\sin(t)\underline{i} + \cos(t)\underline{j}$, $0 \leq t \leq \pi$.

The Cartesian equation of the path of the particle is

- A. $y = \cos\left(\sin^{-1}\left(\frac{x}{2}\right)\right)$, $0 \leq x \leq 2$
- B. $y = \sqrt{1 - \frac{x^2}{4}}$, $0 \leq x \leq 2$
- C. $\frac{x^2}{2} + y^2 = 1$, $-2 \leq x \leq 2$
- D. $\frac{x^2}{4} + y^2 = 1$, $-2 \leq x \leq 2$
- E. $\frac{x^2}{4} + y^2 = 1$, $0 \leq x \leq 2$

Question 21

The velocity of a particle at time t , $t \geq 0$, is given by $\dot{\mathbf{r}}(t) = 3\sin(2t)\mathbf{i} + 4\mathbf{j}$.

The initial position of the particle is given by $\mathbf{r}(0) = \frac{3}{2}\mathbf{i}$.

The position vector $\mathbf{r}(t)$ of the particle at time t is equal to

- A. $-\frac{3}{2}\cos(2t)\mathbf{i} + 4t\mathbf{j}$
- B. $\left(3 - \frac{3}{2}\cos(2t)\right)\mathbf{i} + 4t\mathbf{j}$
- C. $\left(\frac{3}{2} - \frac{3}{2}\cos(2t)\right)\mathbf{i} + 4t\mathbf{j}$
- D. $\frac{3}{2}\cos(2t)\mathbf{i} + 4t\mathbf{j}$
- E. $\left(3 - \frac{3}{2}\cos(2t)\right)\mathbf{i}$

Question 22

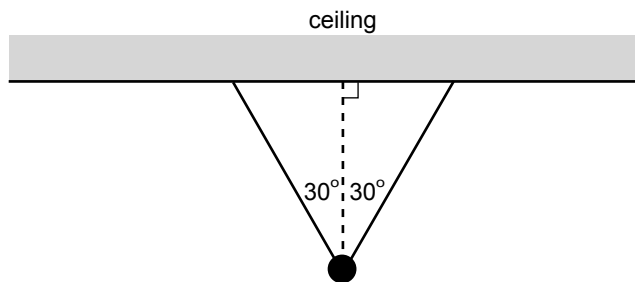
A body of mass 5 kg slides from rest down a smooth plane inclined at an angle of 30° to the horizontal.

The acceleration, in m/s^2 , of the body down the plane has magnitude

- A. $\frac{\sqrt{3}g}{2}$
- B. $\frac{g}{2}$
- C. 0
- D. $\frac{5\sqrt{3}g}{2}$
- E. $\frac{5g}{2}$

Question 23

A 10 kg mass is suspended in equilibrium from a horizontal ceiling by two identical light strings. Each string makes an angle of 30° with the vertical as shown below.



The magnitude, in newtons, of the tension in each string is equal to

- A. $5g$
- B. $10g$
- C. $20g$
- D. $\frac{10g\sqrt{3}}{3}$
- E. $\frac{20g\sqrt{3}}{3}$

Question 24

A body of mass 5 kg is acted upon by three concurrent coplanar forces $\vec{R}$, $\vec{S}$ and $\vec{T}$, where $\vec{R} = 2\vec{i} + \vec{j}$, $\vec{S} = \vec{i} + 10\vec{j}$ and $\vec{T} = 3\vec{i} - 3\vec{j}$. The forces are measured in newtons.

The magnitude of the acceleration of the body, in m/s^2 , is

- A. 2
- B. 4
- C. 6
- D. 8
- E. 10

Question 25

A balloon is rising vertically at a constant speed of 21 metres per second. A stone is dropped from the balloon when it is h metres above the ground. The stone strikes the ground 10 seconds later.

Assuming that air resistance is negligible, the value of h is

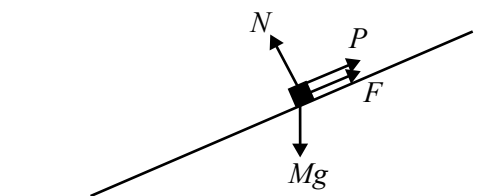
- A. 210
- B. 280
- C. 490
- D. 700
- E. 770

Question 26

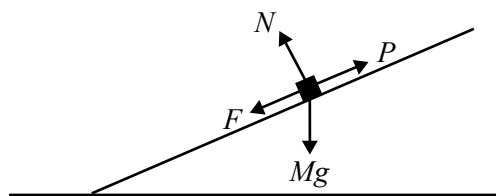
A body of mass M kg is on a rough plane inclined at an angle to the horizontal. The body, which is on the point of sliding down the plane, is held in equilibrium by a force of magnitude P applied parallel to the plane. There is a normal reaction of magnitude N and a frictional force of magnitude F . All forces are measured in newtons.

Which one of the following diagrams shows the forces acting on the body?

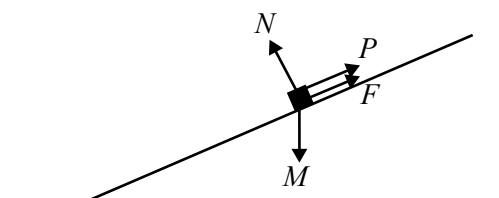
A.



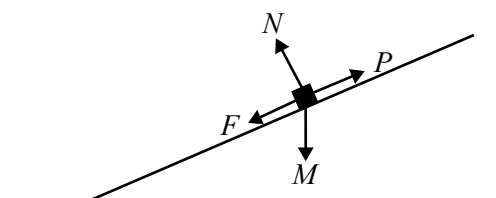
B.



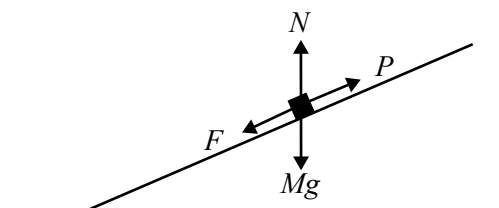
C.



D.



E.



Question 27

A particle is moving in a straight line in such a way that its displacement, x metres, from a fixed origin at time

t seconds is given by $x = 2.5t + 9 \cos\left(\frac{t}{2}\right)$, $t \geq 0$.

If the velocity of the particle at time t seconds is v metres per second, then the minimum value of v is

- A.** -6.5
- B.** -2
- C.** 0
- D.** 2.5
- E.** 7

Question 28

Which one of the following differential equations is satisfied by $y = \sin(2x)$?

- A. $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + y = 4\cos(2x)$
- B. $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - 4y = 4\cos(2x)$
- C. $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 4y = 4\cos(2x)$
- D. $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 4y = 4\cos(2x)$
- E. $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 4y = 4\cos(2x)$

Question 29

A jug of water at a temperature of 20°C is placed in a refrigerator. The temperature inside the refrigerator is maintained at 4°C .

When the jug has been in the refrigerator for t minutes, the temperature of the water in the jug is $y^\circ\text{C}$. The rate at which the water's temperature decreases is proportional to the excess of its temperature over the temperature inside the refrigerator.

If k is a positive constant, a differential equation involving y and t is

- A. $\frac{dy}{dt} = -k(y - 20); \quad t = 0, y = 4$
- B. $\frac{dy}{dt} = -k(y + 4); \quad t = 0, y = 20$
- C. $\frac{dy}{dt} = -k(y - 4); \quad t = 0, y = 16$
- D. $\frac{dy}{dt} = -k(y + 4); \quad t = 0, y = 24$
- E. $\frac{dy}{dt} = -k(y - 4); \quad t = 0, y = 20$

Question 30

A particle moves in a straight line. When its displacement from a fixed origin is x m, its velocity is v m/s and its acceleration is a m/s².

Given that $a = 16x$, and that $v = -5$ when $x = 0$, the relation between v and x is

- A. $v = -4x - 5$
- B. $v = 8x^2 - 5$
- C. $v = -\sqrt{25 + 16x^2}$
- D. $v = \sqrt{25 + 16x^2}$
- E. $v = -\sqrt{25 + 32x^2}$

Working space



Victorian Certificate of Education

2004

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

STUDENT NUMBER

Letter

Figures

Words

SPECIALIST MATHEMATICS

Written examination 1 (Facts, skills and applications)

Monday 1 November 2004

Reading time: 11.45 am to 12.00 noon (15 minutes)

Writing time: 12.00 noon to 1.30 pm (1 hour 30 minutes)

PART II QUESTION AND ANSWER BOOK

This examination has two parts: Part I (multiple-choice questions) and Part II (short-answer questions). Part I consists of a separate question book and must be answered on the answer sheet provided for multiple-choice questions.

Part II consists of this question and answer book.

You must complete **both** parts in the time allotted. When you have completed one part continue immediately to the other part.

Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
5	5	20

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, up to four pages (two A4 sheets) of pre-written notes (typed or handwritten) and an approved scientific and/or graphics calculator (memory may be retained).
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 6 pages.

Instructions

- Detach the formula sheet from the centre of the Part I book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

At the end of the examination

- Place the answer sheet for multiple-choice questions (Part I) inside the front cover of this question and answer book (Part II).

Students are NOT permitted to bring mobile phones and/or any other electronic communication devices into the examination room.

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Instructions for Part II

Answer **all** questions in the spaces provided.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working must be shown.

Where an instruction to **use calculus** is stated for a question, you must show an appropriate derivative or antiderivative.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude $g \text{ m/s}^2$, where $g = 9.8$.

Question 1

- a. Show that, for $0 < x < \frac{1}{2}$, $\frac{d}{dx}(\sin^{-1}(\sqrt{2x})) = \frac{1}{\sqrt{2x(1-2x)}}.$

2 marks

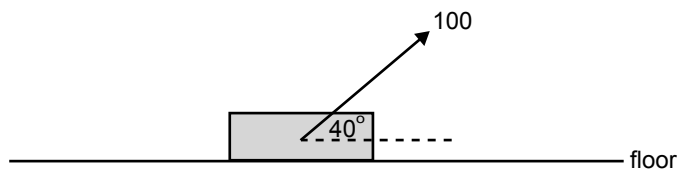
- b. Hence find the exact value of $\int_{\frac{1}{8}}^{\frac{1}{4}} \frac{1}{\sqrt{2x(1-2x)}} dx.$

2 marks

Question 2

A 20 kg crate is pulled across a rough horizontal floor by a force, of magnitude 100 newtons, applied upwards at an angle of 40° to the horizontal.

- a. Complete the following diagram so that it shows **all** the forces acting on the crate.



1 mark

- b. The coefficient of friction between the crate and the floor is 0.34.
Find the acceleration of the crate, in m/s^2 , correct to two decimal places.

4 marks

Question 3

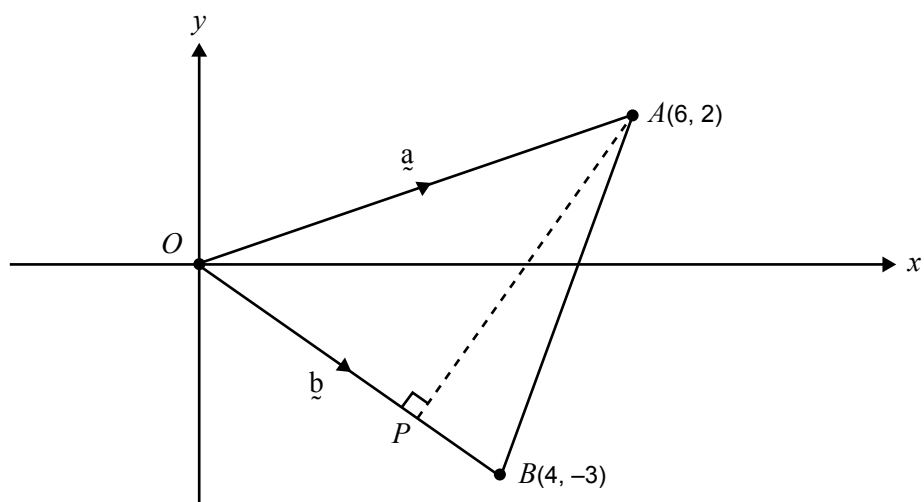
If $f'(x) = 15x\sqrt{2-x}$ and $f(2) = 0$, then $f(x)$ can be written in the form $(ax+b)(2-x)^{\frac{3}{2}}$.
Find the values of a and b .

4 marks

Question 4

Points $O(0, 0)$, $A(6, 2)$ and $B(4, -3)$ form the vertices of a triangle as shown in the diagram below.

The position vectors $\vec{OA} = \underline{a} = 6\hat{i} + 2\hat{j}$ and $\vec{OB} = \underline{b} = 4\hat{i} - 3\hat{j}$ are indicated. AP is an altitude of triangle OAB .



- a. Find the scalar resolute of $\underline{a}$ in the direction of $\underline{b}$.

2 marks

- b. Hence find the length of the altitude AP .

2 marks

Question 5

Let $w = -\frac{\sqrt{3}}{2} - \frac{1}{2}i$.

- a.** Express w in **exact** polar form.

1 mark

- b.** Hence find the least positive integer k for which $w^k = 1$.

2 marks

SPECIALIST MATHEMATICS

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

Specialist Mathematics Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder:	$2\pi rh$
volume of a cylinder:	$\pi r^2 h$
volume of a cone:	$\frac{1}{3}\pi r^2 h$
volume of a pyramid:	$\frac{1}{3}Ah$
volume of a sphere:	$\frac{4}{3}\pi r^3$
area of a triangle:	$\frac{1}{2}bc \sin A$
sine rule:	$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
cosine rule:	$c^2 = a^2 + b^2 - 2ab \cos C$

Coordinate geometry

ellipse:	$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$	hyperbola:	$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$
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Circular (trigometric) functions

$$\cos^2(x) + \sin^2(x) = 1$$

$$1 + \tan^2(x) = \sec^2(x)$$

$$\sin(x+y) = \sin(x) \cos(y) + \cos(x) \sin(y)$$

$$\cos(x+y) = \cos(x) \cos(y) - \sin(x) \sin(y)$$

$$\tan(x+y) = \frac{\tan(x) + \tan(y)}{1 - \tan(x) \tan(y)}$$

$$\cos(2x) = \cos^2(x) - \sin^2(x) = 2 \cos^2(x) - 1 = 1 - 2 \sin^2(x)$$

$$\sin(2x) = 2 \sin(x) \cos(x)$$

$$\cot^2(x) + 1 = \operatorname{cosec}^2(x)$$

$$\sin(x-y) = \sin(x) \cos(y) - \cos(x) \sin(y)$$

$$\cos(x-y) = \cos(x) \cos(y) + \sin(x) \sin(y)$$

$$\tan(x-y) = \frac{\tan(x) - \tan(y)}{1 + \tan(x) \tan(y)}$$

$$\tan(2x) = \frac{2 \tan(x)}{1 - \tan^2(x)}$$

function	$\sin^{-1}$	$\cos^{-1}$	$\tan^{-1}$
domain	$[-1, 1]$	$[-1, 1]$	R
range	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Algebra (Complex numbers)

$$z = x + yi = r(\cos \theta + i \sin \theta) = r \operatorname{cis} \theta$$

$$|z| = \sqrt{x^2 + y^2} = r$$

$$z_1 z_2 = r_1 r_2 \operatorname{cis}(\theta_1 + \theta_2)$$

$$z^n = r^n \operatorname{cis}(n\theta) \text{ (de Moivre's theorem)}$$

$$-\pi < \operatorname{Arg} z \leq \pi$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} \operatorname{cis}(\theta_1 - \theta_2)$$

Calculus

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dx}(e^{ax}) = ae^{ax}$$

$$\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$$

$$\frac{d}{dx}(\sin(ax)) = a \cos(ax)$$

$$\frac{d}{dx}(\cos(ax)) = -a \sin(ax)$$

$$\frac{d}{dx}(\tan(ax)) = a \sec^2(ax)$$

$$\frac{d}{dx}(\sin^{-1}(x)) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\cos^{-1}(x)) = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\tan^{-1}(x)) = \frac{1}{1+x^2}$$

$$\int x^n dx = \frac{1}{n+1} x^{n+1} + c, n \neq -1$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax} + c$$

$$\int \frac{1}{x} dx = \log_e(x) + c, \text{ for } x > 0$$

$$\int \sin(ax) dx = -\frac{1}{a} \cos(ax) + c$$

$$\int \cos(ax) dx = \frac{1}{a} \sin(ax) + c$$

$$\int \sec^2(ax) dx = \frac{1}{a} \tan(ax) + c$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c, a > 0$$

$$\int \frac{-1}{\sqrt{a^2 - x^2}} dx = \cos^{-1}\left(\frac{x}{a}\right) + c, a > 0$$

$$\int \frac{a}{a^2 + x^2} dx = \tan^{-1}\left(\frac{x}{a}\right) + c$$

product rule:

$$\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$$

quotient rule:

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

chain rule:

$$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$$

mid-point rule:

$$\int_a^b f(x) dx \approx (b-a) f\left(\frac{a+b}{2}\right)$$

trapezoidal rule:

$$\int_a^b f(x) dx \approx \frac{1}{2}(b-a)(f(a) + f(b))$$

Euler's method:

$$\text{If } \frac{dy}{dx} = f(x), x_0 = a \text{ and } y_0 = b, \text{ then } x_{n+1} = x_n + h \text{ and } y_{n+1} = y_n + hf(x_n)$$

acceleration:

$$a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v \frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

$$\text{constant (uniform) acceleration: } v = u + at \quad s = ut + \frac{1}{2}at^2 \quad v^2 = u^2 + 2as \quad s = \frac{1}{2}(u+v)t$$

TURN OVER

Vectors in two and three dimensions

$$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$$

$$|\vec{r}| = \sqrt{x^2 + y^2 + z^2} = r$$

$$\vec{r}_1 \cdot \vec{r}_2 = r_1 r_2 \cos \theta = x_1 x_2 + y_1 y_2 + z_1 z_2$$

$$\dot{\vec{r}} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\vec{i} + \frac{dy}{dt}\vec{j} + \frac{dz}{dt}\vec{k}$$

Mechanics

momentum:

$$\vec{p} = m\vec{v}$$

equation of motion:

$$\vec{R} = m\vec{a}$$

friction:

$$F \leq \mu N$$