

Year 2003

VCE

Specialist Mathematics Trial Examination 1

Suggested Solutions

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Question 1 C

Cosine Rule

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Rightarrow a^2 = 8^2 + 7^2 - 2 \times 8 \times 7 \times \cos 110^\circ$$

$$\Rightarrow a^2 = 64 + 49 - 112 \times \cos 110^\circ$$

$$\Rightarrow a^2 = 113 - 112 \times \cos 110^\circ$$

$$\Rightarrow a = \sqrt{113 - 112 \times \cos 110^\circ}$$

$$\text{Since } \sin 20^\circ = -\cos 110^\circ$$

$$\text{Then } a = \sqrt{113 + 112 \sin 20^\circ}$$

Question 2 C

$$y = \frac{x^3 + 4x}{4x^2} = \frac{x}{4} + \frac{1}{x}$$

$\Rightarrow x = 0$ is a vertical asymptote

$\Rightarrow y = \frac{x}{4}$ is an asymptote

The graph has a turning point in the first quadrant and a turning point in the third quadrant.

Question 3 B

The intersection of the two asymptotes for the hyperbola is $(-3, 2)$.

Hence, the equation of the hyperbola is

$$\frac{(x+3)^2}{9} - \frac{(y-2)^2}{4} = 1$$

Question 4 E

$$\text{Let } u = \frac{4}{3x} = \frac{4}{3}x^{-1}$$

$$\Rightarrow \frac{du}{dx} = -\frac{4}{3}x^{-2} = \frac{-4}{3x^2}$$

$$y = \cos^{-1}u$$

$$\Rightarrow \frac{dy}{du} = \frac{-1}{\sqrt{1-u^2}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = \frac{-1}{\sqrt{1-\frac{16}{9x^2}}} \times \frac{-4}{3x^2}$$

$$= \frac{4}{3x^2} \times \frac{1}{\sqrt{\frac{9x^2-16}{9x^2}}}$$

$$= \frac{4}{3x^2} \times \frac{3x}{\sqrt{9x^2-16}}$$

$$= \frac{4}{x\sqrt{9x^2-16}}$$

Question 5 D

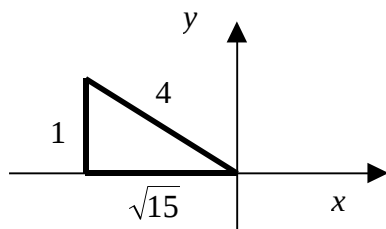
$$\operatorname{cosec}(x) = 4$$

$$\Rightarrow \frac{1}{\sin x} = 4$$

$$\Rightarrow \sin x = \frac{1}{4}$$

$$\Rightarrow \tan x = \frac{-1}{\sqrt{15}} \text{ (second quadrant)}$$

$$\Rightarrow \cot x = -\sqrt{15}$$



Question 6 D

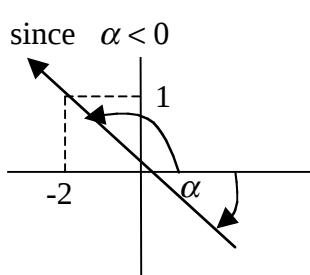
$$y = \tan^{-1}(x) \text{ domain}$$

$$R \text{ range } \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

Hence,

$$y = a \tan^{-1}(bx) + c$$

$$\text{range } \left(-\frac{a\pi}{2} + c, \frac{a\pi}{2} + c\right)$$

Question 7 E $z = x + iy$ $\bar{z} = x - iy$ $\Rightarrow z - \bar{z} = 2iy$ which is not a real number	Question 8 D $\bar{u} = a + bi$ $\Rightarrow \frac{1}{\bar{u}} = \frac{1}{a + bi} \times \frac{a - bi}{a - bi}$ $= \frac{a - bi}{a^2 - b^2 i^2}$ $= \frac{a - bi}{a^2 + b^2}$
Question 9 A $\alpha = 1 - 2i$ $\beta = 1 + 2i$ $\alpha + \beta = 2$ $\alpha\beta = 1 - 4i^2 = 5$ so $z^2 - 2z + 5$ is a factor $\Rightarrow (z + w)(z^2 - 2z + 5) = z^3 + az^2 + bz - 10$ $\Rightarrow w = -2$ $\Rightarrow (z - 2)(z^2 - 2z + 5) = z^3 - 4z^2 + 9z - 10$ $\Rightarrow a = -4$ and $b = 9$	Question 10 D Checking each alternative. A. $2\text{cis}(-150^\circ) = 2\cos(-150^\circ) + i2\sin(-150^\circ) = -\sqrt{3} - i$ B. $2\text{cis}\left(\frac{7\pi}{6}\right) = 2\cos\left(\frac{7\pi}{6}\right) + i2\sin\left(\frac{7\pi}{6}\right) = -\sqrt{3} - i$ C. $-2\text{cis}\left(\frac{\pi}{6}\right) = -2\cos\left(\frac{\pi}{6}\right) - i2\sin\left(\frac{\pi}{6}\right) = -\sqrt{3} - i$ D. $-2\text{cis}\left(\frac{5\pi}{6}\right) = -2\cos\left(\frac{5\pi}{6}\right) - i2\sin\left(\frac{5\pi}{6}\right) = \sqrt{3} + i$ E. $2\text{cis}\left(\frac{-5\pi}{6}\right) = 2\cos\left(\frac{-5\pi}{6}\right) + i2\sin\left(\frac{-5\pi}{6}\right) = -\sqrt{3} - i$ So D is not correct
Question 11 E Using TI-83 calculator mode RAD $\text{fnInt}(Y_1, X, 1, 3) = -0.7312$	Question 12 C $\alpha = \text{Tan}^{-1}\left(-\frac{1}{2}\right) = -\text{Tan}^{-1}\left(\frac{1}{2}\right)$ since $\alpha < 0$ $-\pi < \text{Arg}(z) \leq \pi$ so $\text{Arg}(z) = \pi + \text{Tan}^{-1}\left(-\frac{1}{2}\right)$ 
Question 13 A $\frac{d}{dx}(x \text{Tan}^{-1}(x)) = \text{Tan}^{-1}(x) + \frac{x}{1+x^2}$ $\Rightarrow \int \text{Tan}^{-1}(x) dx + \int \frac{x}{1+x^2} = x \text{Tan}^{-1}(x)$ $\Rightarrow \int \text{Tan}^{-1}(x) dx = x \text{Tan}^{-1}(x) - \int \frac{x}{1+x^2}$ $\Rightarrow \int \text{Tan}^{-1}(x) dx = x \text{Tan}^{-1}(x) - \frac{1}{2} \log_e(1+x^2)$	Question 14 B Centre $c(-5, 5)$ $c = -5 + 5i$ radius $r = 5$ $z - c = r$ $z + 5 - 5i = 5$ not D, E circle also $(z - c)(\bar{z} - \bar{c}) = r^2$ $(z + 5 - 5i)(\bar{z} + 5 + 5i) = 25$ $\Rightarrow \{z: (z + 5 - 5i)(\bar{z} + 5 + 5i) = 25\}$

Question 15 C

Points of intersection

$$4 \sin^2(2x) = 4 \cos^2(2x)$$

$$\Rightarrow \tan^2(2x) = 1$$

$$\Rightarrow \tan(2x) = \pm 1$$

$$\Rightarrow 2x = \frac{\pi}{4}, \frac{3\pi}{4}$$

$$\Rightarrow x = \frac{\pi}{8}, \frac{3\pi}{8}$$

By symmetry (shaded area)

$$\text{Area} = 2 \int_0^{\frac{\pi}{8}} (4 \cos^2(2x) - 4 \sin^2(2x)) dx$$

$$= 8 \int_0^{\frac{\pi}{8}} (\cos^2(2x) - \sin^2(2x)) dx$$

$$= 8 \int_0^{\frac{\pi}{8}} (\cos 4x) dx = 2 \sin 4x \Big|_0^{\frac{\pi}{8}}$$

$$= 2(\sin \frac{\pi}{2} - \sin 0) = 2$$

Question 16 D

$$\vec{a} = -\vec{i} + t\vec{j} + \vec{k}$$

$$\Rightarrow |\vec{a}| = 4 = \sqrt{1 + t^2 + 1}$$

$$\Rightarrow 16 = (\sqrt{2 + t^2})^2$$

$$\Rightarrow t^2 = 14$$

$$\Rightarrow t = \pm\sqrt{14} \text{ both answers OK}$$

Question 17 D

Let $u = 3x$

$$\frac{du}{dx} = 3$$

Terminals $x = \frac{4}{3} \quad u = 4; \quad x = 0 \quad u = 0$

$$\Rightarrow \int_0^{\frac{4}{3}} \frac{dx}{\sqrt{64 - 9x^2}} = \frac{1}{3} \int_0^4 \frac{du}{\sqrt{64 - u^2}}$$

$$= \frac{1}{3} \sin^{-1}\left(\frac{u}{8}\right) \Big|_0^4$$

$$= \frac{1}{3} \sin^{-1} \frac{1}{2} - 0$$

$$= \frac{1}{3} \times \frac{\pi}{6} = \frac{\pi}{18}$$

Question 18 B

$$\vec{a} = \frac{1}{2}(\vec{i} - \vec{j} + z\vec{k})$$

$$\Rightarrow |\vec{a}| = \frac{1}{2} \sqrt{1 + 1 + z^2} = \frac{\sqrt{z^2 + 2}}{2}$$

$$\cos 135^\circ = -\frac{\sqrt{2}}{2} = \frac{z/2}{|\vec{a}|}$$

$$\Rightarrow -\frac{\sqrt{2}}{2} = \frac{z}{\sqrt{z^2 + 2}} \text{ so } z < 0$$

$$\Rightarrow 2z = -\sqrt{2}(\sqrt{z^2 + 2})$$

$$\Rightarrow 4z^2 = 2z^2 + 4$$

$$\Rightarrow z^2 = 2$$

$$\Rightarrow z = -\sqrt{2} \text{ since } z < 0$$

Question 19 A

$$x = \sqrt{t} - 2$$

$$\Rightarrow \sqrt{t} = x + 2$$

$$\Rightarrow t = (x + 2)^2$$

$$y = 3t^2$$

$$\Rightarrow y = 3(x + 2)^2 \quad x \geq -2$$

Question 20 C

$$m = 2\text{kg} \quad a = 5\text{m/s}^2$$

$$F = F_1 + F_2 = ma = 10$$

$$F_1 = -yj \quad \text{and} \quad y > 0$$

$$F_2 = 6\sqrt{2}(-i + j) \times \frac{1}{\sqrt{2}} = 6(-i + j)$$

$$F_1 + F_2 = -6i + (6 - y)j$$

$$\Rightarrow 10 = |F_1 + F_2| = |-6i - 8j|$$

$$\Rightarrow 6 - y = -8 \Rightarrow y = 14$$

Question 21 C

$$S = \int_0^{\frac{7}{4}} \frac{dt}{\sqrt{4t+9}}$$

$$\text{Let } u = 4t + 9$$

$$\frac{du}{dt} = 4$$

$$\text{When } t = \frac{7}{4}, u = 16 \quad \text{When } t = 0, u = 9$$

$$\Rightarrow S = \int_0^{\frac{7}{4}} \frac{dt}{\sqrt{4t+9}} = \int_9^{16} u^{-\frac{1}{2}} du = \left[\frac{1}{2} u^{\frac{1}{2}} \right]_9^{16}$$

$$\Rightarrow S = \frac{1}{2}(\sqrt{16} - \sqrt{9}) = \frac{1}{2}$$

Question 22 C

$$\text{If } r_A(t) = r_B(t)$$

$$t^2 - 5t + 6 = 2t - 4$$

$$\Rightarrow (t - 3)(t - 2) = 2(t - 2)$$

$$\Rightarrow \text{If } t = 2 \text{ } i \text{ components are equal}$$

$$2t - 6 = t^2 - 8t + 15$$

$$\Rightarrow 2(t - 3) = (t - 3)(t - 5)$$

$$\Rightarrow \text{If } t = 3 \text{ } j \text{ components are equal}$$

The two boats do not collide. Option (C) is true because to collide both i and j must be equal at the same time.

Question 23 C

$$\overrightarrow{PQ} = -\overrightarrow{PR}$$

$$\Rightarrow \overrightarrow{PQ} = \overrightarrow{RP}$$

They have a point in common P

so P, Q , and R are collinear.

Option (A) is true.

$$\Rightarrow |\overrightarrow{PQ}| = 1$$

Option (B) is true.

$$\overrightarrow{PQ} \cdot \overrightarrow{QR} = |\overrightarrow{PQ}| |\overrightarrow{QR}| \cos 180^\circ = 1 \times 1 \times -1 = -1$$

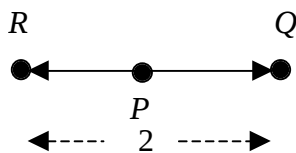
Option (E) is true.

$$\overrightarrow{PQ} \text{ is parallel to } \overrightarrow{RP}.$$

Option (D) is true.

$$\overrightarrow{PQ} \text{ is not perpendicular to } \overrightarrow{QR}.$$

Option (C) is false.



Question 24 B

$$x \frac{dx}{dy} = \sqrt{4x^2 + 9}$$

$$\Rightarrow \frac{dx}{dy} = \frac{\sqrt{4x^2 + 9}}{x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{x}{\sqrt{4x^2 + 9}}$$

Use the Euler PRGM on the TI-83

For $X_0 = 1$ $Y_0 = 2$ $X_N = 2$ $H = 0.5$

Value of $y = 2.3155$

Question 25 A

$$x = 1 + \frac{1}{t} = 1 + t^{-1}$$

$$\frac{dx}{dt} = -t^{-2}$$

$$y = \sqrt{12 + t^2}$$

$$\frac{dy}{dt} = 2 \times \frac{1}{2} \times t \times \frac{1}{\sqrt{12 + t^2}} = \frac{t}{\sqrt{12 + t^2}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = \frac{-t^3}{\sqrt{12 + t^2}}$$

$$\text{For } t = 2, \frac{dy}{dx} = \frac{-8}{\sqrt{16}} = -2$$

Question 26 C

For $x = 0$

$$x < 0 \quad \frac{dy}{dx} > 0$$

$$x > 0 \quad \frac{dy}{dx} < 0$$

$x = 0$ is a local maximum

For $x = 4$

$$x < 4 \quad \frac{dy}{dx} < 0$$

$$x > 4 \quad \frac{dy}{dx} < 0$$

$x = 4$ is a stationary of inflexion

Question 27 E

$$\cos 2x = 0$$

$$\Rightarrow 2x = \pm \frac{\pi}{2}$$

$$\Rightarrow x = \pm \frac{\pi}{4}$$

$$V = \pi \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} y^2 dx = \pi \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} 9 \cos^2 2x dx$$

$$= 18\pi \int_0^{\frac{\pi}{4}} \cos^2 2x dx$$

$$= 9\pi \int_0^{\frac{\pi}{4}} (1 - \cos 4x) dx$$

$$= 9\pi \left[x - \frac{1}{4} \sin 4x \right]_0^{\frac{\pi}{4}}$$

$$= 9\pi \left[\left(\frac{\pi}{4} - \frac{1}{4} \sin \pi \right) - (0 - 0) \right]$$

$$= \frac{9\pi^2}{4}$$

$$\text{Note: } \pi x \text{fnInt}(Y_1^2, X, 0, \frac{\pi}{4}) \times 2 = 22.206$$

Question 28 E

$$\tilde{a}(t) = \frac{3\pi}{2} \cos\left(\frac{\pi t}{2}\right) \tilde{i} - \pi \sin\left(\frac{\pi t}{2}\right) \tilde{j}$$

Integrating

$$\tilde{r}(t) = 3 \sin\left(\frac{\pi t}{2}\right) \tilde{i} + 2 \cos\left(\frac{\pi t}{2}\right) \tilde{j} + \tilde{c}$$

$$\tilde{r}(0) = 2\tilde{j} + \tilde{c} = 6\tilde{j}$$

$$\tilde{c} = 4\tilde{j}$$

$$\tilde{r}(t) = 3 \sin\left(\frac{\pi t}{2}\right) \tilde{i} + (4 + 2 \cos\left(\frac{\pi t}{2}\right)) \tilde{j}$$

$$\tilde{r}(1) = 3\tilde{i} + 4\tilde{j}$$

$$\tilde{p} = m\tilde{r}(1)$$

$$\Rightarrow \tilde{p} = 2(3\tilde{i} + 4\tilde{j})$$

$$\Rightarrow \tilde{p} = 6\tilde{i} + 8\tilde{j}$$

Question 29 B

$$Q = Q(t) = 3(50 + 2t) + C(50 + 2t)^n$$

$$\Rightarrow \frac{dQ}{dt} = 6 + 2Cn(50 + 2t)^{n-1} \dots(1)$$

$$\text{Also } \frac{dQ}{dt} = 12 - \frac{2}{(50 + 2t)} [3(50 + 2t) + C(50 + 2t)^n]$$

$$= 12 - 6 - 2C(50 + 2t)^{n-1}$$

$$= 6 - 2C(50 + 2t)^{n-1} \dots(2)$$

Equating (1) and (2)

$$6 + 2Cn(50 + 2t)^{n-1} = 6 - 2C(50 + 2t)^{n-1}$$

$$\Rightarrow n = -1$$

Question 30 B

Displacement = area under the graph

$$= \int_0^2 (10 - t^2) dt + \frac{1}{2} \times 2 \times 6 + \left(\frac{1}{2} \times 4 \times -12 \right)$$

$$= 10t - \frac{1}{3}t^3 \Big|_0^2 + 6 - 24$$

$$= 20 - \frac{8}{3} + 6 - 24$$

$$= -\frac{2}{3}$$

$$\frac{2}{3} \text{m from starting point}$$

Question 1

i.

Using TI-83

mode PAR RAD

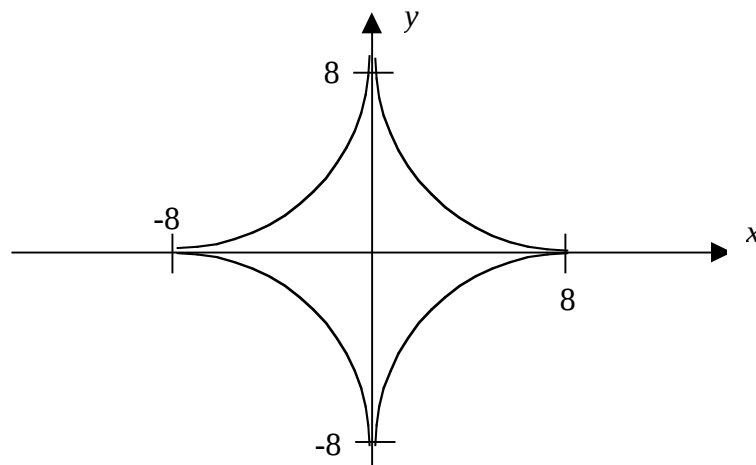
$$X_{IT} = 8(\cos(2T))^3$$

$$Y_{IT} = 8(\sin(2T))^3$$

WINDOW

$$X_{\min} = -10 \quad X_{\max} = 10$$

$$Y_{\min} = -10 \quad Y_{\max} = 10$$



Question 1

ii.

$$x = 8\cos^3 2t$$

$$\Rightarrow \frac{x}{8} = \cos^3 2t$$

$$\Rightarrow \frac{x^{\frac{1}{3}}}{2} = \cos 2t$$

$$y = 8\sin^3 2t$$

$$\Rightarrow \frac{y}{8} = \sin^3 2t$$

$$\Rightarrow \frac{y^{\frac{1}{3}}}{2} = \sin 2t$$

$$\sin^2 2t + \cos^2 2t = 1$$

$$\Rightarrow \frac{y^{\frac{2}{3}}}{4} + \frac{x^{\frac{2}{3}}}{4} = 1$$

$$\Rightarrow x^{\frac{2}{3}} + y^{\frac{2}{3}} = 4$$

$$\Rightarrow n = \frac{2}{3} \quad a = 4$$

Question 1

iii.

$$\tilde{r}(t) = 8\cos^3 2t\tilde{i} + 8\sin^3 2t\tilde{j} \quad t \geq 0$$

$$\Rightarrow \tilde{r}'(t) = -48\cos^2 2t\sin 2t\tilde{i} + 48\sin^2 2t\cos 2t\tilde{j}$$

$$\Rightarrow |\tilde{r}'(t)| = \sqrt{48^2 \cos^4 2t \sin^2 2t + 48^2 \sin^4 2t \cos^2 2t}$$

$$= \sqrt{48^2 \cos^2 2t \sin^2 2t (\cos^2 2t + \sin^2 2t)}$$

$$= 48\cos 2t \sin 2t$$

$$= 24 \sin 4t$$

$$c = 24 \quad b = 4$$

Question 2 i. $P(z) = z^3 + (2 + 3i)z^2 + 5z + 10 + 15i$ $P(-2 - 3i)$ $= (-2 - 3i)^3 + (2 + 3i)(-2 - 3i)^2 + 5(-2 - 3i) + 10 + 15i$ $= -(2 + 3i)^3 + (2 + 3i)^3 - 10 - 15i + 10 + 15i$ $= 0$ Hence, $z + 2 + 3i$ is a factor.	Question 2 ii. $z^3 + (2 + 3i)z^2 + 5z + 10 + 15i = 0$ $\Rightarrow z^2[z + 2 + 3i] + 5[z + 2 + 3i] = 0$ grouping $\Rightarrow (z + 2 + 3i)(z^2 + 5) = 0$ $\Rightarrow (z + 2 + 3i)(z + \sqrt{5}i)(z - \sqrt{5}i) = 0$ $\Rightarrow z = -2 - 3i, \pm \sqrt{5}i$
Question 3 i. When $x = 0, y = 1 = \frac{c}{32}$ Hence, $c = 32$ $-x^2 + bx + 32 = (8 - x)(x + 4) = 0$ $\Rightarrow -x^2 + bx + 32 = -x^2 + 4x + 32 = 0$ Hence, $b = 4$	Question 3 ii. $y = \frac{x + 32}{-x^2 + 4x + 32} = \frac{A}{8 - x} + \frac{B}{x + 4}$ $= \frac{A(x + 4) + B(8 - x)}{(8 - x)(x + 4)}$ Now $x + 32 = A(x + 4) + B(8 - x)$ If $x = -4, 28 = 12B \Rightarrow B = \frac{7}{3}$ If $x = 8, 40 = 12A \Rightarrow A = \frac{10}{3}$ $y = \frac{x + 32}{-x^2 + 4x + 32} = \frac{10}{3(8 - x)} + \frac{7}{3(x + 4)}$
Question 3 iii. Area $= \int_0^4 \frac{x + 32}{-x^2 + 4x + 32} dx = p \log_e 2$ $= \int_0^4 \left(\frac{10}{3(8 - x)} + \frac{7}{3(x + 4)} \right) dx$ $= \frac{1}{3} [7 \log_e (x + 4) - 10 \log_e (8 - x)]_0^4$ $= \frac{1}{3} [7 \log_e 8 - 10 \log_e 4 - 7 \log_e 4 + 10 \log_e 8]$ $= \frac{1}{3} \left[7 \log_e \frac{8}{4} + 10 \log_e \frac{8}{4} \right]$ $= \frac{1}{3} [7 \log_e 2 + 10 \log_e 2]$ $= \frac{17}{3} \log_e 2 = p \log_e 2 \Rightarrow p = \frac{17}{3}$ check fnInt($Y_1, X, 0, 4$) = 3.928	

Question 4

$$\text{Let } u = \frac{2}{x} = 2x^{-1}$$

$$\frac{du}{dx} = -2x^{-2} = -\frac{2}{x^2}$$

$$dx = \frac{-x^2 du}{2}$$

Terminals

$$x = \frac{12}{\pi} \Rightarrow u = 2 \times \frac{\pi}{12} = \frac{\pi}{6}$$

$$x = \frac{6}{\pi} \Rightarrow u = 2 \times \frac{\pi}{6} = \frac{\pi}{3}$$

$$\int_{\frac{6}{\pi}}^{\frac{12}{\pi}} \frac{4 \cos\left(\frac{2}{x}\right)}{x^2} dx = \int_{\frac{\pi}{3}}^{\frac{\pi}{6}} \frac{4 \cos(u)}{x^2} \cdot \frac{-x^2 du}{2}$$

$$= -2 \int_{\frac{\pi}{3}}^{\frac{\pi}{6}} \cos(u) du$$

$$= -2 \sin u \Big|_{\frac{\pi}{3}}^{\frac{\pi}{6}}$$

$$= -2 \left[\sin \frac{\pi}{6} - \sin \frac{\pi}{3} \right]$$

$$= -2 \left[\frac{1}{2} - \frac{\sqrt{3}}{2} \right]$$

$$= \sqrt{3} - 1$$

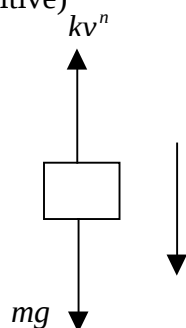
$$\text{check fnInt}(Y_1, X, \frac{6}{\pi}, \frac{12}{\pi}) = 0.732$$

Question 5 i.

By Newton's Law (taking down as positive)

$$\sum F = ma = mg - kv^n$$

$$\Rightarrow a = g - \frac{k}{m}v^n$$



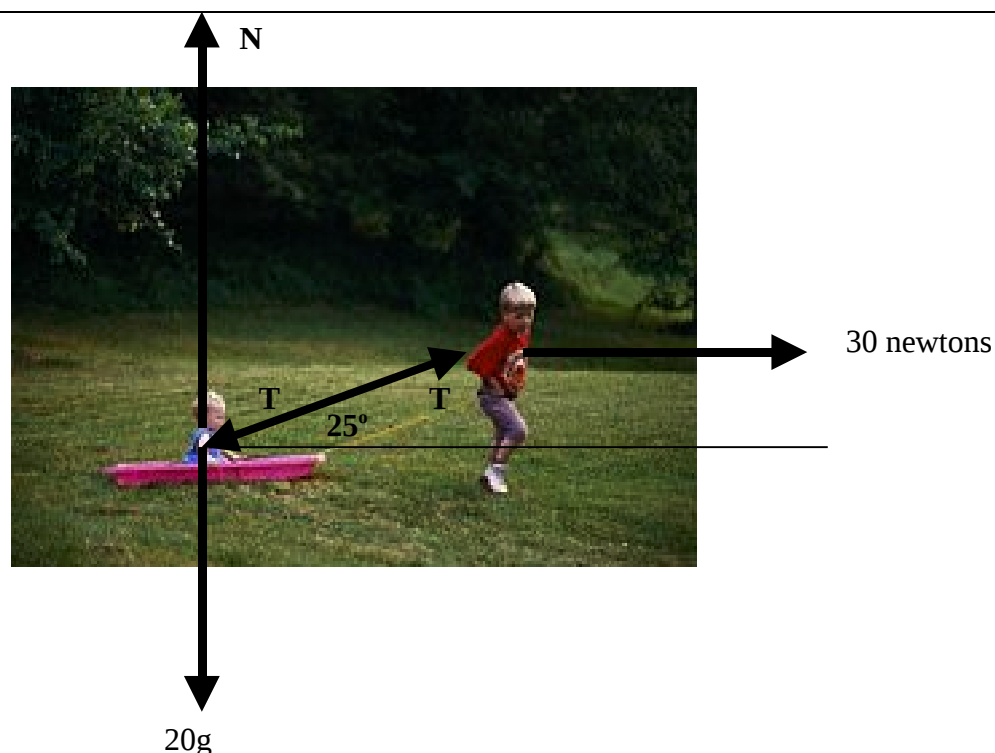
Question 5 ii.

$$a = v \frac{dv}{dx} = g - \frac{k}{m}v^n$$

$$x = \int_0^{v_f} \frac{v dv}{g - \frac{k}{m}v^n} = \int_0^2 \frac{v dv}{9.8 - \frac{0.5}{2}v^3}$$

$$\text{fnInt}(X / (9.8 - 0.25X^3), X, 0, 2) = 0.223 \text{ m}$$

Question 6



Boy: $30 - T \cos 25^\circ = 35a$ (1)

Girl: $T \cos 25^\circ = 20a$ (2)

Add (1) and (2)

$$\Rightarrow 30 = 55a \Rightarrow a = \frac{6}{11} \text{ m/s}^2$$

$$\text{Hence, } T = \frac{20a}{\cos 25^\circ} = \frac{20 \times \frac{6}{11}}{\cos 25^\circ} = 12.04 \text{ Newtons}$$

END OF SUGGESTED SOLUTIONS
2003 Specialist Mathematics Trial Examination 1

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