



Victorian Certificate of Education 2003

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

STUDENT NUMBER

Letter

Figures

Words

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SPECIALIST MATHEMATICS

Written examination 2

(Analysis task)

Wednesday 5 November 2003

Reading time: 11.45 am to 12.00 noon (15 minutes)

Writing time: 12.00 noon to 1.30 pm (1 hour 30 minutes)

QUESTION AND ANSWER BOOK

Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
5	5	60

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, up to four pages (two A4 sheets) of pre-written notes (typed or handwritten) and an approved scientific and/or graphics calculator (memory may be retained).
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 15 pages with a detachable sheet of miscellaneous formulas in the centrefold.
- Working space is provided throughout the book.

Instructions

- Detach the formula sheet from the centre of this book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other electronic communication devices into the examination room.

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Instructions

- Answer **all** questions in the spaces provided.
- A decimal approximation will not be accepted if an **exact** answer is required to a question.
- Where an **exact** answer is required to a question, appropriate working must be shown.
- In questions where more than one mark is available, appropriate working must be shown.
- Where an instruction to **use calculus** is stated for a question, you must show an appropriate derivative or antiderivative.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- Take the **acceleration due to gravity** to have magnitude $g \text{ m/s}^2$, where $g = 9.8$.

Working space

TURN OVER

Later, the sled and snow tractor are parked on the slope with the snow tractor's brakes applied.

Let T newtons be the magnitude of the tension in the tow bar and F newtons be the magnitude of the frictional force between the sled and the ski slope.

- b.** Express T in terms of F .

2 marks

- c.** Find T , correct to the nearest integer, if the coefficient of friction between the sled and the ski slope is

- i.** 0.09

1 mark

- ii.** 0.15

1 mark

Total 8 marks

TURN OVER

Question 2

- a.** Using the fact that $\frac{\pi}{12} = \frac{\pi}{4} - \frac{\pi}{6}$, show that

$$\cos\left(\frac{\pi}{12}\right) = \frac{\sqrt{6} + \sqrt{2}}{4} \quad \text{and} \quad \sin\left(\frac{\pi}{12}\right) = \frac{\sqrt{6} - \sqrt{2}}{4}$$

3 marks

Let $u = \operatorname{cis}\left(\frac{\pi}{12}\right) = \frac{\sqrt{6} + \sqrt{2}}{4} + \frac{\sqrt{6} - \sqrt{2}}{4}i$, and let v be the complex conjugate of u .

- b.** Find the **exact** value of $\text{Arg}(v - u)$.

3 marks

- c. Find the **exact** value of $\text{Arg}\left(\frac{v}{u}\right)$.

2 marks

- d. Let u and v satisfy the quadratic equation $z^2 + az + b = 0$.
Show that $b = 1$ and find the **exact** value of a in surd form.

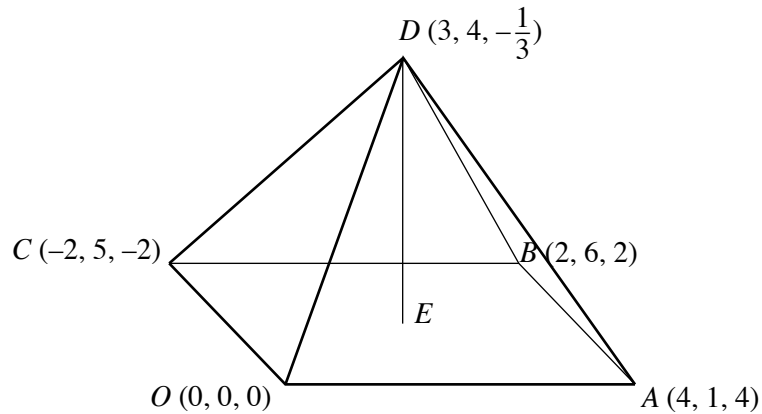
3 marks

Total 11 marks

TURN OVER

Question 3

$OABCD$ is a pyramid. The height of the pyramid is the length of DE where E is the point on the base $OABC$ such that DE is perpendicular to the base.



- a. Show that the base $OABC$ is a rhombus.

3 marks

- b. Use a vector method to find $\angle AOC$ correct to the nearest tenth of a degree.

3 marks

c. i. Show that $q = 0$ and find the **exact** values of p and r .

[illegible]

ii. Hence find the **exact** height of the pyramid.

[illegible]

TURN OVER

Question 4

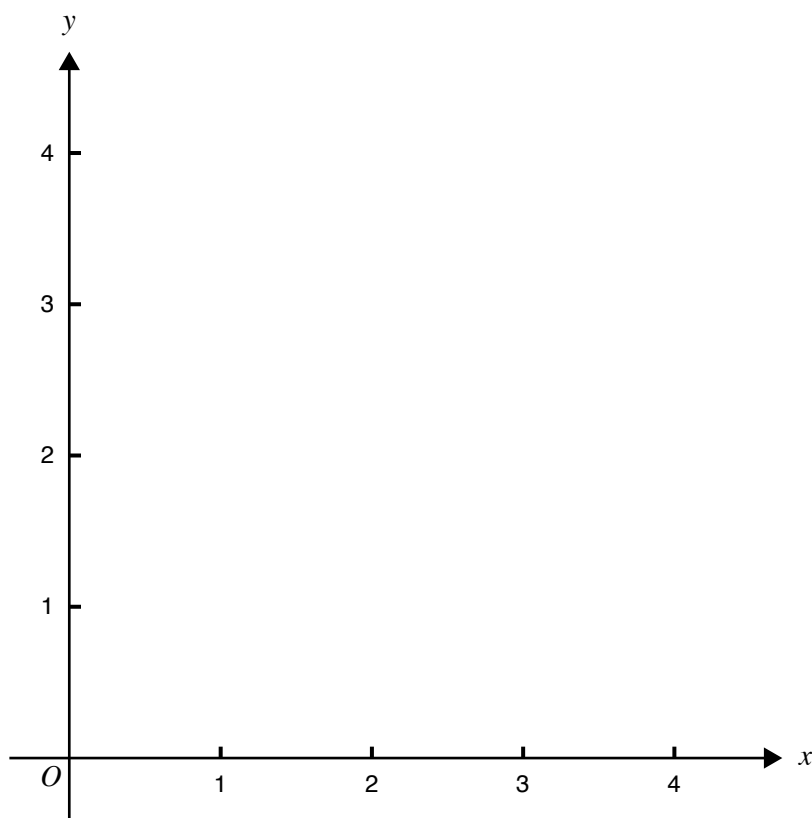
Consider the function $f: [0, 3) \rightarrow \mathbb{R}$ where $f(x) = -2 + 2 \sec\left(\frac{\pi x}{6}\right)$.

- a. Evaluate $f(2)$.

1 mark

Let f^{-1} be the inverse function of f .

- b. On the axes below, sketch the graphs of f and f^{-1} , showing their points of intersection.



2 marks

- c. The rule for f^{-1} can be written as $f^{-1}(x) = a \cos^{-1} \left(\frac{2}{x+2} \right)$.
Find the **exact** value of a .

2 marks

Let A be the magnitude of the area enclosed by the graphs of f and f^{-1} .

- d. Write a definite integral expression for A and evaluate it correct to three decimal places.

2 marks

- e. i. Given that $0 < kx < \frac{\pi}{2}$, show that the derivative of $\log_e \left(\frac{1 + \sin(kx)}{\cos(kx)} \right)$ is $k \sec(kx)$.

3 marks

- ii. Hence find the **exact** value of A , the magnitude of the area enclosed by the graphs of f and f^{-1} .

4 marks

Total 14 marks

Working space

TURN OVER

c. Sketch a graph of y in terms of t from when the baby's bottle is put into the boiling water to when the milk is ready to give to baby Alex.



d. Find the ratio $\frac{a}{b}$ correct to three significant figures.

[illegible]

Total 14 marks