

Year 2002

VCE

Specialist Mathematics

Trial Examination 2



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SPECIALIST MATHEMATICS

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

Specialist Mathematics Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder:	$2\pi rh$
volume of a cylinder:	$\pi r^2 h$
volume of a cone:	$\frac{1}{3}\pi r^2 h$
volume of a pyramid:	$\frac{1}{3}Ah$
volume of a sphere:	$\frac{4}{3}\pi r^3$
area of a triangle:	$\frac{1}{2}bc \sin A$
sine rule:	$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
cosine rule:	$c^2 = a^2 + b^2 - 2ab \cos C$

Coordinate geometry

ellipse:	$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$
hyperbola:	$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

Circular (trigonometric) functions

$$\cos^2 x + \sin^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x$$

$$\sin(x+y) = \sin x \cos y + \cos x \sin y$$

$$\cos(x+y) = \cos x \cos y - \sin x \sin y$$

$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

$$\cos 2x = \cos^2 x - \sin^2 x = 2 \cos^2 x - 1 = 1 - 2 \sin^2 x$$

$$\sin 2x = 2 \sin x \cos x$$

$$\cot^2 x + 1 = \operatorname{cosec}^2 x$$

$$\sin(x-y) = \sin x \cos y - \cos x \sin y$$

$$\cos(x-y) = \cos x \cos y + \sin x \sin y$$

$$\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

function	$\sin^{-1}$	$\cos^{-1}$	$\tan^{-1}$
domain	$[-1, 1]$	$[-1, 1]$	R
range	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Algebra (Complex numbers)

$$z = x + yi = r(\cos \theta + i \sin \theta) = r \operatorname{cis} \theta$$

$$|z| = \sqrt{x^2 + y^2} = r$$

$$z_1 z_2 = r_1 r_2 \operatorname{cis}(\theta_1 + \theta_2)$$

$$z^n = r^n \operatorname{cis} n\theta \quad (\text{de Moivre's theorem})$$

$$-\pi < \operatorname{Arg} z \leq \pi$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} \operatorname{cis}(\theta_1 - \theta_2)$$

Calculus

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dx}(e^{ax}) = ae^{ax}$$

$$\frac{d}{dx}(\log_e x) = \frac{1}{x}$$

$$\frac{d}{dx}(\sin ax) = a \cos ax$$

$$\frac{d}{dx}(\cos ax) = -a \sin ax$$

$$\frac{d}{dx}(\tan ax) = a \sec^2 ax$$

$$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$$

$$\int x^n dx = \frac{1}{n+1} x^{n+1} + c, n \neq -1$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax} + c$$

$$\int \frac{1}{x} dx = \log_e x + c, \text{ for } x > 0$$

$$\int \sin ax dx = -\frac{1}{a} \cos ax + c$$

$$\int \cos ax dx = \frac{1}{a} \sin ax + c$$

$$\int \sec^2 ax dx = \frac{1}{a} \tan ax + c$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a} + c, a > 0$$

$$\int \frac{-1}{\sqrt{a^2 - x^2}} dx = \cos^{-1} \frac{x}{a} + c, a > 0$$

$$\int \frac{a}{a^2 + x^2} dx = \tan^{-1} \frac{x}{a} + c$$

product rule:

$$\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$$

quotient rule:

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

chain rule:

$$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$$

mid-point rule:

$$\int_a^b f(x) dx \approx (b-a) f\left(\frac{a+b}{2}\right)$$

trapezoidal rule:

$$\int_a^b f(x) dx \approx \frac{1}{2}(b-a)(f(a) + f(b))$$

Euler's method:

$$\text{If } \frac{dy}{dx} = f(x), x_0 = a \text{ and } y_0 = b, \text{ then } x_{n+1} = x_n + h \text{ and } y_{n+1} = y_n + hf(x_n)$$

acceleration:

$$a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v \frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2} v^2\right)$$

constant (uniform) acceleration:

$$v = u + at \quad s = ut + \frac{1}{2}at^2 \quad v^2 = u^2 + 2as \quad s = \frac{1}{2}(u+v)t$$

TURN OVER

Vectors in two and three dimensions

$$\underline{r} = x\underline{i} + y\underline{j} + z\underline{k}$$

$$|\underline{r}| = \sqrt{x^2 + y^2 + z^2} = r$$

$$\underline{r}_1 \cdot \underline{r}_2 = r_1 r_2 \cos \theta = x_1 x_2 + y_1 y_2 + z_1 z_2$$

$$\dot{\underline{r}} = \frac{d\underline{r}}{dt} = \frac{dx}{dt} \underline{i} + \frac{dy}{dt} \underline{j} + \frac{dz}{dt} \underline{k}$$

Mechanics

momentum: $\underline{p} = m \underline{v}$

equation of motion: $\underline{R} = m \underline{a}$

friction: $F \leq \mu N$

END OF FORMULA SHEET

STUDENT NUMBER

Letter

Figures									
Words									

VICTORIAN CERTIFICATE OF EDUCATION 2002

SPECIALIST MATHEMATICS

Trial Written Examination 2 (Analysis task)

Reading time: 15 minutes

Total writing time: 1 hour 30 minutes

QUESTION AND ANSWER BOOK

Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>
5	5

Directions to students

Materials

Question and answer book of 11 pages.

Working space is provided throughout the book.

There is a detachable sheet of miscellaneous formula supplied.

You may bring to the examination up to four pages (two A4 sheets) of pre-written notes.

You may use an approved scientific and/or graphics calculator, ruler, protractor, set-square and aids for curve-sketching.

The task

Detach the formula sheet from the book during reading time.

Please ensure that your **student number** in the space provided on the front cover of this book.

Answer **all** questions

The marks allotted to each part of each question are indicated at the end of the part.

There is a total of 60 marks available for the examination.

You need not give numerical answers as decimals unless instructed to do so. Alternative forms may involve, for example, π , e , surds or fractions.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

Where an exact answer is required to a question, appropriate working must be shown and calculus must be used to evaluate derivatives and definite integrals.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude g m/s², where $g = 9.8$.

All written responses should be in English.

Question 1

The flight of a butterfly is given by the parametric equation $x = \sin t, y = \cos 2t, t \geq 0$

- a.** Find the vector equation of the path of the butterfly.

1 mark

- b.** Find the Cartesian equation of the path of the butterfly.

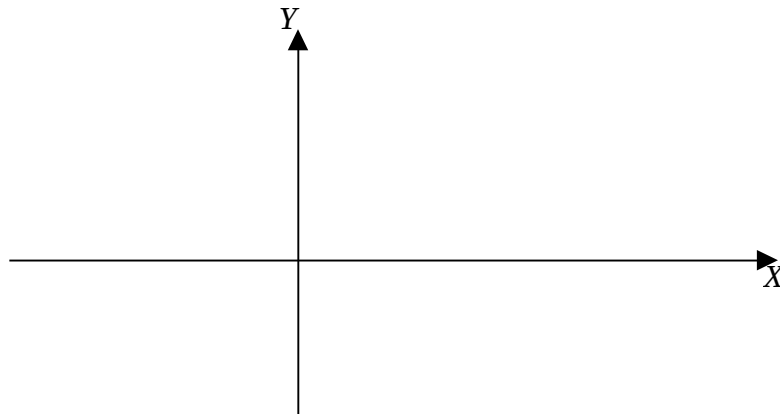
1 mark

- c.** Give the domain and range of this path of the butterfly.

2 marks

Question 1 (continued)

- d. On the axes below, sketch the curve traced out by the butterfly, showing all relevant points.



4 marks

- e. When $t = \frac{\pi}{4}$ the butterfly sees a flower at the point $(0.8, -0.2)$. What is the position vector of the butterfly at this time?

1 mark

Question 1 (continued)

- f. When the butterfly sees the flower, it flies off on a tangent to its original path.
- i. Find the vector $\overrightarrow{AF}$ where A is the point on the path where the butterfly changes course and F is the position of the flower.

2 marks

- ii. Find a vector which represents the butterfly's new path of travel.

2 marks

- g. Will the butterfly reach the flower if it maintains this course?
Give a reason for your answer.

2 marks

(Total = 15 marks)

Question 2

A body of mass m kg is projected vertically upwards from the surface of the earth with a velocity, u . The acceleration of a particle in space is $\vec{a} = \frac{k}{x^2}$ towards the centre of the earth where k is a constant and x is the distance of the body from the centre of the earth. If the acceleration at the earth's surface is g and the radius of the earth is R ,

- a.** Find the value of k in terms of R and g .

1 mark

- b.** Taking the upward direction from the earth as positive, find the force acting on the body.

1 mark

- c.** Show that the velocity, v , of the body at any time, t , can be given by the equation

$$v^2 = u^2 - 2gR^2\left(\frac{1}{R} - \frac{1}{x}\right)$$

5 marks

d. If $u^2 = 2gR$, find t in terms of x

This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and extend across the width of the page. There are no margins, text, or other markings on the paper.

e. What is the minimum velocity of projection, u , in km/sec that the body needs so that it never returns to earth. Take the radius of the earth, $R = 6.4 \times 10^6$ m. Give your answer to the nearest integer.

[illegible]

(Total = 15 marks)

a. Ann, Ben and Chris are standing on three points A, B and C which have position vectors $2\hat{i} + \hat{j}$, $3\hat{i} + 3\hat{j}$ and $5\hat{i} + 2\hat{j}$ respectively. David is asked to stand in a fourth position so that the four friends form a square. Find the position vector for the point where David stands.

This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

3 marks

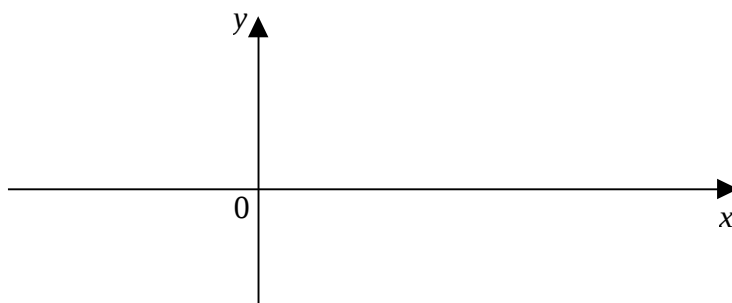
b. Let A and B be endpoints of a diameter of a circle. Let C be a third point on the circumference of the circle. Use a vector method to prove that $\angle ACB = 90^\circ$

This image shows a full page of white paper with horizontal blue or grey ruling lines. The lines are evenly spaced and run across the width of the page, providing a template for handwriting practice. There are no margins, text, or other markings on the page.

(Total = 7 marks)

Question 4

- a. Sketch the curve $x^2 = 10y$ on the axes below.



1 mark

- b. The part of the curve from $y = 0$ to $y = h$ is rotated about the Y axis. Show that the volume of revolution is given by $V = 5\pi h^2$

2 marks

- c. If the volume thus generated represents a pool which is filled with water to a depth of h metres, show that the surface area of the water, S , is equal to $10\pi h$

2 marks

d. If water evaporates from the pool according to the rule $\frac{dV}{dt} = -0.002S$, find the rate of change of height with respect to time, where time is measured in hours.

[illegible]

e. Initially, the pool contains 80π m³ of water. How long will it take for the pool to empty by evaporation?

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

(Total = 12 marks)

Question 5

A boat of mass 5 kg, which is initially at rest, is pushed in the direction of motion by a constant force of 20N exerted by its motor. The water resists the motion with a backwards force whose magnitude is always half the speed, v .

- a.** Draw a diagram showing the forces acting on the boat.

1 mark

- b.** Find the resultant force in the direction of motion in terms of v .

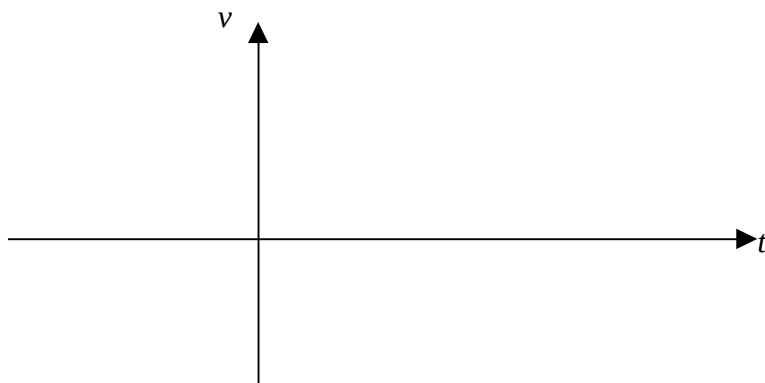
1 mark

- c.** Find the speed of the boat at any time, t .

4 marks

Question 5 (continued)

- d. Sketch the graph of v against t on the axes below.



2 marks

- e. How fast can the boat go?

1 mark

- f. How far does the boat travel in the first 60 seconds? Give your answer in km. to one decimal place.

2 marks

(Total = 11 marks)

END OF QUESTION AND ANSWER BOOK

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