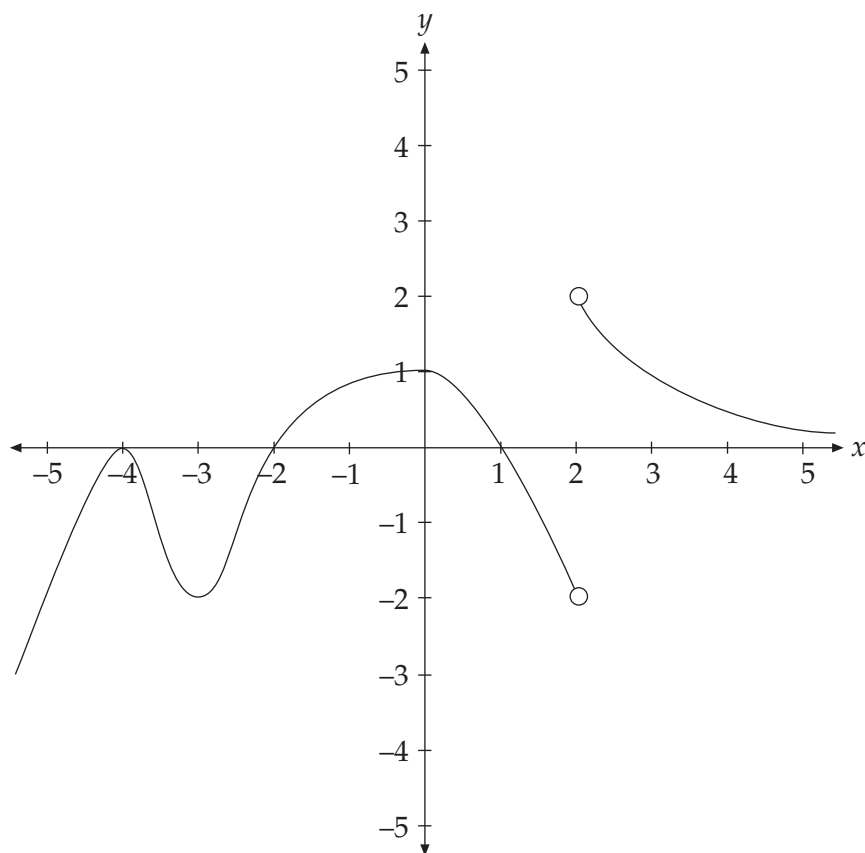


Question 1

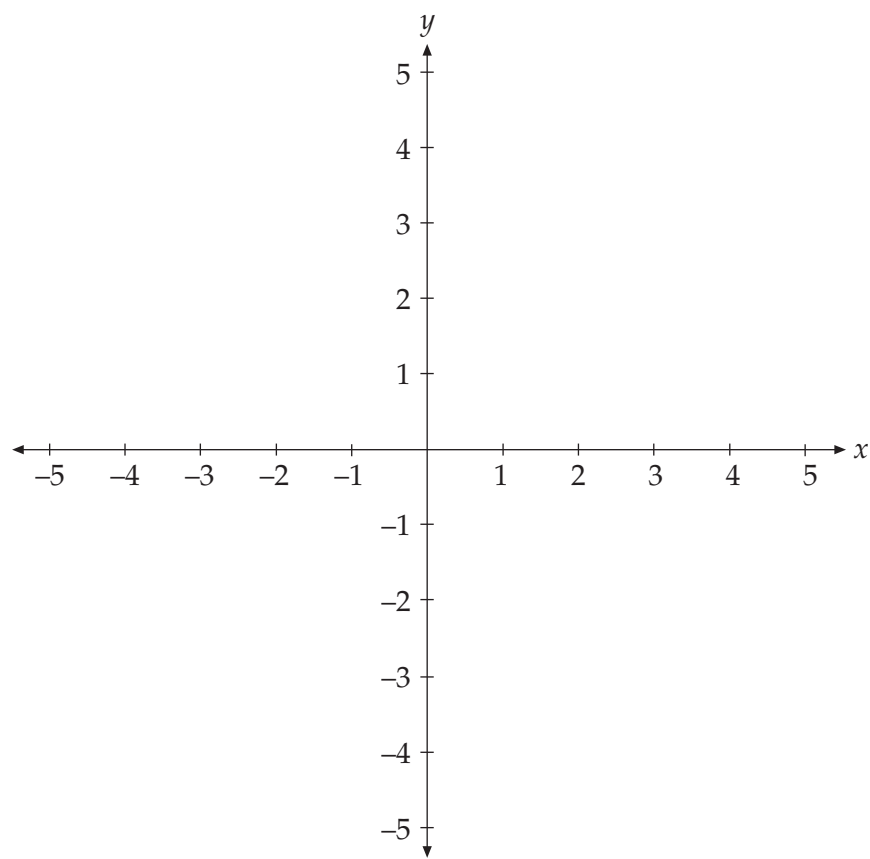
Below is a sketch graph of $y = f'(x)$ where $f'(x)$ is the derivative of the function $y = f(x)$. The curve $y = f'(x)$ has an asymptote at $y = 0$



- a. State the x values for each of the turning points of the curve $y = f(x)$ and by using $f''(x)$ classify them.

3 marks

- b. On the axes below sketch the curve $y = f(x)$ given $f(2) = -1$, $f(0) = 0$, and the graph of $f(x)$ is continuous.



3 marks

Total 6 marks

Question 2

Two sail boats Ragin' ($\vec{r}$) and Starin' ($\vec{s}$) are about to commence a race. Both are heading towards the start line with velocity $\dot{\vec{r}}(t) = -2t\vec{i} + 3\vec{j}$ and $\dot{\vec{s}}(t) = (4t^3 - 9t^2 + 2)\vec{i} + 2t\vec{j}$ (Knots) respectively. As the gun is fired to indicate the start of the race ($t = 0$) their position vectors are $\vec{s} = 2\vec{i}$ and $\vec{r} = 2\vec{i} - 2\vec{j}$.

- a i.** Find the position vector $\vec{s}(t)$ at time t of Starin'?

2 marks

- ii.** Find the position vector $\vec{r}(t)$ at time t of Ragin'?

2 marks

This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

ii. Find the coordinates of the point of the collision.

Page 4

If at the point of collision the angle between the boats is less than 30° and both boats are travelling at a speed of no more than 5 knots they will be able to continue their race.

- c What is the angle between the paths of the sailing boats at collision? Give your answer to the nearest degree.

4 marks

Total 14 marks

Question 3

a i. Show that the derivative of $\text{Cos}^{-1}\left(\frac{x-1}{x+1}\right)$ is $\frac{-1}{\sqrt{x}(x+1)}$

3 marks

ii. Hence, find the **exact** value of $\int_1^3 \frac{1}{\sqrt{x^3} + \sqrt{x}} dx$

2 marks

- b i.** For the functions $u = \text{Cos}^{-1}\left(\frac{x-1}{x+1}\right)$ and $v = \text{Tan}^{-1}\sqrt{x}$, find the value of a where

$$\frac{du}{dx} + a \frac{dv}{dx} = 0$$

3 marks

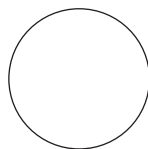
- ii.** Hence, show that $u + av = \pi$ for all values of x , where $\{x: x > 0\}$

2 marks

Total 10 marks

Question 4

A spherical ball is projected vertically upwards. It has an initial velocity of 10 m/s, and a mass of 2 kg. The air resistance is equal to $\frac{2v^2}{g}$ newtons.



- a. On the above diagram show all the forces acting on the ball 1 mark
- b. Show that the acceleration of the sphere, until it reaches its highest point, is $a = -g - \frac{v^2}{g}$.

1 mark

This image shows a single sheet of white paper with horizontal blue or grey ruling lines. The lines are evenly spaced and run across the width of the page. There are approximately 20 lines visible. The paper has a slight shadow on the right side, suggesting it's resting on a surface.

d. Find the maximum height reached by the sphere correct to nearest metre.

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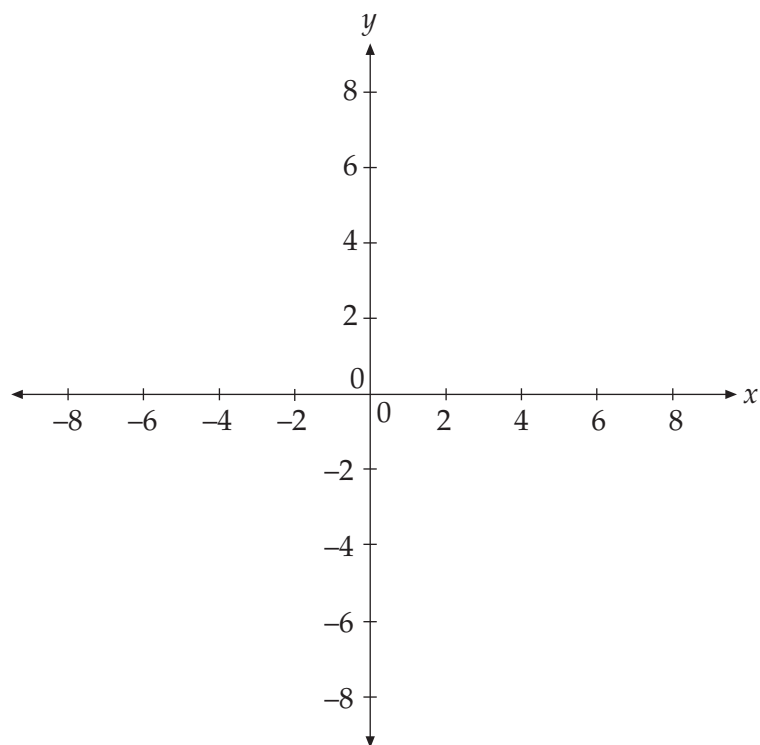
- e. How long does it take for the sphere to reach its maximum height? Give your answer to the nearest tenth of a second.

3 marks

Total 13 marks

Question 5

- a i. On the axes below sketch the graphs of $|z - 6| < 2$ and $|z + 2| \geq |z - 14|$ and hence shade $\{z: |z - 6| < 2\} \cap \{z: |z + 2| \geq |z - 14|\}$.



3 marks

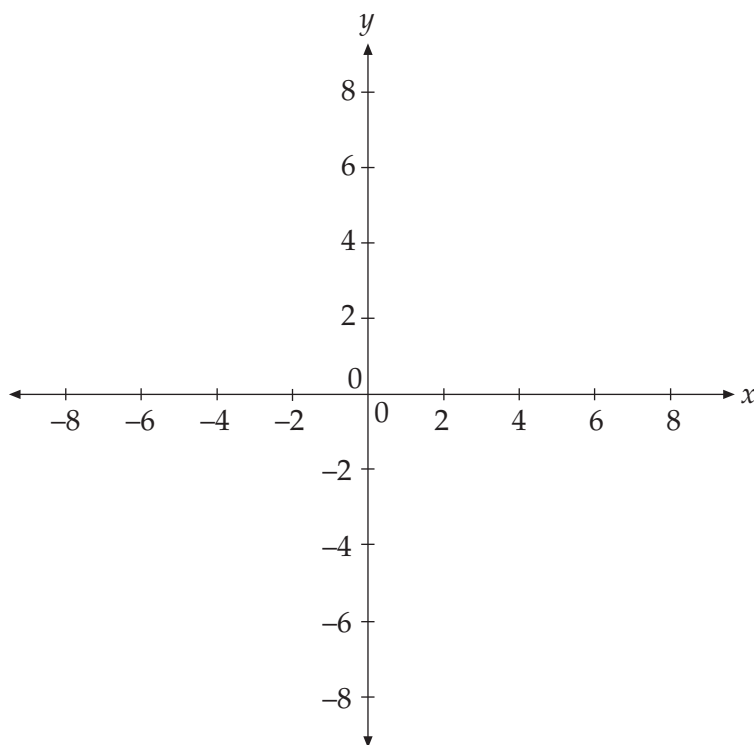
- ii. Find the coordinates of the points of intersection between $\{z: |z - 6| < 2\}$ and $\{z: |z + 2| \geq |z - 14|\}$.

2 marks

b i. Express $u = 3 + 3\sqrt{3}i$ in the form $u = r\text{cis}\theta$.

2 marks

ii. On the axes below sketch the loci $|z - u| = k$ and $|z - 6| = 2$ for the smallest value of k such that the loci intersect at exactly one point.



2 marks

b iii. Write down the value of k and the coordinates of the point of intersection.

2 marks

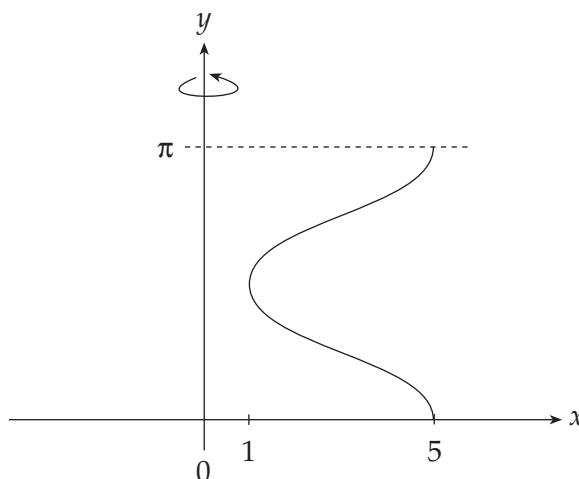
Total 11 marks

Question 6

- a. Find the exact value of the definite integral $\int_0^{2\pi} \sin^2\left(\frac{1}{2}x\right) \cos^2\left(\frac{1}{2}x\right) dx$, showing working.

3 marks

The diagram below shows the graph of the relation $x = 2\cos(2y) + 3$, $0 \leq y \leq \pi$



By rotating the curve about the y -axis, a hollow solid of revolution is produced, having the shape of a vase. Measurements are in centimetres.

- b. Find the exact value of the maximum volume of water that this vase can hold.

4 marks

Total 7 marks