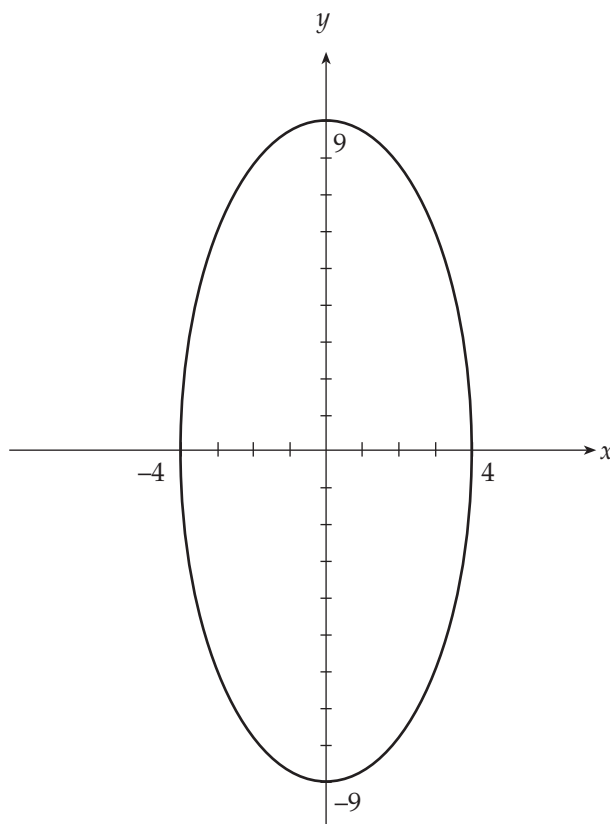


Part I: Multiple-choice questions**Question 1**

The equation of the above ellipse is:

A $\frac{x^2}{2} + \frac{y^2}{3} = 1$

B $\frac{x^2}{4} + \frac{y^2}{9} = 1$

C $\frac{x^2}{16} + \frac{y^2}{81} = 1$

D $\frac{x^2}{4} - \frac{y^2}{9} = 1$

E $\frac{x^2}{16} - \frac{y^2}{81} = 1$

Question 2

The exact value of $\sin^{-1}\left(\cos\left(\frac{7\pi}{6}\right)\right)$ is:

A $-\frac{\pi}{3}$

B $-\frac{\pi}{6}$

C $\frac{\pi}{3}$

D $-\frac{2\pi}{3}$

E $\frac{7\pi}{6}$

Question 3

If $\cos(x) = -\frac{2}{3}$, $\frac{\pi}{2} \leq x \leq \pi$, then $\tan(x)$ is equal to:

A $\frac{\sqrt{5}}{2}$

B $-\frac{\sqrt{5}}{2}$

C $\frac{2}{\sqrt{5}}$

D $-\frac{2}{\sqrt{5}}$

E $\frac{\sqrt{13}}{2}$

Question 4

If $z^4 - 81 = 0$ then z is equal to:

- A $\pm 8i$
- B $\pm 3i$
- C $\pm 3, \pm 3i$
- D ± 3
- E ± 9

Question 5

If $z = 2\text{cis}\left(\frac{5\pi}{6}\right)$ and $w = 5\text{cis}\left(\frac{\pi}{3}\right)$, then $\text{Arg}(zw)$ is equal to:

- A $\frac{7\pi}{6}$
- B $\frac{\pi}{6}$
- C $\frac{5\pi}{18}$
- D $-\frac{\pi}{6}$
- E $-\frac{5\pi}{6}$

Question 6

If $z = 2 + 3i$ and $w = 2 - i$ then $\bar{z}w$ is:

- A $7 + 4i$
- B $4 + 3i$
- C $4 - 3i$
- D $1 - 8i$
- E 7

Question 7

If $z = -\sqrt{3} + i$ then z^4 is equal to:

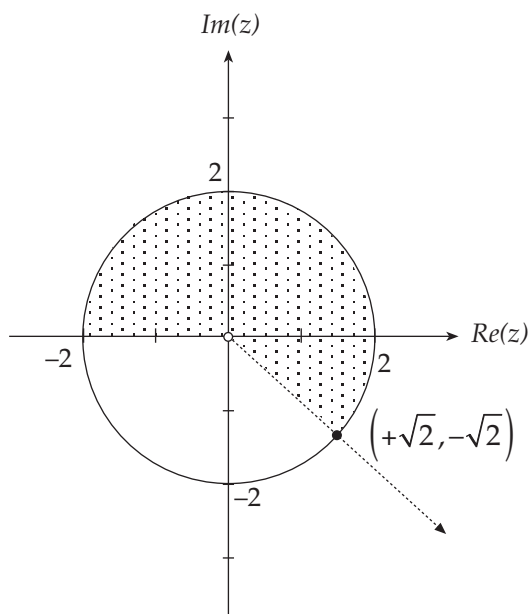
A $16\text{cis}\left(-\frac{2\pi}{3}\right)$

B $16\text{cis}\left(\frac{625\pi}{1296}\right)$

C $16\text{cis}\left(\frac{5\pi}{6}\right)$

D $10\text{cis}\left(\frac{5\pi}{6}\right)$

E $9 + i$

Question 8

Given that $z \in \mathbb{C}$, the shaded region is best described by:

A $\{z : z\bar{z} \leq 2\} \cup \{|z - 2| < |z + 2|\}$

B $\{z : |z| \leq 2\} \cap \{|z - 2i| \geq |z + 2|\}$

C $\{z : z\bar{z} \leq 2\} \cap \left\{\text{Arg}(z) \geq \frac{\pi}{4}\right\}$

D $\{z : |z| \leq 2\} \cap \left\{\text{Arg}(z) \geq -\frac{\pi}{4}\right\}$

E $\{z : z\bar{z} > 2\} \cup \left\{\text{Arg}(z) < -\frac{\pi}{4}\right\}$

Question 9

If $\frac{3x-1}{x^2+x-6}$ is written as the partial fractions $\frac{A}{x+3} + \frac{B}{x-2}$ then,

- A $A = 1$ and $B = 2$
- B $A = 3x$ and $B = 0$
- C $A = -2$ and $B = 3$
- D $A = 2$ and $B = 1$
- E $A = -2$ and $B = -1$

Question 10

The derivative of $x \tan^{-1}(x)$ with respect to x is $\frac{x}{x^2+1} + \tan^{-1}(x)$. It follows that an anti-derivative of $\tan^{-1}(x)$ is:

- A $\int \tan^{-1}(x) dx - \frac{x}{x^2+1}$
- B $x \tan^{-1}(x) - \frac{1}{2} \log_e(x^2+1)$
- C $x \tan^{-1}(x) + \int \frac{x}{x^2+1} dx$
- D $x \tan^{-1}(x) + \log_e(x^2+1)$
- E $\int x \tan^{-1}(x) dx - \int \frac{x}{x^2+1} dx$

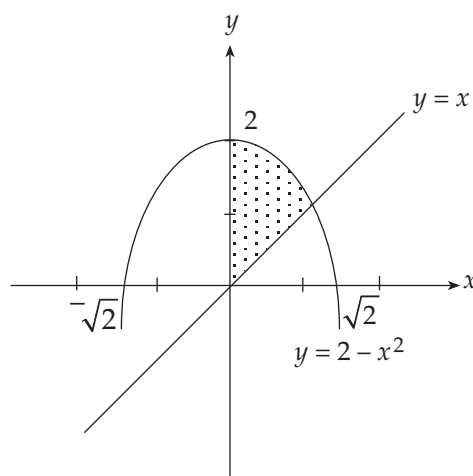
Question 11

Using an appropriate substitution $\int_1^3 (2x+1)\sqrt{2x-1} dx$ is equal to:

- A $\frac{1}{2} \int_1^5 (u+2)\sqrt{u} du$
- B $\int_1^5 (u+2)\sqrt{u} du$
- C $\int_1^3 (u+2)\sqrt{u} du$
- D $\frac{1}{2} \int_1^3 (u+2)\sqrt{u} du$
- E $\frac{1}{2} \int_1^3 (u-2)\sqrt{u} du$

Question 12

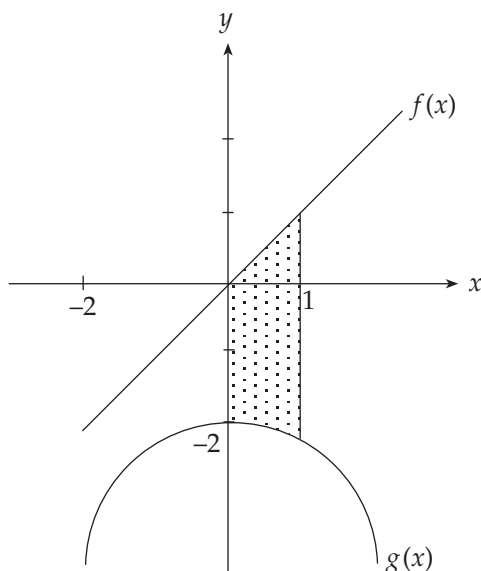
A volume of revolution is found by rotating the area enclosed by the line $y = x$, the parabola $y = 2 - x^2$ and the y -axis about the x -axis.



The volume of revolution formed is equivalent to:

- A $\pi \int \left((2 - x^2)^2 - x^2 \right) dx$
- B $\pi \int_0^1 (4 + x^4 - x^2) dx$
- C $\pi \int_0^1 (2 - x^2)^2 dx$
- D $\pi \int_1^2 (2 - x - x^2) dx$
- E $\pi \int_0^1 \left((2 - x^2)^2 - x^2 \right) dx$

Question 13



Given $g(x) = \frac{-2}{\sqrt{4-x^2}}$ and $f(x) = x$. The shaded region in the diagram is bounded by the y -axis, $g(x)$, $f(x)$ and the line $x = 1$.

The exact value of the area of the shaded region is:

- A $\frac{1}{2} + \frac{\pi}{6}$
- B $\frac{\pi}{6} - \frac{1}{2}$
- C $\frac{\pi}{3} - \frac{1}{2}$
- D $\frac{\pi}{12} + \frac{1}{2}$
- E $\frac{1}{2} + \frac{\pi}{3}$

Question 14

The value of $\int_0^1 (e^{x^2} + e^{-x^2}) dx$, correct to four decimal places, is:

- A 4.9626
- B 3.8271
- C 2.9253
- D 2.9626
- E 8.8126

Question 15

Using the midpoint rule and one interval, $\int_0^1 \cos(2x)dx$ would be approximated by:

- A $\cos(2)$
- B $\frac{\cos(0) + \cos(2)}{2}$
- C $\cos(1)$
- D $\frac{1}{2}\cos(2)$
- E $\frac{1}{2}\cos(1)$

Question 16

A tank initially contains 100 litres of water in which is dissolved 20 kg of salt. A salt solution which contains 0.1 kg of salt per litre of water is poured into the tank at a rate of 2 litres/minute. An amount of the solution is simultaneously withdrawn from the tank at the same rate.

Assuming that the mixture is uniform and x kg is the amount of salt in the tank at t minutes, the differential equation that models this situation is:

- A $\frac{dx}{dt} = 0.1 - x$
- B $\frac{dx}{dt} = 0.2 - x$
- C $\frac{dx}{dt} = (0.2 - 2x) \times \frac{20}{100}$
- D $\frac{dx}{dt} = 0.2 - 2x$
- E $\frac{dx}{dt} = 0.2 - \frac{x}{50}$

Question 17

The solution of the equation $\frac{d^2y}{dx^2} = \cos(2x)$ with $\frac{dy}{dx} = 0$ and $y = 0$ at $x = 0$ is:

A $y = \frac{1}{4}\cos(2x)$

B $y = -\frac{1}{4}\cos(2x) + \frac{1}{4}$

C $y = \frac{1}{4}\cos(2x) - \frac{1}{4}$

D $y = -\frac{1}{4}\cos(2x)$

E $y = -\frac{1}{4}\cos(2x) + \frac{x}{2} + \frac{1}{4}$

Question 18

The angle between $\underline{a} = 2\underline{i} - \underline{j} + 3\underline{k}$ and $\underline{b} = \underline{i} + 3\underline{j} - 2\underline{k}$ is

A 90°

B 60°

C 120°

D 0°

E 30°

Question 19

A boat travels 9 kilometres due north, 6 kilometres in a direction 60° west of north and then 7 kilometres due east. If $\underline{i}$ and $\underline{j}$ are unit vectors in the directions east and north respectively, the position vector of the final position of the boat relative to its starting position is exactly:

A $(7 - 3\sqrt{3})\underline{i} + 12\underline{j}$

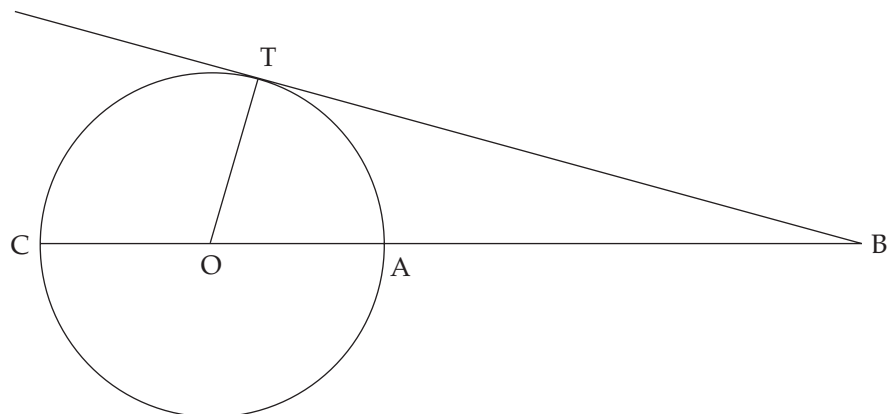
B $11\underline{i} + 3(3 + 3\sqrt{3})\underline{j}$

C $(4 + 3\sqrt{3})\underline{i} + 12\underline{j}$

D $3\underline{k} + 3(3 - \sqrt{3})\underline{j}$

E $15\underline{i} + 13\underline{j}$

Question 20



O is the centre of the circle and BT is a tangent.

Let $\overrightarrow{OT} = \underline{\underline{c}}$ and $\overrightarrow{OB} = \underline{\underline{b}}$.

The dot product $\overrightarrow{OT} \cdot \overrightarrow{TB}$ is equal to:

- A 90
- B $\sqrt{c^2 + b^2}$
- C 0
- D $\underline{\underline{c}} \cdot \underline{\underline{b}}$
- E $\underline{\underline{c}} \cdot (\underline{\underline{c}} + \underline{\underline{b}})$

Question 21

The magnitude of $4\underline{\underline{a}} + \underline{\underline{b}}$ if $\underline{\underline{a}} = 2\underline{\underline{i}} + \underline{\underline{j}} + 5\underline{\underline{k}}$ and $\underline{\underline{b}} = -4\underline{\underline{i}} + \underline{\underline{j}}$ is:

- A 6
- B $2\sqrt{11}$
- C $\sqrt{569}$
- D $4\sqrt{30}$
- E 21

Question 22

Given $\underline{a} = 6\underline{i} - 2\underline{j} + 6\underline{k}$ and $\underline{b} = -6\underline{i} - 2\underline{j} + \underline{k}$ the vector resolute of $\underline{b}$ in the direction of $\underline{a}$ is equal to:

A $-\frac{13}{\sqrt{19}}(6\underline{i} - 2\underline{j} + \underline{k})$

B $-\frac{13}{\sqrt{19}}(-6\underline{i} - 2\underline{j} + \underline{k})$

C $\frac{1}{2\sqrt{19}}(6\underline{i} - 2\underline{j} + 6\underline{k})$

D $-\frac{13}{\sqrt{19}}$

E $-\frac{13}{19}(3\underline{i} - \underline{j} + 3\underline{k})$

Question 23

The position of a body at time $t, t \geq 0$, is given by $\underline{r}(t) = (t-1)\underline{i} + 5(t-1)^2\underline{j}$. The Cartesian equation of the body's path is represented by:

A $y = 5x^2, x \geq 0$

B $y = 5x^2, x \geq -1$

C $x = \pm\sqrt{5}y, y \geq 0$

D $x = \sqrt{5}y, y \leq -1$

E $y = \sqrt{5}x, x \geq -1$

Question 24

The position of a particle is given by: $\underline{r}(t) = (2\sin 2t + 1)\underline{i} + 3e^t\underline{j}$.

The speed of the particle at $t = 0$ is:

A $4\underline{i} + 3\underline{j}$

B 25

C $4\cos 2t\underline{i} + 3e^t\underline{j}$

D 5

E $\underline{i} + 3\underline{j}$

Question 25

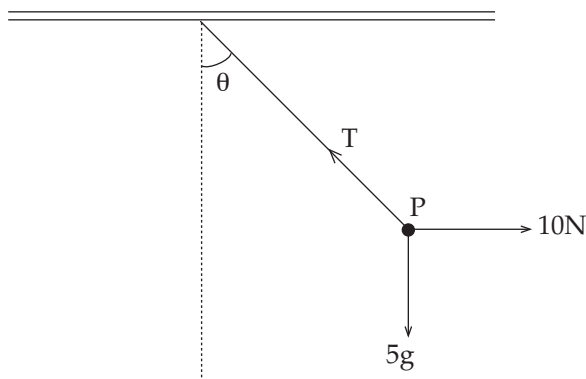
If a particle of mass 4 kg is acted on by two forces $\vec{F}_1 = 2\vec{i} + \vec{j}$ and $\vec{F}_2 = 8\vec{i} - 5\vec{k}$, the acceleration of the particle is:

- A $10\vec{i} + \vec{j} - 5\vec{k}$
- B $6\vec{i} - \vec{j} - 5\vec{k}$
- C $2.5\vec{i} + 0.25\vec{j} - 1.25\vec{k}$
- D 2.806
- E $\sqrt{126}$

Question 26

A light inelastic string suspended from a horizontal beam has a particle, P, of mass 5 kg attached to it. A horizontal force of 10 N is applied to the particle. This means that the string is θ radians from the vertical.

θ is approximately:



- A 11.5346
- B 0.2013
- C 1.3695
- D 78.4654
- E 0.4636

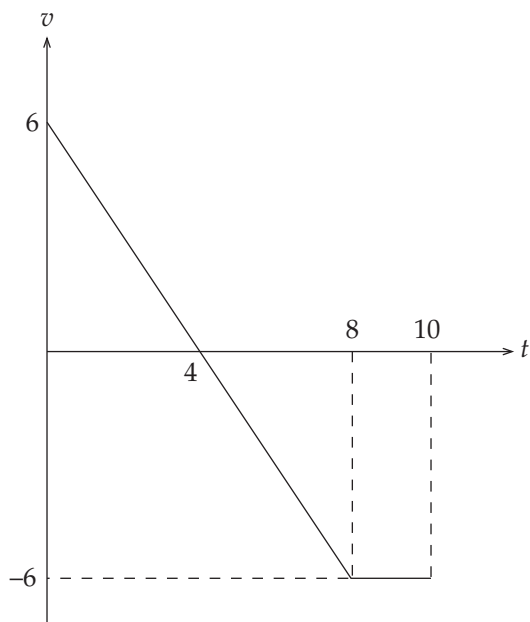
Question 27

A particle moves in a straight line with acceleration of $a = x + 6$ at time t . Given that at $x = 1$, $v = 7$, the velocity of the particle is:

- A $\pm (x + 6)$
- B $x^2 + 12x$
- C $x + 6$
- D $x^2 + 12x + 36$
- E 49

Question 28

The following is the velocity-time graph of a particle over a 10 second interval.

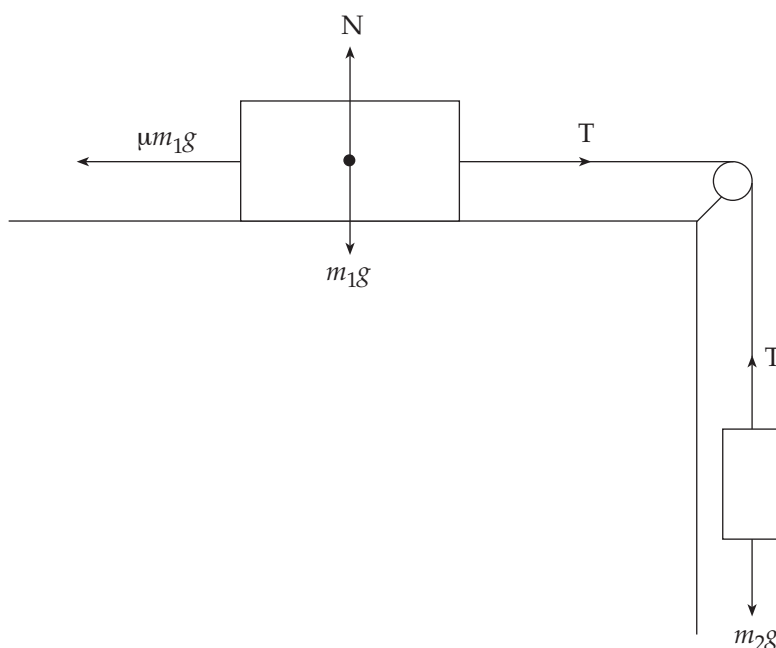


The total distance travelled during the 10 second interval is:

- A -12
- B -24
- C 12
- D 36
- E 60

Question 29

A block of mass m_1 rests on a rough horizontal table. The coefficient of friction between the block and the table is μ . The block is connected to a second block of mass m_2 by a light inelastic string which passes over a smooth pulley. The mass m_2 hangs vertically.

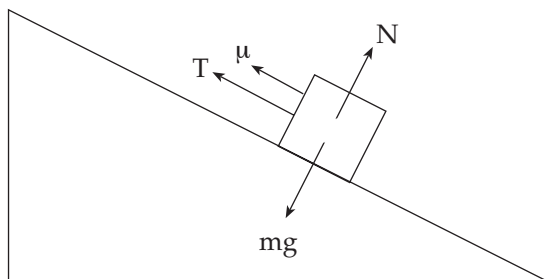
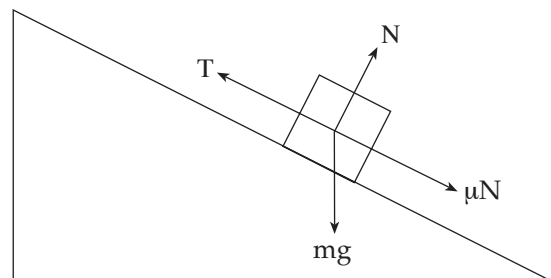
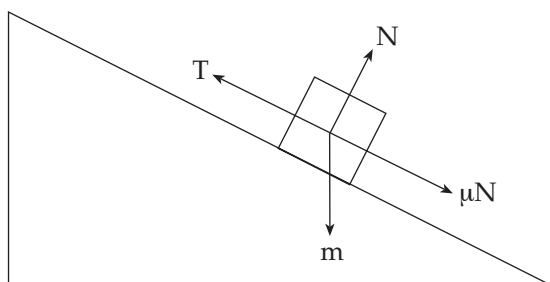
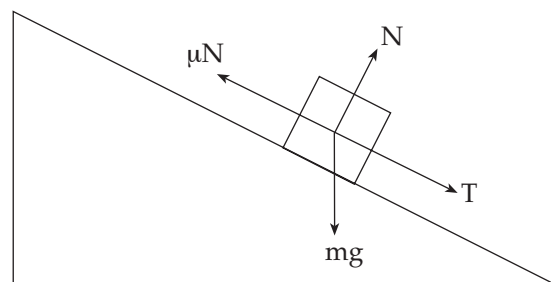
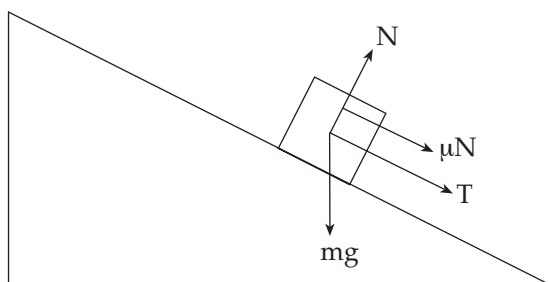


If the block of mass m_1 is accelerating to the right, then,

- A $T - \mu m_1g > m_2g$
- B $T > m_2g$
- C $m_2 < \mu m_1$
- D $m_2 > \mu m_1$
- E $m_1g = m_2g$

Question 30

Terry is dragging a large box of mass m up a ramp. The force she applies to the box is T newtons. The coefficient of friction between the box and the ramp is μ . The normal force exerted by the ramp on the box is N newtons. The diagram which correctly shows all the forces acting on the box is:

A**B****C****D****E**

Part II: Short answer questions**Question 1**

Let $f : \mathbb{R} \rightarrow \mathbb{R}$ where $f(x) = \frac{-1}{1+x^2}$.

- a** **i** Use calculus to find $f'(x)$.

- ii** Find $f'(1)$.

(2 + 1 = 3 marks)

- b** Hence, use Euler's method to estimate $f(1.01)$.

(1 mark)

Total = 4 marks

Question 2

Use calculus to find the exact value of $\int_1^{\sqrt{2}} \frac{1}{\sqrt{4-x^2}} dx$.

Total = 2 marks

Question 3

A particle moves so that its position vector at time t is given by $\underline{r}(t) = 4\cos t \underline{i} + 4\sin t \underline{j} + 3t \underline{k}$, with $t \geq 0$.

- a** Find its initial speed.

(3 marks)

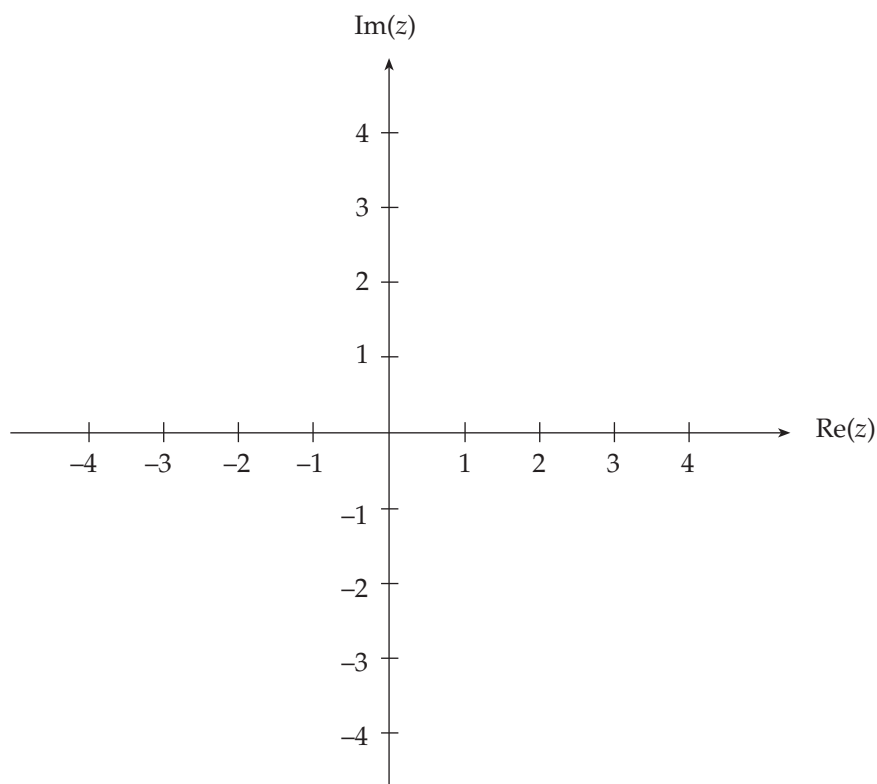
- b** Show that the velocity of the particle is always perpendicular to its acceleration.

(3 marks)

Total = 6 marks

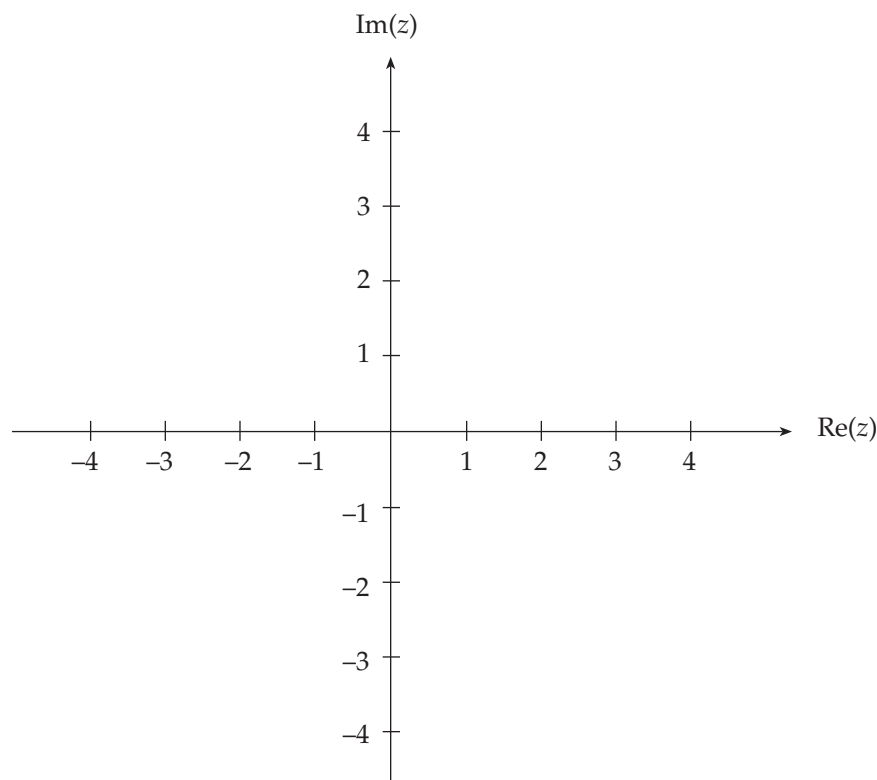
Question 4

- a** Shade the region of the complex plane specified by $\{z : |z - 2 + i| \leq 2\}$.



(2 marks)

- b** Shade the region of the complex plane specified by $\{z : 0 \leq \text{Arg}(z + 1) \leq \frac{\pi}{4}\}$.



(2 marks)

Total = 4 marks

Question 5

Consider the expression $(\cos \theta + i \sin \theta)^3$ where θ is a real number.

- a** Use the expression to show that $\cos 3\theta = \cos^3 \theta - 3\cos \theta \sin^2 \theta$.

(3 marks)

- b** Hence, or otherwise, find a similar expression for $\sin 3\theta$.

(1 mark)

Total = 4 marks