MAV Specialist Mathematics Examination 1
Answers & Solutions

Part I (Multiple-choice) Answers

1. **E**	2. **B**	3. **C**	4. **A**	5. **C**
6. **A**	7. **A**	8. **B**	9. **D**	10. **D**
11. **A**	12. **B**	13. **C**	14. **C**	15. **D**
16. **A**	17. **E**	18. **A**	19. **E**	20. **E**
21. **E**	22. **D**	23. **C**	24. **B**	25. **E**
26. **E**	27. **D**	28. **E**	29. **B**	30. **D**

Question 1 [E]

$$\vec{AB} = \vec{AO} + \vec{OB}$$
$$= \vec{OB} - \vec{OA}$$
$$(4\underset{\sim}{i} - 2\underset{\sim}{j}) - (5\underset{\sim}{i} - 2\underset{\sim}{j} - \underset{\sim}{k})$$
$$= -\underset{\sim}{i} + \underset{\sim}{k}$$

Question 2 [B]

$$|z - 1 + 2i| = 2$$
$$|x + yi - 1 + 2i| = 2$$
$$|(x-1) + (y+2)i| = 2$$
$$\sqrt{(x-1)^2 + (y+2)^2} = 2$$
$$(x-1)^2 + (y+2)^2 = 4$$
circle, centre $(1, -2)$, radius 2

Question 3 [C]

$-1 \le 3x - 1 \le 1$	$3x - 1 \le 1$
$3x - 1 \ge -1$	$3x \le 2$
$3x \ge 0$	$x \le \frac{2}{3}$
$x \ge 0$	

OR

$$-2Sin^{-1}(3x-1) - 2 = -2Sin^{-1}\left[3\left(x - \frac{1}{3}\right)\right] - 2$$

$f(x) = Sin^{-1}(x)$, dom $f = [-1, 1]$

$g(x) = Sin^{-1}(3x)$ Dilated by a factor of $\frac{1}{3}$.

dom $g = \left[-\frac{1}{3}, \frac{1}{3}\right]$

$h(x) = Sin^{-1}\left[3(x - \frac{1}{3})\right]$ is g(x) translated $\frac{1}{3}$ units right.

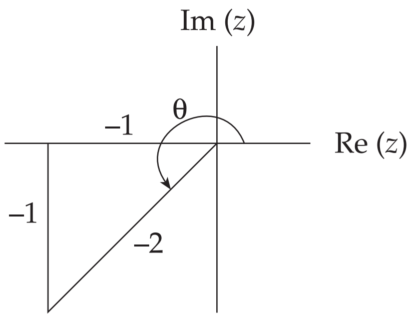
dom $h = \left[0, \frac{2}{3}\right]$

Note: The two (-2)'s in the given function do not effect the domain.

Question 4 [A]

$$\sec^2 x = 1 + \tan^2 x$$
$$= 1 + \frac{4}{25}$$
$$= \frac{29}{25}$$
$$\sec x = -\frac{\sqrt{29}}{5}, \text{ since } \pi < x < \frac{3\pi}{2}$$

Question 5 **[C]**



$\tan\theta = \frac{-1}{-1} = 1$

$\theta = \frac{5\pi}{4}$

$\therefore -1 - i = \sqrt{2}cis\left(-\frac{3\pi}{4}\right)$

Question 6 **[A]**

$$\int \frac{3}{\sqrt{1-4x^2}}\,dx = -3\int \frac{-1}{\sqrt{4\left(\frac{1}{4}-x^2\right)}}\,dx$$

$$= -3\int \frac{-1}{2\sqrt{\left(\frac{1}{2}\right)^2 - x^2}}\,dx$$

$$= -\frac{3}{2}\mathrm{Cos}^{-1}(2x) + c$$

Question 7 **[A]**

$$\int_2^3 \frac{4}{x^2}\log_e\left(\frac{2}{x}\right)dx$$

Let $u = \frac{2}{x} = 2x^{-1}$

$\frac{du}{dx} = -2x^{-2} = \frac{-2}{x^2}$

$-2\frac{du}{dx} = \frac{4}{x^2}$

Terminals: $x = 3,\ u = \frac{2}{3}$

$x = 2,\ u = 1$

$$-2\int_1^{\frac{2}{3}} \log_e(u)\,du$$

Question 8 **[B]**

$$(1+i)^4 = \left(\sqrt{2}\mathrm{cis}\left(\frac{\pi}{4}\right)\right)^4$$

$$= 4\mathrm{cis}(\pi)$$

$$= 4(\cos(\pi) + i\sin(\pi))$$

$$= -4$$

Question 9 **[D]**

If $z = r\mathrm{cis}(\theta)$, then $\bar{z} = r\mathrm{cis}(-\theta)$

Question 10 **[D]**

$$\underset{\sim}{r}(t) = 3e^{-t}\,\underset{\sim}{i} + \frac{3}{2}\cos(2t)\,\underset{\sim}{j}$$

$$\dot{\underset{\sim}{r}}(t) = -3e^{-t}\,\underset{\sim}{i} - \frac{3}{2}\times 3\sin(2t)\,\underset{\sim}{j}$$

$$\dot{\underset{\sim}{r}}(0) = -3\underline{i}$$

$$\left|\dot{\underset{\sim}{r}}(0)\right| = 3$$

$$= 3$$

Question 11 **[A]**

$$Area = \frac{1}{2}\left(1+\frac{8}{9}\right)\frac{1}{2} + \frac{1}{2}\left(\frac{8}{9}+\frac{1}{2}\right)\frac{1}{2}$$

$$= \frac{1}{4}\left(\frac{17}{9} + \frac{25}{18}\right)$$

$$= \frac{1}{4}\left(\frac{59}{18}\right)$$

$$= \frac{59}{72}$$

Question 12 **[B]**

$$\int \frac{1}{x\log_e(5x)}dx$$

$$= \int \frac{\frac{1}{x}}{\log_e(5x)}dx \quad \left[= \int \frac{f'(x)}{f(x)}dx\right]$$

$$= \log_e\left(\log_e(5x)\right) + c$$

OR

In order to determine the correct response to this question, students could systematically differentiate the options.

If $y = \log_e(5x)$, $\frac{dy}{dx} = \frac{1}{x}$

If $y = \log_e(\log_e(5x))$, Let $u = \log_e(5x)$

$y = \log_e(u)$ $\qquad \frac{du}{dx} = \frac{1}{x}$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$= \frac{1}{u} \times \frac{1}{x}$$

$$\frac{1}{x\log_e(5x)}$$

Question 13 **[C]**

Before translation, asymptotes given by

$y = \pm\frac{b}{a}x$ or use $(y-k) = \pm\frac{b}{a}(x-h)$

$y = \pm\frac{6}{3}(x-2)$

$y = \pm 2(x-2)$

Question 14 **[C]**

$$\int_1^4 \frac{x-2}{x^2-4x+7}dx = \frac{1}{2}\int_1^4 \frac{2x-4}{x^2-4x+7}dx$$

$$= \frac{1}{2}\left[\log_e(x^2-4x+7)\right]_1^4$$

$$= \frac{1}{2}\left(\log_e 7 - \log_e 4\right)$$

$$= \frac{1}{2}\log_e\left(\frac{7}{4}\right)$$

$$= \log_e\left(\sqrt{\frac{7}{4}}\right)$$

$$= \log_e\left(\frac{\sqrt{7}}{2}\right)$$

Question 15 **[D]**

$\underset{\sim}{a} = 2\underset{\sim}{i} - 3\underset{\sim}{j} + 4\underset{\sim}{k}$, $|\underset{\sim}{a}| = \sqrt{16+9+4} = \sqrt{29}$

$\underset{\sim}{b} = 3\underset{\sim}{i} + 2\underset{\sim}{j} + \underset{\sim}{k}$, $|\underset{\sim}{b}| = \sqrt{9+4+1} = \sqrt{14}$

$$\cos\theta = \frac{\underset{\sim}{a}\,.\,\underset{\sim}{b}}{|\underset{\sim}{a}| \times |\underset{\sim}{b}|}$$

$$= \frac{2(3) - 3(2) + 4(1)}{\sqrt{29} \times \sqrt{14}}$$

$$= \frac{4}{\sqrt{406}}$$

$$\theta = \text{Cos}^{-1}\left(\frac{4}{\sqrt{406}}\right) = 78.50°$$

Question 16 **[A]**

If l grams dissolved, then $(8-l)$ grams undissolved.

Hence $\frac{dl}{dt} = \frac{25}{100}(8-l)$

$$= \frac{8-l}{4}$$

Question 17 **[E]**

$$y = \sin(3x-1)$$

$$\frac{dy}{dx} = 3\cos(3x-1)$$

$$\frac{d^2y}{dx^2} = -9\sin(3x-1)$$

$$9y + 3\frac{dy}{dx} + \frac{d^2y}{dx^2}$$

$$= 9\sin(3x-1) + 9\cos(3x-1) - 9\sin(3x-1)$$

$$= 9\cos(3x-1)$$

Question 18 **[A]**

$$uv = 3 \times 2cis\left(\frac{5\pi}{6} + \frac{3\pi}{4}\right)$$

$$= 6cis\left(\frac{10\pi + 9\pi}{12}\right)$$

$$= 6cis\left(\frac{19\pi}{12}\right)$$

$$= 6cis\left(-\frac{5\pi}{12}\right) \text{ since } -\pi < \theta < \pi$$

Question 19 **[E]**

Method 1: Consider total motion.

$s = 40\text{m},\ a = 9.8\,\text{m/s}^2,\ u = -15\,\text{m/s}$

$s = ut + \frac{1}{2}at^2$

$40 = -15t + 4.9t^2$

$4.9t^2 - 15t - 40 = 0$

Quadratic formula $\Rightarrow t = 4.77\,\text{sec}$, since $t > 0$

Method 2: Consider upwards and downwards motions separately.

Upward motion:

$u = 15\,\text{m/s},\ v = 0\,\text{m/s},\ a = -9.8\ \text{m/s}^2$

$t = \frac{v-u}{a} = \frac{-15}{-9.8} = 1.53$ seconds

$v^2 - u^2 = 2as$

$s = \frac{-u^2}{2a} = \frac{-15^2}{-19.6} = 11.48$

Downward motion:

$s = 40 + 11.48 = 51.48$ metres,

$u = 0\ \text{m/s},\ a = 9.8\ \text{m/s}^2$

$s = ut + \frac{1}{2}at^2$

$51.48 = 4.9t^2$

$t = 3.24$ seconds, since $t > 0$

Total time $= 1.53 + 3.24 = 4.77$ seconds

Question 20 **[E]**

$\frac{dy}{dx} = \frac{1}{x^2},\quad y_{n+1} = y_n + hf'(x_n),\quad h = 0.2$

x	y
1	3
1.2	$3 + 0.2\left(\frac{1}{1^2}\right) = 3.2$
1.4	$3.2 + 0.2\left(\frac{1}{1.2^2}\right) = 3.339$

Question 21 **[E]**

$$\left|-2\underset{\sim}{i} + 6\underset{\sim}{j} - 9\underset{\sim}{k}\right| = \sqrt{4 + 36 + 81}$$

$$= 11$$

$$\frac{22}{11}\left(-2\underset{\sim}{i} + 6\underset{\sim}{j} - 9\underset{\sim}{k}\right)$$

$$= 2\left(-2\underset{\sim}{i} + 6\underset{\sim}{j} - 9\underset{\sim}{k}\right)$$

$$= -4\underset{\sim}{i} + 12\underset{\sim}{j} - 18\underset{\sim}{k}$$

Option E: $4\underset{\sim}{i} - 12\underset{\sim}{j} + 18\underset{\sim}{k} = -2(-2\underset{\sim}{i} + 6\underset{\sim}{j} - 9\underset{\sim}{k})$

Question 22 **[D]**

$\left|\underset{\sim}{b}\right| = \sqrt{5} \qquad \underset{\sim}{a} . \hat{\underset{\sim}{b}} = \frac{1}{\sqrt{5}}((2 \times 1) + (-3 \times -2))$

$$= \frac{8}{\sqrt{5}}$$

Question 23 **[C]**

$a = v\frac{dv}{dx},\quad \frac{dv}{dx} = \frac{1}{2}$

$$= \frac{x}{2} \times \frac{1}{2}$$

$$= \frac{x}{4}$$

Question 24 **[B]**

$\dot{\underset{\sim}{r}} = \underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k}$

$\underset{\sim}{r} = t\underset{\sim}{i} - 2t\underset{\sim}{j} + t\underset{\sim}{k} + \underset{\sim}{c}$

$t = 0, \underset{\sim}{r} = 2\underset{\sim}{i} - 3\underset{\sim}{k}, \Rightarrow \underset{\sim}{c} = 2\underset{\sim}{i} - 3\underset{\sim}{k}$

$\underset{\sim}{r} = (t+2)\underset{\sim}{i} - 2t\underset{\sim}{j} + (t-3)\underset{\sim}{k}$

Question 25 **[E]**

$y = Sin^{-1}\left(\frac{2}{x}\right)$ Let $u = \frac{2}{x} = 2x^{-1}$

$$\frac{du}{dx} = -2x^{-2} = -\frac{2}{x^2}$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$= \frac{1}{\sqrt{1-u^2}} \times \frac{-2}{x^2}$$

$$= \frac{-2}{x^2\sqrt{1-\frac{4}{x^2}}}$$

$$= \frac{-2}{x^2\sqrt{\frac{x^2-4}{x^2}}}$$

$$= \frac{-2}{x^2\,\frac{\sqrt{x^2-4}}{x}}$$

$$= \frac{-2}{x\sqrt{(x^2-4)}}$$

Question 26 **[E]**

$\frac{dv}{dt} = 3e^{-0.2t} + 2$

$v = \int\left(3e^{-0.2t} + 2\right)dt$

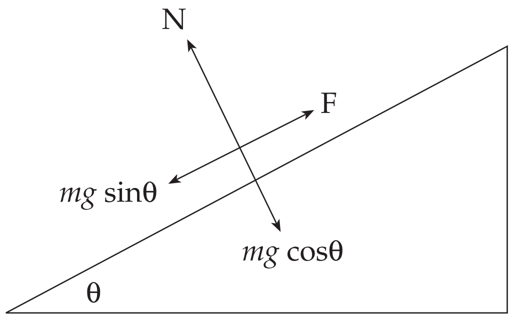
$v = -15e^{-0.2t} + 2t + c$

Starts from rest ($t = 0$, $v = 0$), $c = 15$

Hence $v = -15e^{-0.2t} + 2t + 15$

When $t = 2$, $v = 8.95$

Question 27 **[D]**



$N = mg\cos\theta, \;\; F = \mu N = \mu\, mg\cos\theta$

$R = mg\sin\theta - F,$

$= mg\sin\theta - \mu\, mg\cos\theta$

$a = \frac{R}{m} = g\sin\theta - \mu\, g\cos\theta$

Question 28 **[E]**

$R = N - 4g$

$N - 4g = 4 \times 3$

$N = 12 + 4g$

$= 51.2$

Question 29 **[B]**

$\Delta\underset{\sim}{p} = \underset{\sim}{p}_{final} - \underset{\sim}{p}_{initial}, \;\; \underset{\sim}{p} = m \times \underset{\sim}{v}$

$= 4(5) - 4(8)$

$= -12$

Question 30 **[D]**

Long diagonal : $\overrightarrow{AC} = \overrightarrow{AO} + \overrightarrow{OD}$

$= \overrightarrow{OD} - \overrightarrow{OA}$

$= \underset{\sim}{d} - \underset{\sim}{a}$

Short diagonal : $\overrightarrow{OB} = \underset{\sim}{b}$

Diagonals are perpendicular if $\left(\underset{\sim}{d} - \underset{\sim}{a}\right) \cdot \underset{\sim}{b} = 0$

Part II Solutions

Question 1

a. $\dfrac{1}{9-x^2} \equiv \dfrac{a}{3-x} + \dfrac{b}{3+x}$

$\equiv \dfrac{a(3+x)+b(3-x)}{(3-x)(3+x)}$

$\therefore \quad 1 \equiv a(3+x)+b(3-x)$ **[A]**

If $x = 3$, $1 = 6a$, $a = \dfrac{1}{6}$

If $x = -3$, $1 = 6b$, $b = \dfrac{1}{6}$

$\therefore \dfrac{1}{9-x^2} = \dfrac{1}{6(3-x)} + \dfrac{1}{6(3+x)}$ **[A]**

b. $\displaystyle\int \frac{2}{\sqrt{9-x^2}}\,dx = 2Sin^{-1}\left(\frac{x}{3}\right)$ **[A]**

c. $V = \displaystyle\int_a^b \left(\pi y^2\right) dx$

$y = \dfrac{1}{\sqrt{9-x^2}} + 1$

$y^2 = \dfrac{1}{9-x^2} + \dfrac{2}{\sqrt{9-x^2}} + 1$

$V = \pi\displaystyle\int_0^2 \left(\frac{1}{9-x^2} + \frac{2}{\sqrt{9-x^2}} + 1\right) dx$ **[M]**

$= \pi\displaystyle\int_0^2 \left\{\frac{1}{6}\left(\frac{1}{3-x} + \frac{1}{3+x}\right) + \frac{2}{\sqrt{9-x^2}} + 1\right\} dx$

$= \pi\left[\dfrac{1}{6}\log_e\left(\dfrac{3+x}{3-x}\right) + 2Sin^{-1}\left(\dfrac{x}{3}\right) + x\right]_0^2$ **[M]**

$= \pi\left[\left(\dfrac{1}{6}\log_e 5 + 2Sin^{-1}\dfrac{2}{3} + 2\right) - 0\right]$

≈ 11.71 cubic units **[A]**

Question 2

$h'(x) = \dfrac{x}{\sqrt{1-x}}$ Let $u = 1-x$

$\dfrac{du}{dx} = -1$

$x = 1-u$

$h(x) = -\displaystyle\int \frac{1-u}{u^{\frac{1}{2}}}\,du = \int (-u^{-\frac{1}{2}} + u^{\frac{1}{2}})\,du$ **[M]**

$= -2u^{\frac{1}{2}} + \dfrac{2}{3}u^{\frac{3}{2}} + c$

$= \dfrac{2}{3}(1-x)\sqrt{(1-x)} - 2\sqrt{(1-x)} + c$ **[M]**

$= \dfrac{2}{3}\sqrt{(1-x)}(1-x-3) + c$

$= -\dfrac{2}{3}\sqrt{(1-x)}(x+2) + c$

Since $h(-3) = \dfrac{4}{3}$,

$\dfrac{4}{3} = -\dfrac{2}{3}(-1)\sqrt{4} + c$, $c = 0$ **[A]**

$h(x) = -\dfrac{2}{3}(x+2)\sqrt{(1-x)}$

Question 3

a. $w = 1 - \sqrt{3}i$

$\text{Arg } w = -\dfrac{\pi}{3}$ **[A]**

b. $\arg\left(z^2 w\right) = \left(2 \times \dfrac{3\pi}{4}\right) - \dfrac{\pi}{3}$

$= \dfrac{3\pi}{2} - \dfrac{\pi}{3}$

$= \dfrac{7\pi}{6}$

$\text{Arg}\left(z^2 w\right) = -\dfrac{5\pi}{6}$ **[A]**

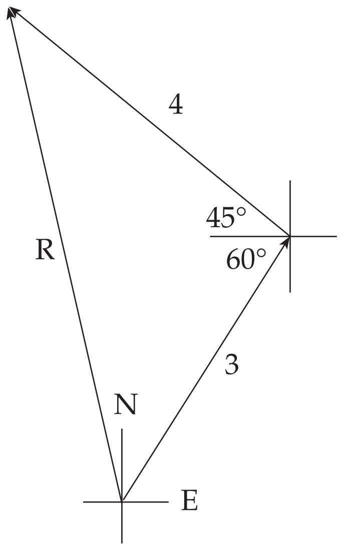
c. $\dfrac{z}{w} = \dfrac{3}{2}\text{cis}\left(\dfrac{3\pi}{4} - \left(-\dfrac{\pi}{3}\right)\right)$

$= \dfrac{3}{2}\text{cis}\left(\dfrac{9\pi}{12} + \dfrac{4\pi}{12}\right)$

$= \dfrac{3}{2}\text{cis}\left(\dfrac{13\pi}{12}\right)$

$= \dfrac{3}{2}\text{cis}\left(-\dfrac{11\pi}{12}\right)$ **[A]**

Question 4



a. $R^2 = 3^2 + 4^2 - 2(4)(3)\cos(105°)$

Diagram **[M]**

$R \approx 5.59$ Newtons **[A]**

b. Let the angle between the 3 Newton force and the Resultant be θ.

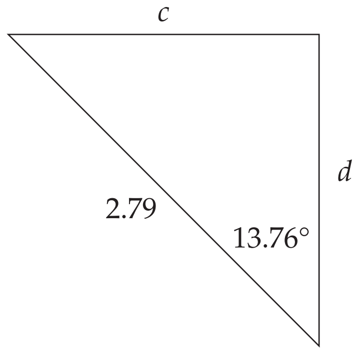
By sine rule $\dfrac{5.59}{\sin(105°)} = \dfrac{4}{\sin\theta}$ **[M]**

$$\sin\theta = \frac{4\sin(105°)}{5.59}$$

$$\theta = 43.755°$$

Hence angle between the Resultant and North is $43.755° - 30° = 13.755°$

$a = \dfrac{F}{m} = 2.79\ \text{m/s}^2$



$\sin(13.755°) = \dfrac{c}{2.59} \Leftrightarrow c = 2.79\sin(13.755°)$ **[M]**

$$c = 0.664$$

$\cos(13.755°) = \dfrac{d}{2.79} \Leftrightarrow d = 2.79\cos(13.755°)$

$$d = 2.71$$

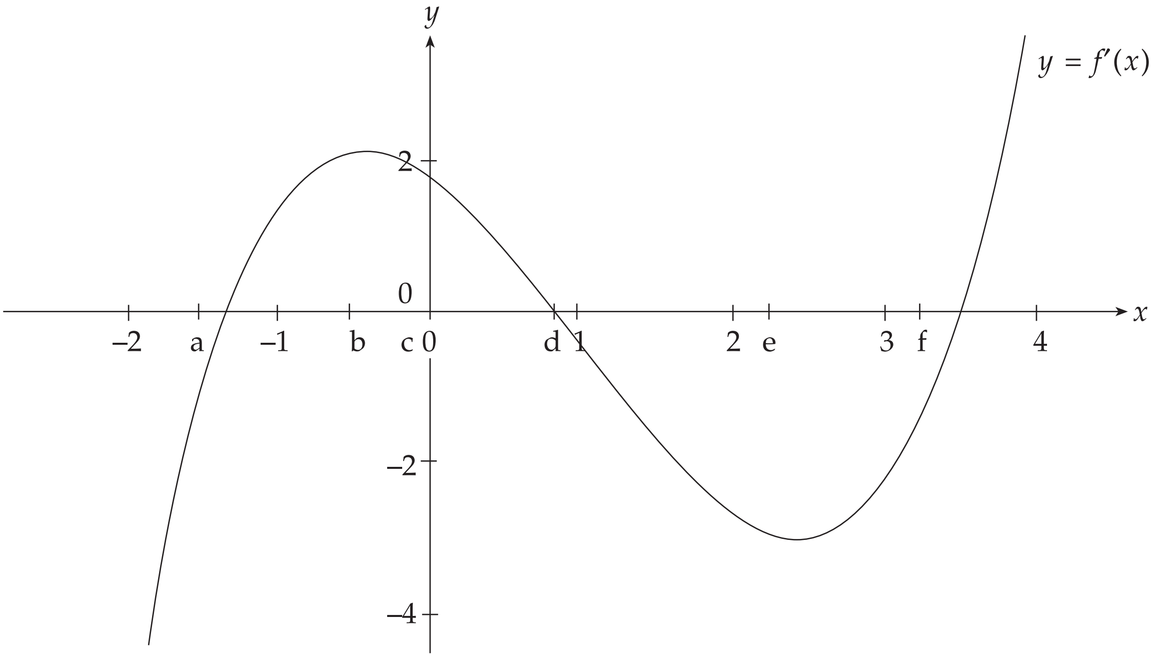
Hence $\underset{\sim}{a} = -0.66\,\underset{\sim}{i} + 2.71\,\underset{\sim}{j}$ **[A]**

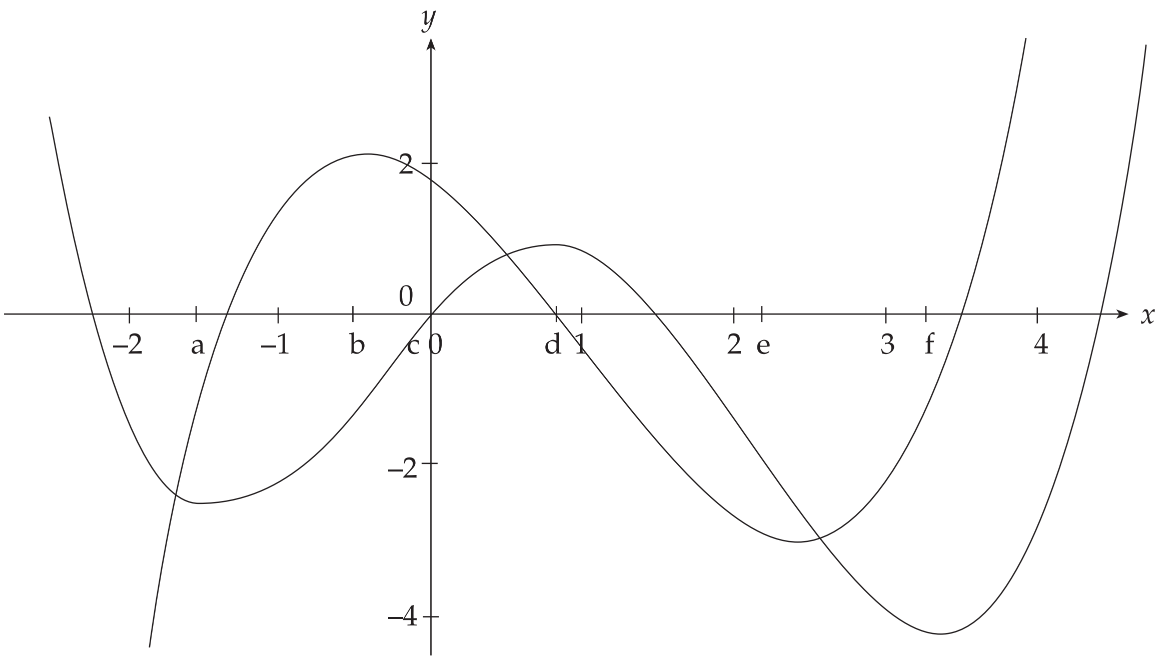
OR

$$\begin{aligned}\sum \underset{\sim}{F} &= 3\cos 60°\,\underset{\sim}{i} + 3\sin 60°\,\underset{\sim}{j} - 4\cos 45°\,\underset{\sim}{i} + 4\sin 45°\,\underset{\sim}{j}\\ &= \frac{3}{2}\underset{\sim}{i} + \frac{3\sqrt{3}}{2}\underset{\sim}{j} - \frac{4}{\sqrt{2}}\underset{\sim}{i} + \frac{4}{\sqrt{2}}\underset{\sim}{j}\\ &= \left(\frac{3}{2} - \frac{4}{\sqrt{2}}\right)\underset{\sim}{i} + \left(\frac{3\sqrt{3}}{2} + \frac{4}{\sqrt{2}}\right)\underset{\sim}{j}\end{aligned}$$

$$\begin{aligned}\therefore \underset{\sim}{a} &= \frac{\sum \underset{\sim}{F}}{m} = \frac{1}{2}\left(\frac{3}{2} - \frac{4}{\sqrt{2}}\,\underset{\sim}{i}\right) + \frac{1}{2}\left(\frac{3\sqrt{3}}{2} + \frac{4}{\sqrt{2}}\right)\underset{\sim}{j}\\ &\approx -0.66\,\underset{\sim}{i} + 2.71\,\underset{\sim}{j}\end{aligned}$$

Question 5





Local minima at 'a' and 'f'; local maximum at 'd'. **[A]**

Points of inflexion at 'b' and 'e'. **[A]**

Correct general shape. **[A]**