

Specialist Mathematics Formulas

Mensuration

area of a trapezium:	$\frac{1}{2} (a + b)h$
curved surface area of a cylinder:	$2\pi rh$
volume of a cylinder:	$\pi r^2 h$
volume of a cone:	$\frac{1}{3} \pi r^2 h$
volume of a pyramid:	$\frac{1}{3} Ah$
volume of a sphere:	$\frac{4}{3} \pi r^3$
area of a triangle:	$\frac{1}{2} bc \sin A$
sine rule:	$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
cosine rule:	$c^2 = a^2 + b^2 - 2ab \cos C$

Coordinate geometry

ellipse:	$\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$
hyperbola:	$\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1$

Circular (trigonometric) functions

$\cos^2 x + \sin^2 x = 1$	
$1 + \tan^2 x = \sec^2 x$	$\cot^2 x + 1 = \operatorname{cosec}^2 x$
$\sin(x + y) = \sin x \cos y + \cos x \sin y$	$\sin(x - y) = \sin x \cos y - \cos x \sin y$
$\cos(x + y) = \cos x \cos y - \sin x \sin y$	$\cos(x - y) = \cos x \cos y + \sin x \sin y$
$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$	$\tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$
$\cos 2x = \cos^2 x - \sin^2 x = 2 \cos^2 x - 1 = 1 - 2 \sin^2 x$	
$\sin 2x = 2 \sin x \cos x$	$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$

function	$\sin^{-1}$	$\cos^{-1}$	$\tan^{-1}$
domain	$[-1, 1]$	$[-1, 1]$	R
range	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Algebra (Complex numbers)

$z = x + yi = r(\cos \theta + i \sin \theta) = r \operatorname{cis} \theta$	
$ z = \sqrt{x^2 + y^2} = r$	$-\pi < \operatorname{Arg} z \leq \pi$
$z_1 z_2 = r_1 r_2 \operatorname{cis}(\theta_1 + \theta_2)$	$\frac{z_1}{z_2} = \frac{r_1}{r_2} \operatorname{cis}(\theta_1 - \theta_2)$
$z^n = r^n \operatorname{cis} n\theta$ (de Moivre's theorem)	

Calculus

$$\frac{d}{dx} (x^n) = nx^{n-1}$$

$$\frac{d}{dx} (e^{ax}) = ae^{ax}$$

$$\frac{d}{dx} (\log_e x) = \frac{1}{x}$$

$$\frac{d}{dx} (\sin ax) = a \cos ax$$

$$\frac{d}{dx} (\cos ax) = -a \sin ax$$

$$\frac{d}{dx} (\tan ax) = a \sec^2 ax$$

$$\frac{d}{dx} (\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx} (\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx} (\tan^{-1} x) = \frac{1}{1+x^2}$$

$$\int x^n dx = \frac{1}{n+1} x^{n+1} + c, n \neq -1$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax} + c$$

$$\int \frac{1}{x} dx = \log_e x + c, \text{ for } x > 0$$

$$\int \sin ax dx = -\frac{1}{a} \cos ax + c$$

$$\int \cos ax dx = \frac{1}{a} \sin ax + c$$

$$\int \sec^2 ax dx = \frac{1}{a} \tan ax + c$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a} + c, a > 0$$

$$\int \frac{-1}{\sqrt{a^2 - x^2}} dx = \cos^{-1} \frac{x}{a} + c, a > 0$$

$$\int \frac{a}{a^2 + x^2} dx = \tan^{-1} \frac{x}{a} + c$$

product rule:

$$\frac{d}{dx} (uv) = u \frac{dv}{dx} + v \frac{du}{dx}$$

quotient rule:

$$\frac{d}{dx} \left(\frac{u}{v} \right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

chain rule:

$$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$$

mid-point rule:

$$\int_a^b f(x) dx \approx (b-a) f\left(\frac{a+b}{2}\right)$$

trapezoidal rule:

$$\int_a^b f(x) dx \approx \frac{1}{2} (b-a) (f(a) + f(b))$$

Euler's method:

$$\text{If } \frac{dy}{dx} = f(x), x_0 = a \text{ and } y_0 = b, \text{ then } x_{n+1} = x_n + h \text{ and } y_{n+1} = y_n + h f(x_n)$$

acceleration:

$$a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v \frac{dv}{dx} = \frac{d}{dx} \left(\frac{1}{2} v^2 \right)$$

$$\text{constant (uniform) acceleration: } v = u + at \quad s = ut + \frac{1}{2} at^2 \quad v^2 = u^2 + 2as \quad s = \frac{1}{2} (u + v)t$$

Vectors in two and three dimensions

$$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$$

$$|\vec{r}| = \sqrt{x^2 + y^2 + z^2} = r$$

$$\vec{r}_1 \cdot \vec{r}_2 = r_1 r_2 \cos \theta = x_1 x_2 + y_1 y_2 + z_1 z_2$$

$$\dot{\vec{r}} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\vec{i} + \frac{dy}{dt}\vec{j} + \frac{dz}{dt}\vec{k}$$

Mechanics

momentum: $\vec{p} = m\vec{v}$

equation of motion: $\vec{R} = m\vec{a}$

friction: $F \leq \mu N$