

MAV Specialist Mathematics Examination 2

Answers & Solutions

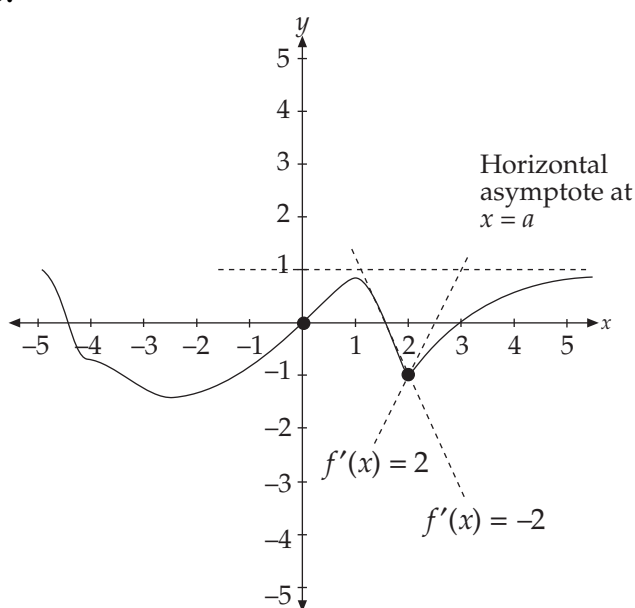
Question 1

- a. At $x = -4$, $f'(x) = 0$ and $f''(x) = 0$ thus point of inflexion **A1**

At $x = -2$, $f'(x) = 0$ and $f''(x) > 0$ thus local minimum **A1**

at $x = 1$, $f'(x) = 0$ and $f''(x) < 0$ thus local maximum **A1**

b.



Shape **A1**
 Correct location of turning points **A1**
 Recognition of asymptotic behaviour **A1**

Question 2

- a i. $\dot{\underline{s}}(t) = (4t^3 - 9t^2 + 2)\underline{i} + 2t\underline{j}$
 $\underline{s}(t) = (t^4 - 3t^3 + 2t)\underline{i} + t^2\underline{j} + \underline{c}$ **M1**

At $t = 0$, $\underline{s} = 2\underline{i} \Rightarrow \underline{c} = 2\underline{i}$

$\underline{s}(t) = (t^4 - 3t^3 + 2t + 2)\underline{i} + t^2\underline{j}$ **A1**

- ii. $\dot{\underline{r}}(t) = -2t\underline{i} + 3\underline{j}$
 $\underline{r}(t) = -t^2\underline{i} + 3t\underline{j} + \underline{c}$ **M1**

At $t = 0$, $\underline{r} = 2\underline{i} - 2\underline{j} \Rightarrow \underline{c} = 2\underline{i} - 2\underline{j}$

$\underline{r}(t) = (2 - t^2)\underline{i} + (3t - 2)\underline{j}$ **A1**

- b i. Boats collide if they have the same position at the same time, ie $\underline{r}(t) = \underline{s}(t)$ for some value of t .

$$t^4 - 3t^3 + 2t + 2 = 2 - t^2 \quad (1)$$

$$\& \quad t^2 = 3t - 2 \quad (2) \quad \text{M1}$$

From (2)

$$t^2 - 3t + 2 = 0$$

$$(t - 1)(t - 2) = 0$$

$$t = 1 \text{ or } t = 2 \quad \text{M1}$$

Substituting $t = 1$ into (1)

$$\text{LHS} = 1 - 3 + 2 + 2$$

$$= 2$$

$$\text{RHS} = 2 - 1$$

$$= 1$$

$$\text{LHS} \neq \text{RHS} \quad \text{M1}$$

Substituting $t = 2$ into (1)

$$\text{LHS} = 16 - 24 + 4 + 2$$

$$= -2$$

$$\text{RHS} = 2 - 4$$

$$= -2$$

$$\text{RHS} = \text{LHS} \quad \text{M1}$$

Thus $t = 2$ satisfies both (1) and (2), which implies both Ragin' and Starin' are at the same position at the same time and hence collide. **A1**

ii. Collision at $t = 2$

$$\begin{aligned}\underline{r}(2) &= (2-4)\underline{i} + (6-2)\underline{j} \\ &= -2\underline{i} + 4\underline{j}\end{aligned}$$

Point of collision $(-2, 4)$ **A1**

c. $\dot{\underline{r}}(2) = -4\underline{i} + 3\underline{j}$ **M1**

$\dot{\underline{s}}(2) = -2\underline{i} + 4\underline{j}$ **M1**

$$\begin{aligned}\cos \theta &= \frac{\dot{\underline{s}} \cdot \dot{\underline{r}}}{|\dot{\underline{s}}| |\dot{\underline{r}}|} \\ &= \frac{8+12}{\sqrt{20} \times \sqrt{25}} = \frac{20}{10\sqrt{5}}\end{aligned}$$
 M1

$$\begin{aligned}\theta &= \cos^{-1}\left(\frac{2}{\sqrt{5}}\right) \\ &= 26.56 \approx 27^\circ\end{aligned}$$
 A1

Question 3

a i. $y = \cos^{-1}\left(\frac{x-1}{x+1}\right)$

$$u = \frac{x-1}{x+1} \quad \frac{du}{dx} = \frac{x+1-(x-1)}{(x+1)^2} = \frac{2}{(x+1)^2}$$
 M1

$y = \cos^{-1}u$

$\frac{dy}{du} = \frac{-1}{\sqrt{1-u^2}}$

$$\frac{dy}{dx} = \frac{-1}{\sqrt{1-\left(\frac{x-1}{x+1}\right)^2}} \times \frac{2}{(x+1)^2}$$
 M1

$$= \frac{-1}{\sqrt{\left(\frac{x+1}{x+1}\right)^2 - \left(\frac{x-1}{x+1}\right)^2}} \times \frac{2}{(x+1)^2}$$

$$= \frac{-1}{\sqrt{\frac{(x+1-x+1)(x+1+x-1)}{(x+1)^2}}} \times \frac{2}{(x+1)^2}$$

$$= -\frac{(x+1)}{2\sqrt{x}} \times \frac{2}{(x+1)^2}$$

$$= \frac{-1}{\sqrt{x} (x+1)}$$
 A1

$$\begin{aligned}\text{ii. } \int_1^3 \frac{1}{\sqrt{x^3} + \sqrt{x}} dx \\ &= -\int_1^3 \frac{-1}{\sqrt{x^3} + \sqrt{x}} dx \\ &= -\left[\cos^{-1}\left(\frac{x-1}{x+1}\right)\right]_1^3 \\ &= -\cos^{-1}\left(\frac{1}{2}\right) + \cos^{-1}(0) \\ &= \frac{\pi}{6}\end{aligned}$$
 M1 **A1**

$$\begin{aligned}\text{b i. } \frac{du}{dv} &= \frac{-1}{\sqrt{x}(x+1)} \\ v &= \tan^{-1}\sqrt{x} \\ \frac{dv}{dx} &= \frac{1}{1+x} \times \frac{1}{2\sqrt{x}} \\ &= \frac{1}{2\sqrt{x}(x+1)}\end{aligned}$$
 M1

$$\begin{aligned}\frac{du}{dx} &= -2 \times \frac{dv}{dx} \\ \Rightarrow \frac{du}{dx} + 2 \times \frac{dv}{dx} &= 0 \\ \therefore a &= 2\end{aligned}$$
 M1 **A1**

$$\begin{aligned}\text{ii. } \frac{du}{dx} + 2 \frac{dv}{dx} &= 0 \\ \int \frac{du}{dx} dx + 2 \int \frac{dv}{dx} dx &= \int 0 dx \\ \Rightarrow u + 2v &= c \\ \cos^{-1}\left(\frac{x-1}{x+1}\right) + 2\tan^{-1}\sqrt{x} &= c\end{aligned}$$
 C1

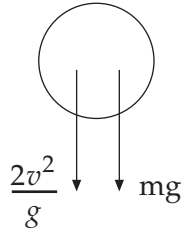
For $x = 1$

$$\begin{aligned}\cos^{-1}0 + 2\tan^{-1}1 \\ &= \frac{\pi}{2} + \frac{\pi}{2} = \pi \\ \therefore c &= \pi\end{aligned}$$
 A1

Note: Answer is given

Question 4

a & b.



A1

$$F = -\left(2g + \frac{2v^2}{g}\right)$$

$$2a = -\left(2g + \frac{2v^2}{g}\right)$$

Need to see working

$$a = -g - \frac{v^2}{g}$$

Answer given A1

c. $a = -g - \frac{v^2}{g}$

$$v \frac{dv}{dx} = -g - \frac{v^2}{g}$$

M1

$$\begin{aligned} \frac{dv}{dx} &= -\frac{g}{v} - \frac{v}{g} \\ &= \frac{g^2 + v^2}{-gv} \end{aligned}$$

$$\frac{dx}{dv} = \frac{-gv}{g^2 + v^2}$$

$$= -\frac{g}{2} \left[\frac{2v}{g^2 + v^2} \right]$$

M1

$$x = -\frac{g}{2} \log_e(v^2 + g^2) + c$$

M1

At $x = 0$ $v = 10$

$$c = \frac{g}{2} \log_e 196.04$$

A1

$$v = \sqrt{196.04e^{\frac{-2x}{g}} - g^2}$$

C1

d. $-\frac{2x}{g} = \log_e \left(\frac{v^2 + g^2}{196.04} \right)$

M1

At $v = 0$

$$-\frac{2x}{g} = \log_e \left(\frac{g^2}{196.04} \right)$$

$$x = \frac{g}{2} \log_e \left(\frac{g^2}{196.04} \right)$$

M1 A1

$\therefore x = 3 \text{ m}$

e. $a = -g - \frac{v^2}{g}$

$$\frac{dv}{dt} = \frac{g^2 + v^2}{-g}$$

$$\frac{dt}{dv} = \frac{-g}{g^2 + v^2}$$

M1

$$t = -\tan^{-1} \frac{v}{g} + c$$

M1

At $t = 0$, $v = 10$

$$t = -\tan^{-1} \frac{v}{g} + \tan^{-1} \frac{10}{g}$$

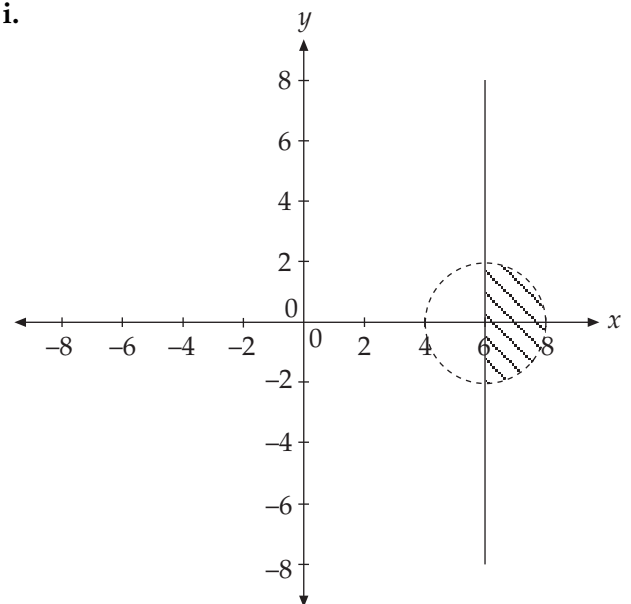
Maximum height at $v = 0$

$$t = \tan^{-1} \frac{10}{g} = 0.8 \text{ seconds}$$

A1

Question 5

a i.



Circle of radius 2 with centre $z = 6$,
Dotted line

M1

Line parallel to $\text{Im}(z)$ axis passing
through 6, Solid line

M1

Region Shaded

M1

ii. $z_1 = 6 + 2i$ or $(6, 2)$

A1

$z_1 = 6 - 2i$ or $(6, -2)$

A1

b i. $r^2 = 9 + 27$

M1

$r = 6$

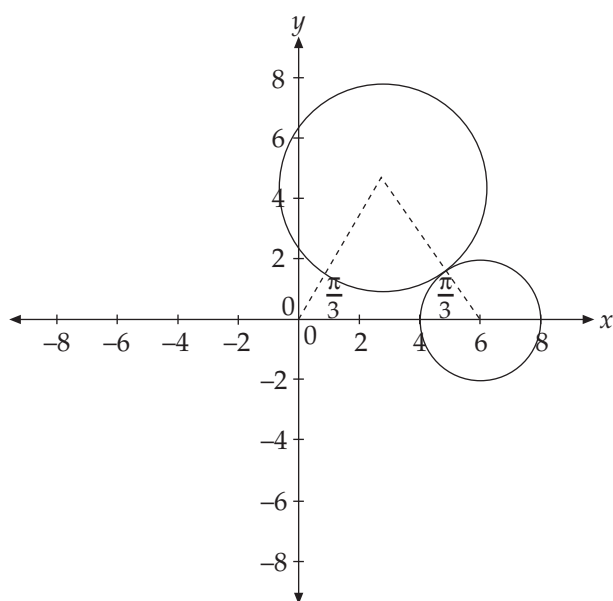
$\theta = \tan^{-1} \frac{3\sqrt{3}}{3}$

$\theta = \frac{\pi}{3}$

$u = 6\text{cis} \frac{\pi}{3}$

A1

ii.



Circle with centre at $6\text{cis} \frac{\pi}{6}$

M1

One point of intersection with the radius of both circles on the same line

M1

iii. $k + 2 = 6$

A1

$\therefore k = 4$

By similar triangles, point of intersection

$z = 5 + \sqrt{3}i$ or $(5, \sqrt{3})$

C1

Question 6

a. $\int_0^{2\pi} \left(\sin\left(\frac{1}{2}x\right) \cos\left(\frac{1}{2}x\right) \right)^2 dx = \int_0^{2\pi} \left(\frac{1}{2} \sin\left(2 \cdot \frac{1}{2}x\right) \right)^2 dx$ M1

$$= \frac{1}{4} \int_0^{2\pi} \sin^2 x dx$$

$$= \frac{1}{4} \int_0^{2\pi} \frac{1}{2} (1 - \cos(2x)) dx$$
 A1

$$= \frac{1}{8} \left[x - \frac{1}{2} \sin(2x) \right]_0^{2\pi} = \frac{\pi}{4}$$
 A1

b. $V = \pi \int_0^{\pi} (2 \cos(2y) + 3)^2 dy$ A1

$$= \pi \int_0^{\pi} (4 \cos^2(2y) + 12 \cos(2y) + 9) dy$$
 M1

$$= \pi \left[\frac{1}{2} \sin(4y) + 6 \sin(2y) + 11y \right]_0^{\pi}$$

$$= \pi(11\pi) = 11\pi^2$$
 M1

$\therefore$ the maximum volume is $11\pi^2 \text{ cm}^3$. A1