

Part 1: Multiple-choice questions**Question 1**

If $u = 3 - i$ and $v = 3 + i$ then $\frac{u}{v}$ is equal to

- A. $\frac{5}{4 - 3i}$
- B. $\frac{4 - 3i}{5}$
- C. $\frac{4 - 3i}{4}$
- D. $-i$
- E. -1

Question 2

The angle between the vectors $\vec{a} = 2\vec{i} - \vec{j}$ and $\vec{b} = 3\vec{i} + 2\vec{j}$ is closest to

- A. 1°
- B. 7°
- C. 30°
- D. 60°
- E. 86°

Question 3

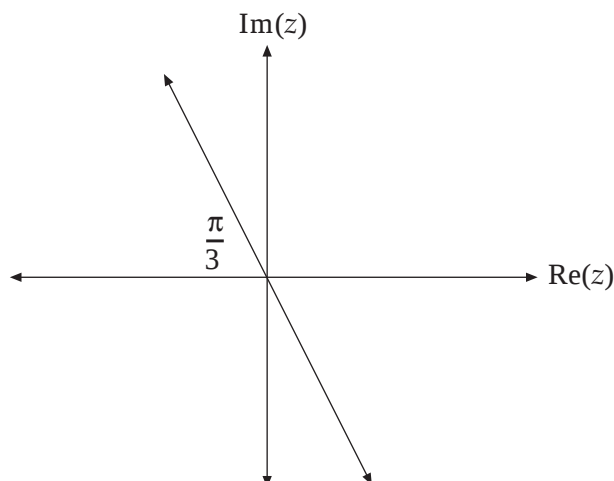
If $f(x) = 2x^2 + 7x - 4$, then the graph of $\frac{1}{f(x)}$ has

- A. asymptotes at $x = -4$ and $x = \frac{1}{2}$
- B. asymptotes at $x = 4$ and $x = -\frac{1}{2}$
- C. x -intercepts at $x = -4$ and $x = \frac{1}{2}$
- D. a maximum at the point $\left(-\frac{7}{4}, \frac{81}{8}\right)$
- E. a minimum at the point $\left(-\frac{7}{4}, -\frac{8}{81}\right)$

Question 4

The Argand diagram shown is the graph of

- A. $\left\{z: \operatorname{Arg} z = \frac{\pi}{3}\right\}$
- B. $\left\{z: \operatorname{Arg} z = -\frac{\pi}{3}\right\}$
- C. $\left\{z: \operatorname{Arg} z = \frac{2\pi}{3}\right\}$
- D. $\left\{z: \operatorname{Im} z - \sqrt{3} \operatorname{Re} z = 0\right\}$
- E. $\left\{z: \operatorname{Im} z + \sqrt{3} \operatorname{Re} z = 0\right\}$

**Question 5**

$(1+i)^5$ can be expressed in polar form as

- A. $2\sqrt{2}\operatorname{cis}\left(\frac{5\pi}{4}\right)$
- B. $4\sqrt{2}\operatorname{cis}\left(-\frac{3\pi}{4}\right)$
- C. $32\operatorname{cis}\left(\frac{\pi}{4}\right)$
- D. $32\operatorname{cis}\left(-\frac{3\pi}{4}\right)$
- E. $4\sqrt{2}\operatorname{cis}\left(\frac{\pi}{4}\right)$

Question 6

If $\sec^{-1}x = 4.2$, then the value of x is

- A. undefined
- B. -0.49
- C. -0.87
- D. -1.15
- E. -2.04

Question 7

An antiderivative of $\frac{2}{1-3x}$, $x > \frac{1}{3}$ is

A. $-\frac{2}{3}\log_e(1-3x)$, $x < \frac{1}{3}$

B. $-\frac{2}{3}\log_e(1-3x)$, $x > \frac{1}{3}$

C. $2\log_e(1-3x)$, $x > \frac{1}{3}$

D. $\frac{6}{(1-3x)^2}$, $x > \frac{1}{3}$

E. $-\frac{3}{2}\log_e(1-3x)$, $x > \frac{1}{3}$

Question 8

The derivative of $x\sin^{-1}(2x)$ with respect to x is $\frac{2x}{\sqrt{1-4x^2}} + \sin^{-1}(2x)$. It follows that an antiderivative of $\sin^{-1}(2x)$ is

A. $\int x\sin^{-1}(2x)dx - \int \frac{2x}{\sqrt{1-4x^2}}dx$

B. $\int x\sin^{-1}(2x)dx + \int \frac{2x}{\sqrt{1-4x^2}}dx$

C. $x\sin^{-1}(2x) - \int \frac{2x}{\sqrt{1-4x^2}}dx$

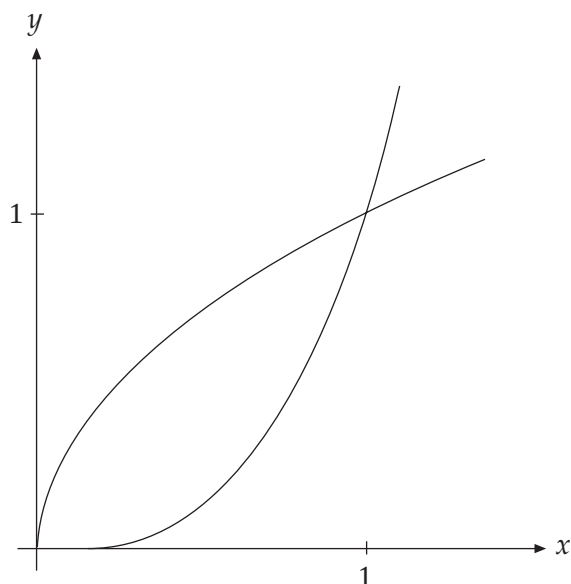
D. $x\sin^{-1}(2x) + \int \frac{2x}{\sqrt{1-4x^2}}dx$

E. $x\sin^{-1}(2x) - \frac{2x}{\sqrt{1-4x^2}}$

Question 9

The region enclosed by the curves with equations $f(x) = \sqrt{x}$ and $g(x) = x^3$ is rotated about the x -axis. The **exact value** of the solid obtained is given by

- A. $\int_0^1 \pi (\sqrt{x} - x^3)^2 dx$
- B. $\int_0^1 \pi x dx$
- C. $\int_0^1 \pi x^6 dx$
- D. $\int_0^1 \pi x dx - \int_0^1 \pi x^6 dx$
- E. $\int_0^1 \pi \sqrt{x} dx - \int_0^1 \pi x^3 dx$

**Question 10**

The position vector, $\vec{r}(t)$, in metres, of a particle at time t is given by $\vec{r}(t) = 2t \vec{i} + 5 \cos(2t) \vec{j}$. The Cartesian equation of the path followed by the particle is

- A. $x = 2t$
- B. $y = 5 \cos(2t)$
- C. $y = 5x$
- D. $y = 5 \cos(x)$
- E. $y = 5 \cos\left(\frac{x}{2}\right)$

Question 11

The trapezoidal rule with two equal intervals is used to approximate the area enclosed by the curve, $f(x) = \sqrt{x} - 1$ and the x -axis between the lines $x = 2$ and $x = 4$. The value of this estimate, correct to three decimal places is

- A. 1.439
- B. 1.448
- C. 1.452
- D. 2.878
- E. 3.439

Question 12

A unit vector parallel to $2\hat{i} - 3\hat{j}$ is

A. $\sqrt{13} \begin{pmatrix} 2\hat{i} - 3\hat{j} \end{pmatrix}$

B. $\frac{1}{\sqrt{13}} \begin{pmatrix} 2\hat{i} - 3\hat{j} \end{pmatrix}$

C. $-\frac{1}{\sqrt{13}} \begin{pmatrix} 2\hat{i} + 3\hat{j} \end{pmatrix}$

D. $\frac{1}{\sqrt{5}} \begin{pmatrix} 2\hat{i} - 3\hat{j} \end{pmatrix}$

E. $\frac{1}{\sqrt{13}} \begin{pmatrix} 2\hat{i} + 3\hat{j} \end{pmatrix}$

Question 13

The equation of the hyperbola shown below is

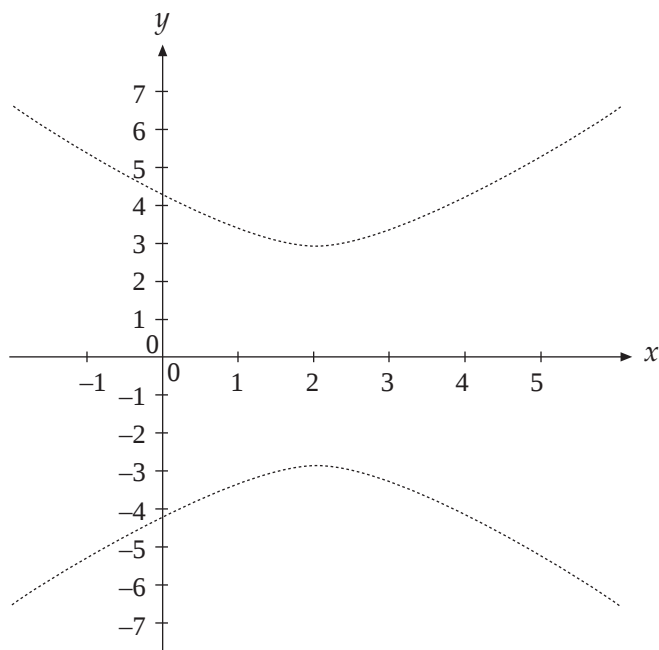
A. $\frac{y^2}{3} - \frac{(x-2)^2}{4} = 1$

B. $\frac{y^2}{9} - \frac{(x+2)^2}{4} = 1$

C. $\frac{y^2}{9} - \frac{(x-2)^2}{4} = 1$

D. $\frac{x^2}{4} - \frac{y^2}{9} = 1$

E. $\frac{x^2}{4} - \frac{y^2}{3} = 1$



Question 14

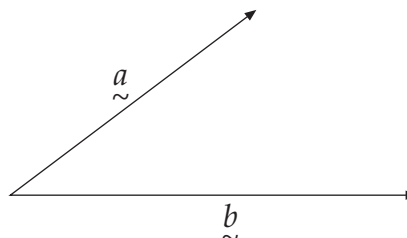
$\frac{5x}{(x-1)^2}$ expressed in partial fractions has the form

- A. $\frac{a}{x-1} + \frac{b}{x-1}$
- B. $\frac{a}{(x-1)^2} + \frac{b}{(x-1)^2}$
- C. $\frac{a}{x-1} + \frac{b}{(x-1)^2}$
- D. $\frac{a}{x-1} + \frac{b}{x+1}$
- E. $\frac{a}{x-1} + \frac{bx+c}{(x-1)^2}, b \neq 0$

Question 15

The vector projection of $\vec{a}$ perpendicular to $\vec{b}$ is given by

- A. $\left(\vec{a} \cdot \hat{\vec{b}} \right) \hat{\vec{b}}$
- B. $\vec{a} - \left(\vec{a} \cdot \hat{\vec{b}} \right) \hat{\vec{b}}$
- C. $\vec{a} - \left(\vec{a} \cdot \hat{\vec{b}} \right) \hat{\vec{b}}$
- D. $\left(\vec{a} \cdot \hat{\vec{b}} \right) \hat{\vec{b}} - \vec{a}$
- E. $\left(\vec{a} \cdot \hat{\vec{b}} \right) \hat{\vec{b}} - \vec{a}$

**Question 16**

Which of the following is **not** a solution of the differential equation $\frac{d^2y}{dx^2} + y = 4e^x$

- A. $y = 2e^x - \sin x$
- B. $y = 2e^x - \cos x$
- C. $y = e^{2x} + \cos x$
- D. $y = 2e^x + \sin x$
- E. $y = 2e^x + \cos x$

Question 17

If $y = \text{Cos}^{-1}\left(\frac{7}{x}\right)$, $x > 7$, then $\frac{dy}{dx} =$

A. $-\frac{1}{\sqrt{49-x^2}}$

B. $-\frac{7x}{\sqrt{x^2-49}}$

C. $-\frac{x}{\sqrt{x^2-49}}$

D. $\frac{7x}{\sqrt{x^2-49}}$

E. $\frac{7}{x\sqrt{x^2-49}}$

Question 18

The graph of $y = \text{cosec}\left(x - \frac{\pi}{4}\right) + 2$, for $0 \leq x \leq 2\pi$, has turning points at

A. $\left(\frac{\pi}{4}, 3\right)$ and $\left(\frac{\pi}{4}, 3\right)$

B. $\left(\frac{3\pi}{4}, 1\right)$ and $\left(\frac{3\pi}{4}, -1\right)$

C. $\left(\frac{3\pi}{4}, 3\right)$ and $\left(\frac{7\pi}{4}, 1\right)$

D. $\left(\frac{\pi}{4}, 1\right)$ and $\left(\frac{5\pi}{4}, -1\right)$

E. $\left(\frac{\pi}{4}, 3\right)$ and $\left(\frac{5\pi}{4}, 1\right)$

Question 19

An object is moving such that its position vector $\vec{r}(t)$ is given by $\vec{r}(t) = 4\sin(2t)\vec{i} + 3t\vec{j}$. The magnitude of the velocity vector in m/s at $t = \frac{\pi}{6}$ seconds is

- A. $\sqrt{41}$
- B. $\sqrt{57}$
- C. $\sqrt{12 + \frac{\pi^2}{4}}$
- D. 5
- E. 7

Question 20

Euler's method, with a step size of 0.2 is used to solve the differential equation $\frac{dy}{dx} = x \log_e x$ with $y = 3$ at $x = 1$. The value obtained for y at $x = 1.4$, correct to four decimal places, is

- A. 3.0365
- B. 3.0438
- C. 3.0665
- D. 3.0875
- E. 3.1380

Question 21

The radius of a circular oil slick is increasing at a rate of 2 m/min. When the radius is 8 m, the rate of increase in m^2/min of the area of the oil slick is

- A. 4π
- B. 8π
- C. 16π
- D. 32π
- E. 128π

Question 22

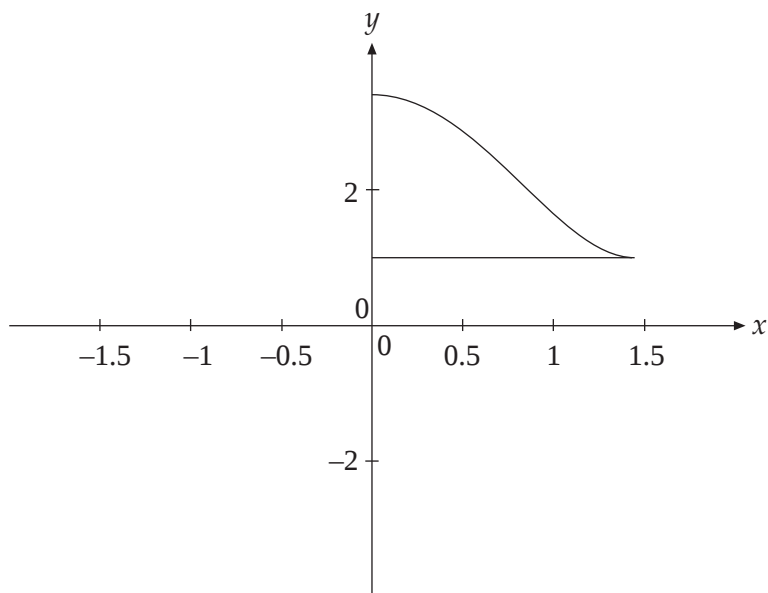
Luiji leaves his campsite, C, and walks in a direction of NE for 4 km to B. He then heads N50°W until he reaches his destination at D. Luiji can tell from his compass that his destination of D is at a bearing of N20°W relative to his previous campsite at C. The distance, d km from C to D is given by

- A. $\frac{d}{\sin 45^\circ} = \frac{4}{\sin 50^\circ}$
- B. $\frac{d}{\sin 95^\circ} = \frac{4}{\sin 20^\circ}$
- C. $\frac{d}{\sin 95^\circ} = \frac{4}{\sin 40^\circ}$
- D. $\frac{d}{\sin 85^\circ} = \frac{4}{\sin 50^\circ}$
- E. $\frac{d}{\sin 85^\circ} = \frac{4}{\sin 30^\circ}$

Question 23

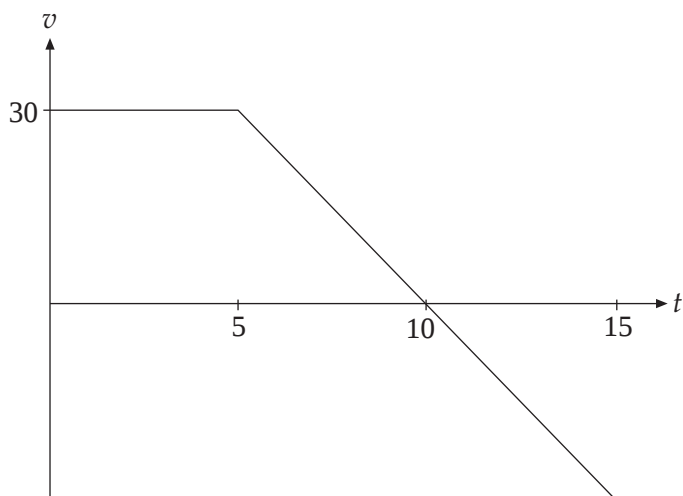
The region enclosed by the curve $y = \frac{3}{1+x^2}$, the straight line $y = 1$ and the y -axis, is rotated about the y -axis to form a solid of revolution. The volume of this solid, in cubic units is given by

- A. $\pi \int_0^{\sqrt{2}} \left(\frac{9}{(1+x^2)^2} \right) dx$
- B. $\pi \int_0^{\sqrt{2}} \left(\frac{9}{(1+x^2)^2} - 1 \right) dx$
- C. $\pi \int_0^{\sqrt{2}} \left(\frac{3}{y} - 1 \right) dy$
- D. $\pi \int_1^3 \left(\frac{3}{y} - 1 \right) dy$
- E. $\pi \int_1^3 \left(\frac{3}{(1+y^2)^2} \right) dy$

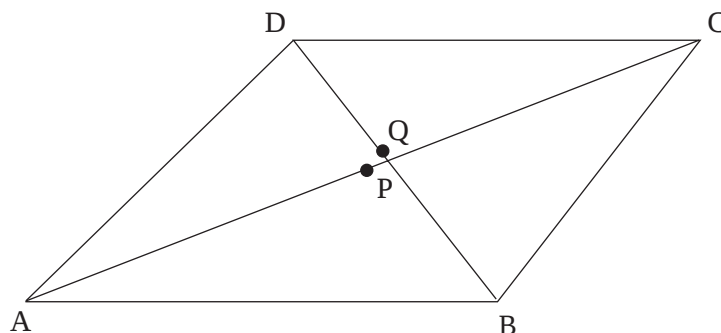


Question 24

The velocity-time graph for a particle moving in a straight line is shown below. The total distance, in metres, travelled by the particle in 15 seconds is



- A. 275
- B. 175
- C. 30
- D. 50
- E. 300

Question 25

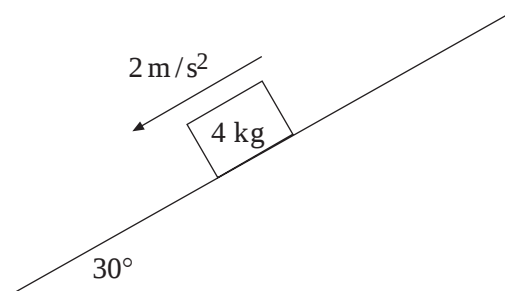
If P is the mid-point of $\vec{AC}$ and Q is the mid-point of $\vec{BD}$, in order to prove that the diagonals of a parallelogram bisect each other, it needs to be shown that

- A. $\vec{AP} \cdot \vec{AQ} = 0$
- B. $\vec{AP} \cdot \vec{PQ} = 0$
- C. $\vec{AP} = \vec{AQ}$
- D. $|\vec{BD}| = \frac{1}{2}|\vec{AC}|$
- E. $\vec{AP} = \vec{BQ}$

Question 26

A body of mass 2 kg is acted on by two forces, one of magnitude 3 Newtons acting due South and the other of magnitude 4 Newtons acting due East. The magnitude, in m/s^2 , of the resulting acceleration is

- A. 2.5
- B. $2\sqrt{5}$
- C. $2\sqrt{7}$
- D. 14
- E. 24

Question 27

A block of mass 4.0 kg slides down a rough slope inclined at 30° to the horizontal. The block is sliding with an acceleration of 2 m/s^2 . The magnitude of the acceleration due to gravity is 9.8 m/s^2 . The magnitude of the frictional force between the block and the rough surface is

- A. 11.6
- B. 19.6
- C. 25.9
- D. 27.6
- E. 31.2

Question 28

A person of mass 64 kg stands in a lift accelerating downwards at a magnitude of 1.5 m/s^2 . The magnitude of the acceleration due to gravity is 9.8 m/s^2 . The reaction force, in Newtons, of the lift floor on the person is closest to

- A. 96
- B. 531
- C. 544
- D. 627
- E. 723

Question 29

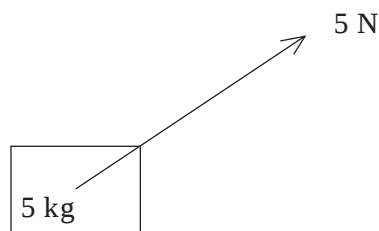
A particle of mass 3 kg is moving with a velocity of $3\hat{i} + 4\hat{j}$ m/s. The magnitude in kg m/s, of the momentum of the particle is

- A. 9
- B. 12
- C. 15
- D. 21
- E. 75

Question 30

A box of mass 5 kg is pulled at a constant speed along a rough horizontal surface, by a force of 5 newtons acting at an angle of 30° to the horizontal. The acceleration due to gravity is 9.8 m/s^2 . The magnitude of the frictional force is closest to

- A. 0
- B. 2.5
- C. 4.3
- D. 5
- E. 9.3



PART II

Short-answer Questions

Question 1

Using the substitution $u = \sqrt{x}$, find $\int \frac{\sqrt{x}}{x-4} dx$

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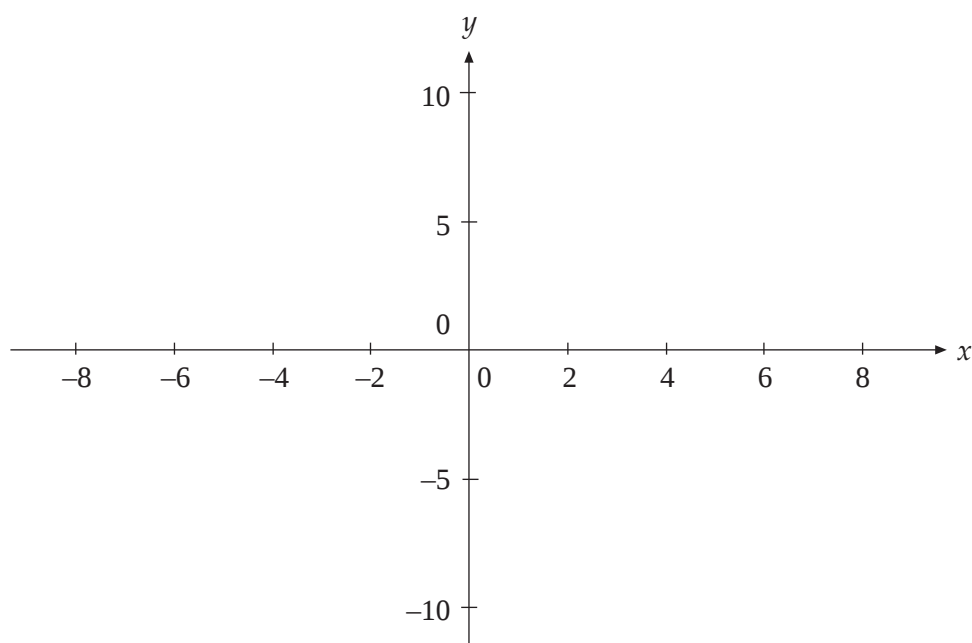
4 marks

Question 2

- a. Show that the subset of the complex plane defined by $\{z: |z + 1| + |z - 1| = 6\}$ can be expressed in Cartesian form as $\frac{x^2}{9} + \frac{y^2}{8} = 1$

3 marks

- b. Sketch $\{z: |z + 1| + |z - 1| < 6\}$, shading the required region.



2 marks

Question 3

A cricket ball is thrown upwards from the top of a 15 metre high building, with a speed of 4 m/s. Find, correct to one decimal place, the speed of the cricket ball when it strikes the ground.

2 marks

Question 4

a. Show that $\int (\tan x) dx = -\log_e |\cos x| + c$

1 mark

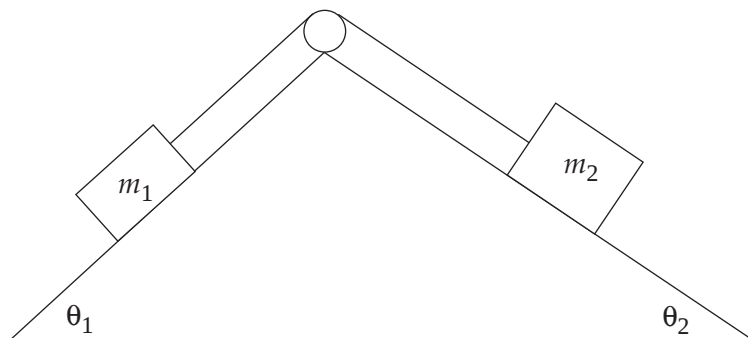
b. Hence find $\int \tan^3(x) dx$

2 marks

Question 6

Two particles of mass m_1 and m_2 are connected by a light string that passes over a smooth pulley respectively. The coefficients of friction of the two surfaces are μ_1 and μ_2 .

If the system is on the point of moving to the left, i.e. is about to slide down the plane, express $\frac{m_1}{m_2}$ in terms of $\theta_1, \theta_2, \mu_1$ and μ_2 .



3 marks