
Part I – Multiple-choice answers

1.	D	7.	C	13.	C	19.	D	25.	D
2.	D	8.	A	14.	B	20.	B	26.	C
3.	B	9.	E	15.	B	21.	D	27.	D
4.	E	10.	E	16.	C	22.	C	28.	D
5.	A	11.	D	17.	B	23.	D	29.	B
6.	A	12.	D	18.	C	24.	D	30.	A

Part I- Multiple-choice solutions

Question 1

The graph of $y = 3x^2 + \frac{5}{x}$ has a vertical asymptote given by $x = 0$. Its other asymptote is

given by $y = 3x^2$ so reject option E. The graph has a local minimum where $x = \sqrt[3]{\frac{5}{6}}$ and not

where $x = 1$ so reject option B. The graph is defined for $y < 0$ so reject option C.

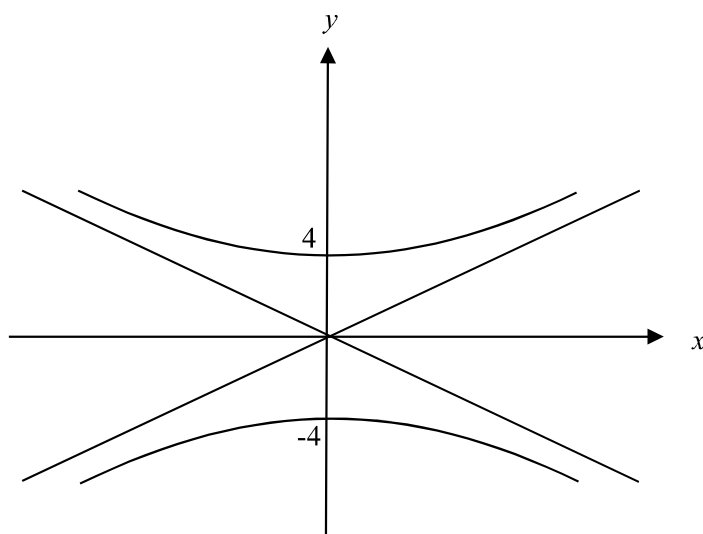
Also, the graph has an x -intercept but not at $x = -1$ so reject option A.

Only option D is correct since $x = 0$ is an asymptote and hence there are no y -intercepts.

The answer is D.

Question 2

The graph of the relation $\frac{y^2}{16} - \frac{x^2}{25} = 1$ is the hyperbola shown below.



It intersects with the y -axis twice.

The answer is D.

Question 3

$$y = \cos^{-1}\left(\frac{x}{2}\right)$$

$$= \cos^{-1}(u) \text{ where } u = \frac{x}{2} \text{ and so } \frac{du}{dx} = \frac{1}{2}$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} \text{ (Chain rule)}$$

$$= \frac{-1}{\sqrt{1-u^2}} \cdot \frac{1}{2}$$

$$= \frac{-1}{2\sqrt{1-\left(\frac{x}{2}\right)^2}}$$

$$= \frac{-1}{2\sqrt{\frac{4-x^2}{4}}}$$

$$= \frac{-1}{\sqrt{4-x^2}}$$

At $x = 0$

$$\frac{dy}{dx} = \frac{-1}{2}$$

The answer is B.

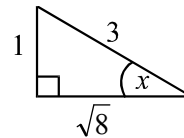
Question 4

$$\sin(x) = -\frac{1}{3}, \quad \frac{3\pi}{2} \leq x \leq 2\pi$$

$$\cos(x) = \frac{\sqrt{8}}{3} \text{ since in the fourth quadrant cos is positive}$$

$$\begin{aligned} \text{So } \sec(x) &= \frac{3}{2\sqrt{2}} \\ &= \frac{3\sqrt{2}}{4} \end{aligned}$$

The answer is E.



Question 5

$$\begin{aligned}
 u &= 2 - i, & v &= \bar{u} + 1 \\
 & & &= 2 + i + 1 \\
 & & &= 3 + i
 \end{aligned}$$

$$\begin{aligned}
 \text{So, } \frac{u}{v} &= \frac{2-i}{3+i} \\
 &= \frac{2-i}{3+i} \times \frac{3-i}{3-i} \\
 &= \frac{6-5i-1}{10} \\
 &= \frac{5-5i}{10} \\
 &= \frac{1}{2} - \frac{i}{2}
 \end{aligned}$$

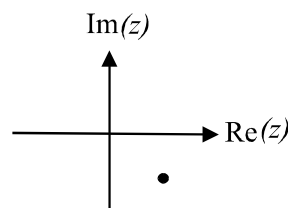
The answer is A.

Question 6

$$\begin{aligned}
 r &= \sqrt{3+1} \\
 &= 2
 \end{aligned}$$

Note that the question asks for “a” polar form.

$$\begin{aligned}
 \text{Now } \text{Arg}(\sqrt{3}-i) &= \tan^{-1}\left(\frac{-1}{\sqrt{3}}\right) \quad (\text{in } 4^{\text{th}} \text{ quadrant}) \\
 &= -\frac{\pi}{6}
 \end{aligned}$$



This is the value of $\text{Arg}(z)$ in the range $(-\pi, \pi]$.

So a polar form of $\sqrt{3}-i$ could be $2\text{cis}\left(\frac{-\pi}{6}\right)$ or $2\text{cis}\left(\frac{-13\pi}{6}\right)$ and so on.

In this case the only correct option is A.

The answer is A.

Question 7

We have a semicircle.

Now $\{z : |z| \leq 2\}$ describes a circle with radius 2 units.

Also, $\{z : \text{Im}(z) \geq 0\}$ describes the top half of the complex plane including the real axis.

Hence $\{z : |z| \leq 2\} \cap \{z : \text{Im}(z) \geq 0\}$ describes the semicircle we have. Note that

$\{z : |z| \leq 2\} \cap \{z : \text{Im}(z) > 0\}$ is close but excludes the Real axis between -2 and 2 which is included in the diagram.

The answer is C.

Question 8

Since $P(z)$ is a cubic polynomial with real coefficients and one of its solutions is $1+i$ then another of its solutions must be $1-i$, that is, the conjugate of $1+i$ (conjugate root theorem).

The other solution must be real.

Note that $z-1+i$ is a factor not a solution, as is $z-2$.

The only feasible answer is 3.

The answer is A.

Question 9

Let $z = r\text{cis}\theta$

So $z^3 = \text{cis}\left(\frac{\pi}{2}\right)$

becomes $(r\text{cis}\theta)^3 = \text{cis}\left(\frac{\pi}{2}\right)$

$$r^3 \text{cis}(3\theta) = \text{cis}\left(\frac{\pi}{2}\right) \quad (\text{De Moivre's Theorem})$$

So, $r = 1$ and $3\theta = \frac{\pi}{2} + 2k\pi, \quad k \in J$

$$\theta = \frac{\pi}{6} + \frac{2k\pi}{3}$$

If $k = 0, \quad \theta = \frac{\pi}{6}$

If $k = -1, \quad \theta = \frac{\pi - 4\pi}{6}$
 $= \frac{-\pi}{2}$

If $k = 1, \quad \theta = \frac{\pi + 4\pi}{6}$
 $= \frac{5\pi}{6}$

So the three solutions are $\text{cis}\left(\frac{\pi}{6}\right), \text{cis}\left(\frac{-\pi}{2}\right)$, and $\text{cis}\left(\frac{5\pi}{6}\right)$.

The answer is E.

Question 10

$$\begin{aligned}
& \int \frac{x}{\sqrt{3-x}} dx \\
&= \int \frac{3-u}{\sqrt{u}} \cdot -1 \frac{du}{dx} dx && \text{let } u = 3 - x \\
&= -1 \int \left(3u^{-\frac{1}{2}} - u^{\frac{1}{2}} \right) du && \frac{du}{dx} = -1 \\
& && x = 3 - u \\
&= -1 \left(2 \times 3u^{\frac{1}{2}} - \frac{2u^{\frac{3}{2}}}{\frac{3}{2}} \right) + c \\
&= \frac{2}{3} u^{\frac{3}{2}} - 6u^{\frac{1}{2}} + c \\
&= \frac{2}{3} (3-x)^{\frac{3}{2}} - 6(3-x)^{\frac{1}{2}} + c
\end{aligned}$$

Note “an antiderivative” means c takes on a particular value. In this case $c = 0$.
The answer is E.

Question 11

$$\begin{aligned}
\int \frac{-2}{\sqrt{4-x^2}} dx &= -2 \int \frac{1}{\sqrt{4-x^2}} dx \\
&= -2 \text{Sin}^{-1} \left(\frac{x}{2} \right) + c
\end{aligned}$$

OR

$$\begin{aligned}
\int \frac{-2}{\sqrt{4-x^2}} dx &= 2 \int \frac{-1}{\sqrt{4-x^2}} dx \\
&= 2 \text{Cos}^{-1} \left(\frac{x}{2} \right) + c
\end{aligned}$$

Only the second answer is offered.
The answer is D.

Question 12

With the integration techniques available to us in this course, we are not able to antiderivate $\sqrt{9-x^2}$.

Use a graphics calculator instead.

The answer, correct to 4 decimal places is 8.4633.

The answer is D.

Question 13

$$f'(x) = \sin^2(3x)$$

$$\text{So, } f(x) = \int \sin^2(3x) dx$$

$$= \frac{1}{2} \int (1 - \cos 6x) dx$$

$$= \frac{1}{2} \left(x - \frac{1}{6} \sin(6x) \right) + c$$

$$\text{When } x = \frac{\pi}{6}, \quad f(x) = \frac{\pi}{12}$$

$$\text{So, } \frac{\pi}{12} = \frac{1}{2} \left(\frac{\pi}{6} - \frac{1}{6} \sin(\pi) \right) + c$$

$$\frac{\pi}{12} = \frac{1}{2} \left(\frac{\pi}{6} - 0 \right) + c$$

$$\frac{\pi}{12} = \frac{\pi}{12} + c$$

$$c = 0$$

$$f(x) = \frac{x}{2} - \frac{1}{12} \sin(6x)$$

The answer is C.

Question 14

$$y = \log_e(e^{2x})$$

$$\frac{dy}{dx} = \frac{2e^{2x}}{e^{2x}}$$

$$= 2$$

$$\frac{d^2y}{dx^2} = 0$$

$$\text{So } \frac{d^2y}{dx^2} = \frac{dy}{dx} - 2$$

Alternatively,

$$y = \log_e(e^{2x})$$

$$= 2x$$

$$\frac{dy}{dx} = 2$$

$$\frac{d^2y}{dx^2} = 0$$

$$\text{So, } \frac{d^2y}{dx^2} = \frac{dy}{dx} - 2$$

The answer is B.

Question 15

$$\begin{aligned}
 \text{Total Area} &= 3 \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} (2 \cos(2x) - \cos(2x)) dx \quad \text{by symmetry} \\
 &= 3 \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \cos(2x) dx \\
 &= 6 \int_0^{\frac{\pi}{4}} \cos(2x) \quad \text{again by symmetry}
 \end{aligned}$$

Note that in option D, the second term should be negative and similarly in option E.
The answer is B.

Question 16

On the graph of $y = f'(x)$, $f'(-2) = 0$ and $f'(1) = 0$. So at $x = -2$ and at $x = 1$ on the graph of $y = f'(x)$ we have a stationary point.

For $x < -2$, $f'(x) > 0$ and for $-2 < x < 1$, $f'(x) > 0$ so at $x = -2$ on the graph of $y = f'(x)$, we must have a stationary point of inflection.

For $-2 < x < 1$, $f'(x) > 0$ and for $x > 1$, $f'(x) < 0$ so at $x = 1$ on the graph of $y = f'(x)$, we must have a local maximum.

There cannot be a stationary point of inflection at $x = 0$ since $f'(0) \neq 0$.

The answer is C.

Question 17

Using the formula sheet, we have, if $\frac{dy}{dx} = f(x)$, $x_0 = a$ and $y_0 = b$,

then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x_n)$

Now $x_0 = 0$ and $y_0 = 0$

$$\begin{aligned}
 \text{and } \frac{dy}{dx} &= f(x) \\
 &= \log_e(2x+1)
 \end{aligned}$$

Also, $h = 0.1$

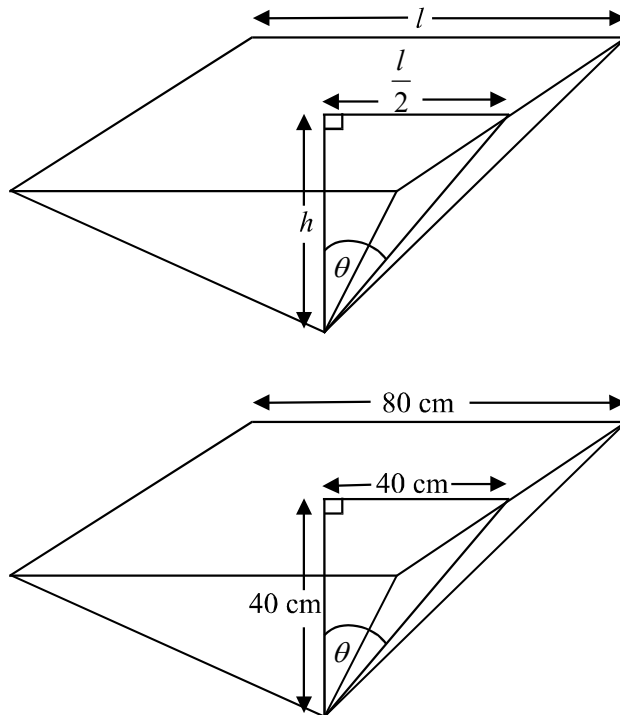
$$\begin{aligned}
 \text{So, } x_1 &= 0 + 0.1 \quad \text{and} \quad y_1 = 0 + 0.1 \times f(0) \\
 &= 0.1 \quad \quad \quad = 0
 \end{aligned}$$

$$\begin{aligned}
 x_2 &= 0.1 + 0.1 \quad \quad \quad y_2 = 0 + 0.1 \times f(0.1) \\
 &= 0.2 \quad \quad \quad = 0.1 \log_e(1.2)
 \end{aligned}$$

$$\text{So, when } x = 0.2, y = \frac{\log_e(1.2)}{10}$$

The answer is B.

Question 18



$$V = \frac{1}{3} Ah$$

$$= \frac{1}{3} \times l^2 h \text{ where } l \text{ is the sidelength of the square}$$

In the triangle drawn, $\tan \theta = 1$

$$\text{so, } \frac{l}{2} = h, \quad l = 2h$$

$$\text{So } V = \frac{1}{3} \times 4h^2 \times h$$

$$= \frac{4h^3}{3}$$

$$\frac{dV}{dh} = 4h^2$$

$$\text{Now, } \frac{dh}{dt} = \frac{dh}{dV} \cdot \frac{dV}{dt}$$

$$= \frac{1}{4h^2} \cdot -10$$

$$= \frac{-5}{2h^2}$$

The answer is C.

Question 19

$$\begin{aligned}
 a &= v(2v+1) \\
 v \frac{dv}{dx} &= v(2v+1) \\
 \frac{dv}{dx} &= 2v+1 \\
 \frac{dx}{dv} &= \frac{1}{2v+1} \\
 x &= \int \frac{1}{2v+1} dv \\
 x &= \frac{1}{2} \log_e(2v+1) + c
 \end{aligned}$$

When $x=0$, $v=1$

$$\begin{aligned}
 0 &= \frac{1}{2} \log_e(3) + c \\
 x &= \frac{1}{2} \log_e(2v+1) - \frac{1}{2} \log_e(3) \\
 x &= \frac{1}{2} \log_e\left(\frac{2v+1}{3}\right)
 \end{aligned}$$

The answer is D.

Question 20

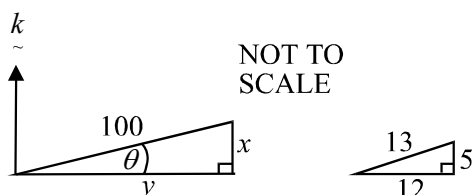
$$\begin{aligned}
 \vec{OP} &= 3\vec{i} + 2\vec{j} - \vec{k} \\
 \vec{OQ} &= 2\vec{i} + \vec{j} - \vec{k} \\
 \vec{PQ} &= \vec{PO} + \vec{OQ} \\
 &= -3\vec{i} - 2\vec{j} + \vec{k} + 2\vec{i} + \vec{j} - \vec{k} \\
 &= -\vec{i} - \vec{j} \\
 |\vec{PQ}| &= \sqrt{1+1} \\
 &= \sqrt{2}
 \end{aligned}$$

The answer is B.

Question 21

Note that since the surveyor is travelling up at a gradient of $\frac{5}{12}$ he won't travel 100 m in the $\tilde{i} - \tilde{j}$ plane.

Consider the vertical component.



From the diagram we have $\frac{x}{5} = \frac{100}{13}$ since the triangles are similar.

$$x = \frac{500}{13}$$

Also note from the diagram that $\frac{y}{12} = \frac{100}{13}$

$$y = \frac{1200}{13}$$

This is the distance travelled in the $\tilde{i} - \tilde{j}$ plane.

In the $\tilde{i} - \tilde{j}$ plane,

$$\sin 30^\circ = \text{opp} \div \frac{1200}{13} \quad \text{and} \quad \cos 30^\circ = \text{adj} \div \frac{1200}{13}$$

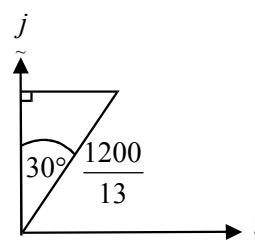
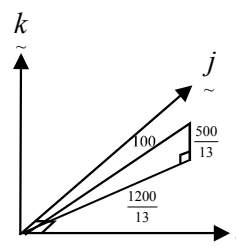
$$\frac{1}{2} = \text{opp} \times \frac{13}{1200} \quad \frac{\sqrt{3}}{2} = \text{adj} \times \frac{13}{1200}$$

$$\text{opp} = \frac{600}{13}$$

$$\text{adj} = \frac{600\sqrt{3}}{13}$$

$$\text{So } \vec{OP} = \frac{600}{13} \tilde{i} + \frac{600\sqrt{3}}{13} \tilde{j} + \frac{500}{13} \tilde{k}$$

The answer is D.



Question 22

$$\underline{\underline{u}} = n \underline{\underline{i}}, \quad \underline{\underline{v}} = \underline{\underline{i}} - \underline{\underline{j}}$$

$$\underline{\underline{u}} \bullet \underline{\underline{v}} = n$$

$$\underline{\underline{u}} \bullet \underline{\underline{v}} = |\underline{\underline{u}}| |\underline{\underline{v}}| \cos \theta \quad \text{where } \theta \text{ is the angle between } \underline{\underline{u}} \text{ and } \underline{\underline{v}}$$

$$= \sqrt{n^2} \sqrt{1+1} \cos \theta$$

$$\text{So } n = \sqrt{2} n \cos \theta$$

$$\frac{1}{\sqrt{2}} = \cos \theta$$

$$\theta = \frac{\pi}{4}$$

The answer is C.

Question 23

$$\underline{\underline{a}} \bullet \underline{\underline{b}} = 0 \quad \text{since } \angle RPQ = 90^\circ.$$

So option A is correct.

Option B is correct. (triangle rule for addition of vectors)

$$\text{Consider } \underline{\underline{a}} \bullet \underline{\underline{a}} + \underline{\underline{b}} \bullet \underline{\underline{b}} = \underline{\underline{c}} \bullet \underline{\underline{c}}$$

$$\begin{aligned} LS &= \underline{\underline{a}} \bullet \underline{\underline{a}} + \underline{\underline{b}} \bullet \underline{\underline{b}} \\ &= |\underline{\underline{a}}| |\underline{\underline{a}}| \cos(0^\circ) + |\underline{\underline{b}}| |\underline{\underline{b}}| \cos(0^\circ) \\ &= |\underline{\underline{a}}|^2 + |\underline{\underline{b}}|^2 \\ &= |\underline{\underline{c}}|^2 \quad (\text{Pythagoras}) \end{aligned}$$

$$\begin{aligned} RS &= \underline{\underline{c}} \bullet \underline{\underline{c}} \\ &= |\underline{\underline{c}}| |\underline{\underline{c}}| \cos(0^\circ) \\ &= |\underline{\underline{c}}|^2 \\ &= LS \end{aligned}$$

So option C is correct.

$$\begin{aligned} & \left(\underline{\underline{a}} - \underline{\underline{b}} \right) \bullet \left(\underline{\underline{a}} + \underline{\underline{b}} \right) \\ &= \underline{\underline{a}} \bullet \underline{\underline{a}} - \underline{\underline{b}} \bullet \underline{\underline{b}} \\ &= |\underline{\underline{a}}| |\underline{\underline{a}}| \cos(0^\circ) - |\underline{\underline{b}}| |\underline{\underline{b}}| \cos(0^\circ) \\ &= |\underline{\underline{a}}|^2 - |\underline{\underline{b}}|^2 \\ &= 0 \quad \text{since } |\underline{\underline{a}}| = |\underline{\underline{b}}| \end{aligned}$$

So option D is incorrect. Option E is correct since the sum of side lengths PR and PQ must be greater than sidelength QR . The answer is D.

Question 24

$$\begin{aligned}
 \left| 2\hat{i} + \hat{j} - \hat{k} \right| &= \sqrt{4 + 1 + 1} \\
 &= \sqrt{6} \\
 \left| -\hat{i} + 2\hat{j} + \hat{k} \right| &= \sqrt{1 + 4 + 1} \\
 &= \sqrt{6}
 \end{aligned}$$

Now the vector resolute of $\hat{a}$ perpendicular to $\hat{b}$ is $\hat{a} - (\hat{a} \cdot \hat{b})\hat{b}$.

So, the vector resolute of $2\hat{i} + \hat{j} - \hat{k}$ perpendicular to $-\hat{i} + 2\hat{j} + \hat{k}$ is

$$\begin{aligned}
 &2\hat{i} + \hat{j} - \hat{k} - \left(\left(2\hat{i} + \hat{j} - \hat{k} \right) \cdot \frac{1}{\sqrt{6}} \left(-\hat{i} + 2\hat{j} + \hat{k} \right) \right) \frac{1}{\sqrt{6}} \left(-\hat{i} + 2\hat{j} + \hat{k} \right) \\
 &= 2\hat{i} + \hat{j} - \hat{k} - \left(\frac{1}{\sqrt{6}} (-2 + 2 - 1) \right) \times \frac{1}{\sqrt{6}} \left(-\hat{i} + 2\hat{j} + \hat{k} \right) \\
 &= 2\hat{i} + \hat{j} - \hat{k} + \frac{1}{6} \left(-\hat{i} + 2\hat{j} + \hat{k} \right) \\
 &= \frac{11}{6}\hat{i} + \frac{8}{6}\hat{j} - \frac{5}{6}\hat{k} \\
 &= \frac{1}{6} \left(11\hat{i} + 8\hat{j} - 5\hat{k} \right)
 \end{aligned}$$

The answer is D.

Question 25

$$\begin{aligned}
 \underline{r}(t) &= \tan(t)\hat{i} + 3\hat{j} + e^{4t}\hat{k} \\
 \underline{v}(t) &= \sec^2(t)\hat{i} + 4e^{4t}\hat{k} \\
 \underline{v}(0) &= \sec^2(0)\hat{i} + 4e^0\hat{k} \\
 &= \frac{1}{\cos^2(0)}\hat{i} + 4\hat{k} \\
 &= \hat{i} + 4\hat{k}
 \end{aligned}$$

The motion of the particle initially is in the direction of $\hat{i} + 4\hat{k}$.

The answer is D.

Question 26

$$\underline{a}(t) = 6t^2 \underline{i} + \cos(2t) \underline{j}, \quad t \geq 0$$

$$\underline{v}(t) = 2t^3 \underline{i} + \frac{1}{2} \sin(2t) \underline{j} + \underline{c}$$

When $t = 0$, $\underline{v} = \underline{0}$

$$\underline{0} = 0 \underline{i} + 0 \underline{j} + \underline{c}$$

$$\text{So } \underline{v}(t) = 2t^3 \underline{i} + \frac{1}{2} \sin(2t) \underline{j}$$

The momentum of the particle, $m \underline{v}$, is given by

$$8t^3 \underline{i} + 2 \sin(2t) \underline{j}$$

Note that momentum is a vector quantity.
The answer is C.

Question 27

$$\underline{R} = m \underline{a}$$

$$\underline{F}_1 + \underline{F}_2 = 5 \underline{a}$$

$$5 \underline{i} + (n+5) \underline{j} = 5(\underline{i} + 2 \underline{j})$$

$$5 \underline{i} + (n+5) \underline{j} = 5 \underline{i} + 10 \underline{j}$$

$$\text{So } n = 5$$

The answer is D.

Question 28

The box is stationary.

Resolving horizontally, we have

$$Fr = 5 \cos(30^\circ)$$

$$Fr = \frac{5\sqrt{3}}{2}$$

So options A and C are not correct.

Note, since the box is not on the point of moving across the table,

$$Fr \neq \mu N$$

In fact $Fr < \mu N$.

So option D is correct.

Resolving vertically, we have

$$N + 5 \sin(30^\circ) = 12g$$

$$N = 12g - \frac{5}{2}$$

So option B and option E are incorrect.

The answer is D.

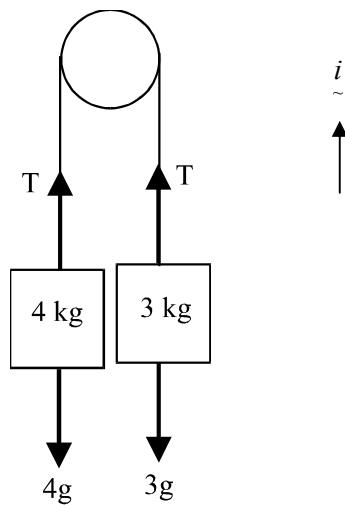
Question 29

The gravitational forces acting on each particle are $m_A g$ and $m_B g$. This eliminates options A and D.

The friction forces act in the opposite direction to the force F . This eliminates option E. The tension force in the connection runs in both directions. This eliminates option C.

Only option B shows all forces correctly.

The answer is B.

Question 30

Around the 3kg box.

$$\vec{R} = m \vec{a}$$

$$(T - 3g)\hat{i} = 3a\hat{i}$$

$$\text{So } a = \frac{T - 3g}{3}$$

The answer is A.

PART II**Question 1**

When George finally passes the stationary red car, each has covered the same distance since George was originally overtaken.

Let the original speed of the red car be v .

$$\text{So, } v \times 10 + \frac{1}{2} \times (30 - 10) \times v = 20 \times 30 \quad \text{(1 mark)}$$

$$10v + 10v = 600$$

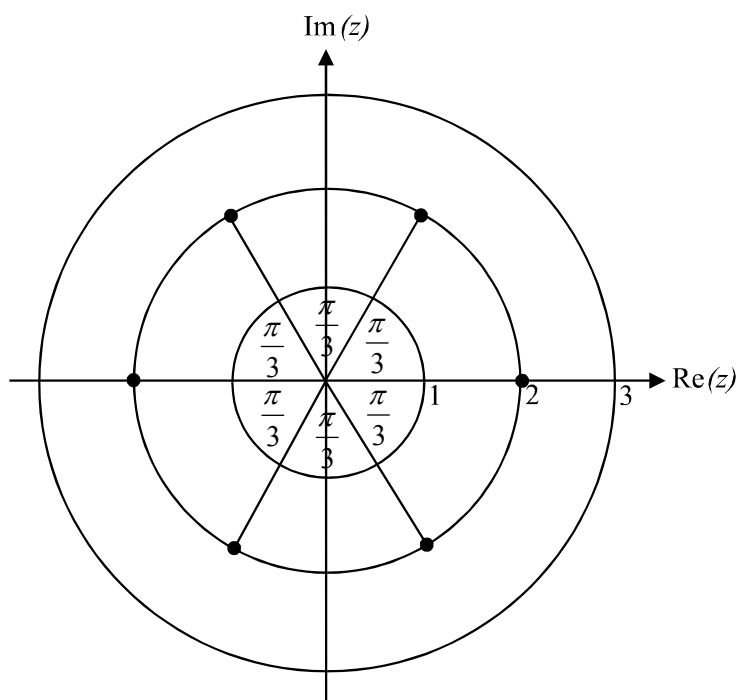
$$v = 30$$

The red car was travelling at 30 m/s.

(1 mark)

Question 2

- a. The 6 solutions to the equation $z^6 - 64 = 0$ are spaced evenly around a circle with radius $64^{\frac{1}{6}} = 2$. Since one has already been given to us the others must be spaced at intervals of $2\pi \div 6 = \frac{\pi}{3}$ apart as indicated in the diagram below.



(1 mark)

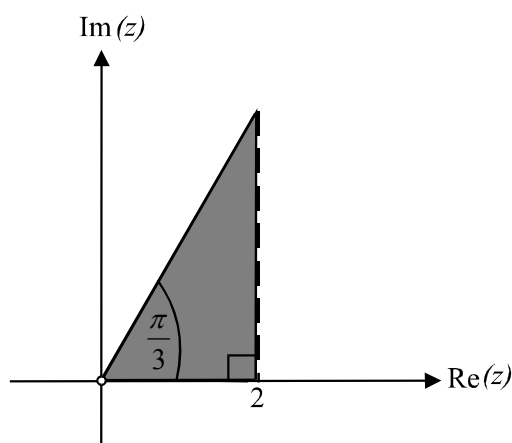
b.



area required

boundary included

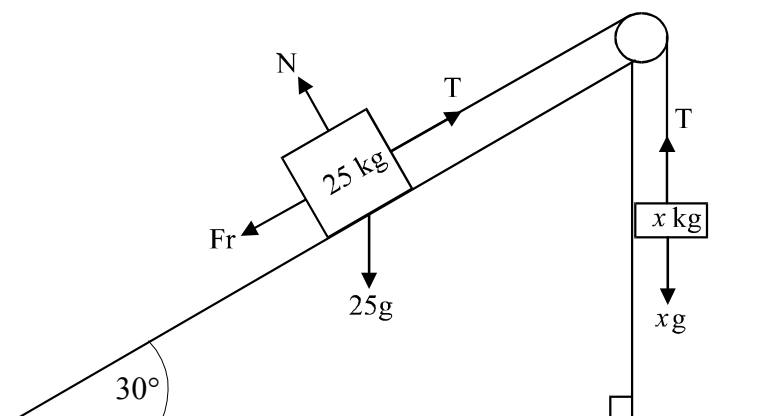
boundary excluded



(1 mark) correct area
(1 mark) correct boundaries

Question 3

a.



(1 mark)

b. The system is at the point of moving so $\tilde{R} = 0$.

Resolving around the 25kg object we have
 $T = Fr + 25g \sin(30^\circ)$ and $N = 25g \cos(30^\circ)$

$$= \mu N + \frac{25g}{2} = \frac{25\sqrt{3}g}{2}$$

$$= 0.6 \times \frac{25\sqrt{3}g}{2} + \frac{25g}{2}$$

$$= 7.5\sqrt{3}g + 12.5g$$

(1 mark) for correct resolution

(1 mark)

Resolving around the x kg object we have

$$T = xg$$

$$\text{So } x = 7.5\sqrt{3} + 12.5$$

$$= 25.49 \text{ (correct to 2 decimal places)}$$

(1 mark)

Question 4

a. $\sqrt{2} \sin\left(2x + \frac{\pi}{4}\right) = \cos(2x)$

$$\sqrt{2} \left(\sin(2x) \cos\left(\frac{\pi}{4}\right) + \cos(2x) \sin\left(\frac{\pi}{4}\right) \right) = \cos(2x) \quad \text{(1 mark)}$$

$$\sqrt{2} \left(\sin(2x) \times \frac{1}{\sqrt{2}} + \cos(2x) \times \frac{1}{\sqrt{2}} \right) = \cos(2x)$$

$$\sin(2x) + \cos(2x) = \cos(2x)$$

$$\sin(2x) = 0$$

$$2x = 0, \pi, 2\pi, 3\pi, 4\pi$$

$$x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$$

Domain $0 \leq x \leq 2\pi$

so $0 \leq 2x \leq 4\pi$

(1 mark)

b. $\int_0^{\frac{\pi}{6}} \sin^2(x) \cos^3(x) dx$

$$= \int_0^{\frac{\pi}{6}} \sin^2(x) \cos^2(x) \cos(x) dx$$

$$= \int_0^{\frac{\pi}{6}} \sin^2(x) (1 - \sin^2(x)) \cos(x) dx$$

$$= \int_0^{\frac{\pi}{6}} (\sin^2(x) - \sin^4(x)) \cos(x) dx \quad \text{(1 mark)}$$

$$= \int_0^{\frac{1}{2}} (u^2 - u^4) \frac{du}{dx} dx \quad \begin{array}{l} \text{(1 mark) for integrand} \\ \text{(1 mark) for terminals} \end{array}$$

$$= \left[\frac{u^3}{3} - \frac{u^5}{5} \right]_0^{\frac{1}{2}}$$

$$= \left\{ \left[\frac{\left(\frac{1}{2}\right)^3}{3} - \frac{\left(\frac{1}{2}\right)^5}{5} \right] - 0 \right\}$$

$$= \frac{1}{24} - \frac{1}{160}$$

$$= \frac{20 - 3}{480}$$

$$= \frac{17}{480}$$

(1 mark)

$$u = \sin(x)$$

$$\frac{du}{dx} = \cos(x)$$

$$x = \frac{\pi}{6}, u = \frac{1}{2}$$

$$x = 0, u = 0$$

Question 5

a. Area required

$$= \frac{\frac{\pi}{3} - 0}{2 \times 2} \left(f(0) + 2f\left(\frac{\pi}{6}\right) + f\left(\frac{\pi}{3}\right) \right) \quad (1 \text{ mark})$$

$$= \frac{\pi}{12} \left(\tan(0) + 2 \tan\left(\frac{\pi}{6}\right) + \tan\left(\frac{\pi}{3}\right) \right)$$

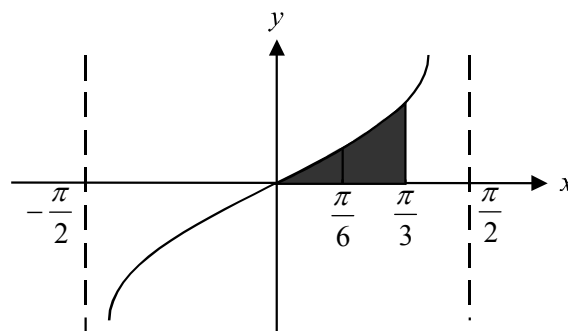
$$= \frac{\pi}{12} \left(0 + \frac{2}{\sqrt{3}} + \sqrt{3} \right)$$

$$= \frac{\pi}{12} \left(\frac{2+3}{\sqrt{3}} \right)$$

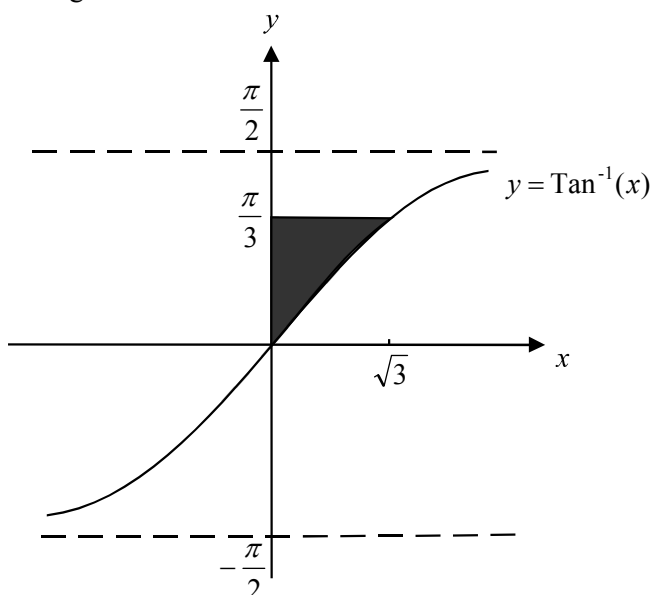
$$= \frac{\pi}{12} \times \frac{5}{\sqrt{3}}$$

$$= \frac{5\sqrt{3}\pi}{36} \text{ square units}$$

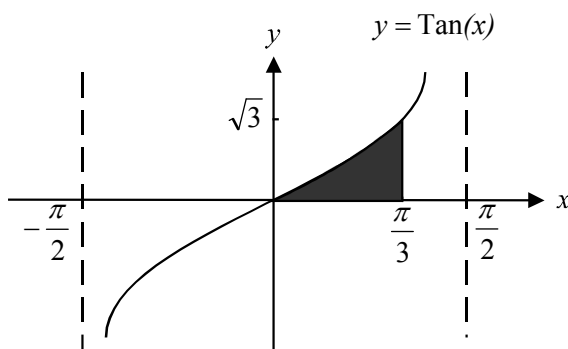
(1 mark)



b. Draw a diagram.



The shaded area above is equal to the shaded area shown in the diagram below on the graph of its inverse function, $y = \tan(x)$.



$$\text{Area required} = \int_0^{\frac{\pi}{3}} \tan(x) \, dx \quad \text{(1 mark)}$$

$$= \int_0^{\frac{\pi}{3}} \tan(x) \, dx, \quad x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

$$= \int_0^{\frac{\pi}{3}} \frac{\sin(x)}{\cos(x)} \, dx$$

$$= \int_1^{\frac{1}{2}} \frac{1}{u} \cdot -\frac{du}{dx} \, dx \quad \text{(1 mark)}$$

$$= \int_{\frac{1}{2}}^1 \frac{1}{u} \, du$$

$$= [\log_e(u)]_{\frac{1}{2}}^1$$

$$= \log_e(1) - \log_e\left(\frac{1}{2}\right)$$

$$= 0 - \log_e\left(\frac{1}{2}\right)$$

$$= \log_e(2)$$

Area required is $\log_e(2)$ square units.

(1 mark)

(Do NOT express this answer as a decimal approximation since you have been asked for an exact value. Also, if you have time, check your answer using your graphics calculator.)

Total 20 marks