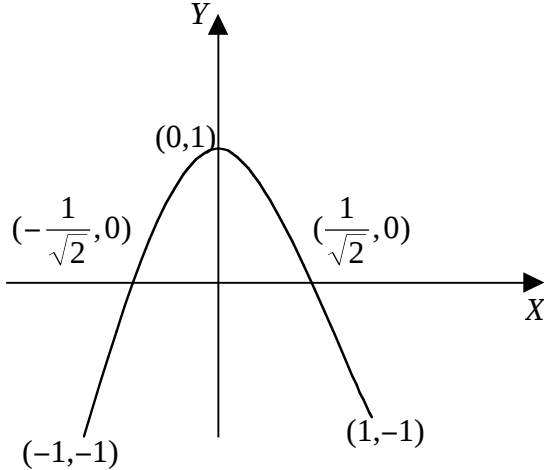


Question 1

a. $\vec{r} = x\hat{i} + y\hat{j}$ $\vec{r} = \sin t\hat{i} + \cos 2t\hat{j}$ (1 mark)	b. $y = \cos 2t$ $y = 1 - 2\sin^2 t$ $y = 1 - 2x^2$ (1 mark)
c. $-1 \leq \cos 2t \leq 1$ $-1 \leq y \leq 1$ $-1 \leq \sin t \leq 1$ $-1 \leq x \leq 1$ Domain : $-1 \leq x \leq 1$ (1 mark) Range : $-1 \leq y \leq 1$ (1 mark)	d.  When $x = -1, y = 1 - 2 = -1$ When $x = 1, y = 1 - 2 = -1$ When $y = 0, 1 - 2x^2 = 0$ $2x^2 = 1$ $x^2 = \frac{1}{2}$ $x = \pm \frac{1}{\sqrt{2}}$ • shape (1 mark) • x intercepts (1 mark) • y intercepts (1 mark) • end points (1 mark)

Question 1 (continued)

e. $r = \sin \frac{\pi}{4} \hat{i} + \cos \frac{\pi}{2} \hat{j}$ $r = \frac{1}{\sqrt{2}} \hat{i}$ (1 mark)	f.(i) $\overrightarrow{OF} = 0.8\hat{i} - 0.2\hat{j}$ $\overrightarrow{OA} = \frac{1}{\sqrt{2}}\hat{i}$ $\overrightarrow{AF} = \overrightarrow{AO} + \overrightarrow{OF}$ $\overrightarrow{AF} = \overrightarrow{OF} - \overrightarrow{OA} \text{ (1 mark)}$ $\overrightarrow{AF} = 0.8\hat{i} - 0.2\hat{j} - \frac{1}{\sqrt{2}}\hat{i}$ $\overrightarrow{AF} = 0.09\hat{i} - 0.2\hat{j} \text{ (1 mark)}$
f.(ii) New path of travel is along the tangent to the curve. $\dot{r} = \cos t \hat{i} - 2 \sin 2t \hat{j} \text{ (1 mark)}$ Initial point along tangent is when $t = \frac{\pi}{4}$ $\therefore$ a vector $\overrightarrow{AP}$ which represents the butterfly's new path is $\overrightarrow{AP} = k \left(\frac{1}{\sqrt{2}} \hat{i} - 2\hat{j} \right) \text{ where } k \text{ is a constant}$ $\overrightarrow{AP} = k \left(\frac{\sqrt{2}}{2} \hat{i} - 2\hat{j} \right)$ $\overrightarrow{AP} = k(0.7\hat{i} - 2\hat{j}) \text{ (1 mark)}$	g. No, because the position vectors $\overrightarrow{AF}$ and $\overrightarrow{AP}$ are not parallel since $0.09\hat{i} - 0.2\hat{j} \neq k(0.7\hat{i} + 2\hat{j})$ where k is a constant. (2 marks)

Question 2

a. $g = \frac{k}{R^2}$ $k = gR^2$ (1 mark)	b. $\vec{F} = m\vec{a}$ $\vec{F} = \frac{-mk}{x^2}$ (1 mark)
c. $\vec{a} = \frac{-k}{x^2} = \frac{-gR^2}{x^2}$ $\frac{d}{dx}\left(\frac{1}{2}v^2\right) = \frac{-gR^2}{x^2} \quad (1 \text{ mark})$ $\frac{1}{2}v^2 = \int -gR^2 x^{-2} dx$ $\frac{1}{2}v^2 = \frac{gR^2}{x} + c \quad (1 \text{ mark})$ At the surface of the earth $x = R, v = u$ (1 mark) $\Rightarrow \frac{1}{2}u^2 = \frac{gR^2}{R} + c$ $\Rightarrow c = \frac{1}{2}u^2 - gR \quad (1 \text{ mark})$ Hence, $\frac{1}{2}v^2 = \frac{gR^2}{x} + \frac{1}{2}u^2 - gR$ $v^2 = \frac{2gR^2}{x} + u^2 - 2gR$ $v^2 = u^2 - 2gR^2\left(\frac{1}{R} - \frac{1}{x}\right) \quad (1 \text{ mark})$	d. $v^2 = 2gR - 2gR^2\left(\frac{1}{R} - \frac{1}{x}\right)$ $v^2 = 2gR - 2gR + \frac{2gR^2}{x}$ $v^2 = \frac{2gR^2}{x} \quad (1 \text{ mark})$ $v = \sqrt{2gR^2} \times x^{-\frac{1}{2}}$ $\frac{dx}{dt} = \sqrt{\frac{2gR^2}{x}} \quad (1 \text{ mark})$ $\frac{dt}{dx} = \frac{x^{\frac{1}{2}}}{\sqrt{2gR^2}} \quad (1 \text{ mark})$ Antidiff. both sides w.r.t. x $t = \frac{2}{3\sqrt{2gR^2}} x^{\frac{3}{2}} + c \quad (1 \text{ mark})$ When $t = 0, x = R$ $0 = \frac{2}{3\sqrt{2gR^2}} R^{\frac{3}{2}} + c$ $c = -\frac{2R^{\frac{1}{2}}}{3\sqrt{2g}}$ $\Rightarrow t = \frac{2}{3\sqrt{2g} \times R} \times x^{\frac{3}{2}} - \frac{2\sqrt{R}}{3\sqrt{2g}}$ $\Rightarrow t = \frac{2}{3\sqrt{2g}} \left[\frac{x^{\frac{3}{2}}}{R} - \sqrt{R} \right] \quad (1 \text{ mark})$

Question 2 (continued)

e.

For body never to return to earth

$$x \rightarrow \infty$$

$$v^2 = u^2 - 2gR^2\left(\frac{1}{R} - \frac{1}{x}\right)$$

As $x \rightarrow \infty$

$$v^2 = u^2 - 2gR^2 \times \frac{1}{R}$$

$$v^2 = u^2 - 2gR \quad (1 \text{ mark})$$

For body not to return to earth, $v > 0$

$$\Rightarrow u^2 - 2gR > 0$$

$$\Rightarrow u^2 > 2gR$$

$$\Rightarrow u > \sqrt{2gR} \quad (1 \text{ mark})$$

$$u > \sqrt{2 \times 9.8 \times 6.4 \times 10^6}$$

$$u > 11.2 \times 10^3 \text{ m/sec}$$

$$u > 11.2 \text{ km/sec}$$

$$u = 12 \text{ km/sec} \quad (1 \text{ mark})$$

Question 3

a.

$$\overrightarrow{AB} = \hat{i} + 2\hat{j}$$

$$|\overrightarrow{AB}| = \sqrt{5}$$

$$\overrightarrow{BC} = 2\hat{i} - \hat{j}$$

$$|\overrightarrow{BC}| = \sqrt{5}$$

$$\overrightarrow{AB} \cdot \overrightarrow{BC} = (\hat{i} + 2\hat{j}) \cdot (2\hat{i} - \hat{j})$$

$$\overrightarrow{AB} \cdot \overrightarrow{BC} = 2 - 2$$

$$\overrightarrow{AB} \cdot \overrightarrow{BC} = 0$$

$\therefore AB$ is perpendicular to BC (1 mark)

$$\overrightarrow{DC} = \overrightarrow{AB}$$

$$\overrightarrow{DC} = \hat{i} + 2\hat{j} \text{ (1 mark)}$$

$$\overrightarrow{DC} = \overrightarrow{DO} + \overrightarrow{OC}$$

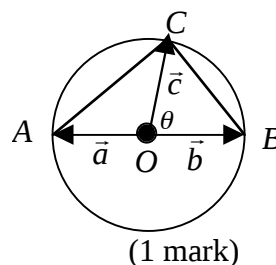
$$\overrightarrow{DC} = \overrightarrow{OC} - \overrightarrow{OD}$$

$$\overrightarrow{OD} = \overrightarrow{OC} - \overrightarrow{DC}$$

$$\overrightarrow{OD} = 5\hat{i} + 2\hat{j} - \hat{i} - 2\hat{j}$$

$$\overrightarrow{OD} = 4\hat{i} \text{ (1 mark)}$$

b.



$$\overrightarrow{AC} = \overrightarrow{AO} + \overrightarrow{OC}$$

$$\overrightarrow{AC} = \vec{c} - \vec{a}$$

$$\overrightarrow{BC} = \overrightarrow{BO} + \overrightarrow{OC}$$

$$\overrightarrow{BC} = \vec{c} - \vec{b}$$

$$\overrightarrow{AC} \cdot \overrightarrow{BC} = (\vec{c} - \vec{a}) \cdot (\vec{c} - \vec{b}) \text{ (1 mark)}$$

$$\overrightarrow{AC} \cdot \overrightarrow{BC} = \vec{c} \cdot \vec{c} - \vec{c} \cdot \vec{b} - \vec{a} \cdot \vec{c} + \vec{a} \cdot \vec{b}$$

$$\overrightarrow{AC} \cdot \overrightarrow{BC} = c^2 - bc \cos \theta - ac \cos(180 - \theta)^0 + ab \cos 180^0$$

$$\overrightarrow{AC} \cdot \overrightarrow{BC} = c^2 - bc \cos \theta + ac \cos \theta - ab$$

But, $a = b = c$ (equal radii)

$$\therefore \overrightarrow{AC} \cdot \overrightarrow{BC} = c^2 - c^2 \cos \theta + c^2 \cos \theta - c^2$$

$$\therefore \overrightarrow{AC} \cdot \overrightarrow{BC} = 0 \text{ (1 mark)}$$

$$\therefore |\overrightarrow{AC}| |\overrightarrow{BC}| \cos \angle ACB = 0$$

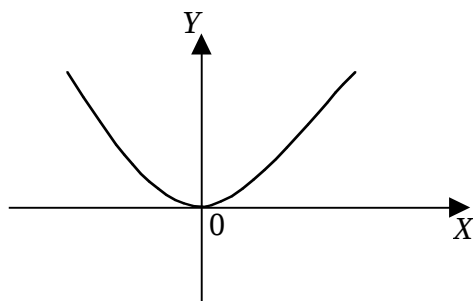
But $|\overrightarrow{AC}| \neq 0$ and $|\overrightarrow{BC}| \neq 0$

$$\therefore \cos \angle ACB = 0$$

$$\Rightarrow \angle ACB = 90^0 \text{ (1 mark)}$$

Question 4

a.



(1 mark)

b.

$$V = \int_0^h \pi x^2 dy$$

$$V = \int_0^h 10\pi y dy \quad (1 \text{ mark})$$

$$V = 5\pi y^2 \Big|_0^h$$

$$V = 5\pi h^2 \quad (1 \text{ mark})$$

c.

Surface area is a circle.

$$S = \pi r^2 \text{ where } r = x$$

$$\therefore S = \pi x^2 \quad (1 \text{ mark})$$

$$\Rightarrow S = \pi \times 10y$$

$$\text{But } y = h$$

$$\therefore S = 10\pi h$$

(1 mark)

d.

$$\frac{dh}{dt} = \frac{dh}{dv} \frac{dv}{dt} \quad (1 \text{ mark})$$

$$\frac{dv}{dh} = 10\pi h \quad (1 \text{ mark})$$

$$\frac{dh}{dt} = \frac{1}{10\pi h} \times -0.002 \times 10\pi h \quad (1 \text{ mark})$$

$$\frac{dh}{dt} = -0.002 \text{ m/hr}$$

Suggested Solutions

Question 4 (continued)

e.

$$V = 5\pi h^2$$

Initially, $V = 80\pi$

$$\therefore 80\pi = 5\pi h^2$$

$$h^2 = 16$$

$$\therefore h = 4 \text{ m initially, when } t = 0 \text{ (1 mark)}$$

$$\frac{dh}{dt} = -0.002$$

$$h = \int -0.002 dt$$

$$h = -0.002t + c \text{ (1 mark)}$$

$$4 = c$$

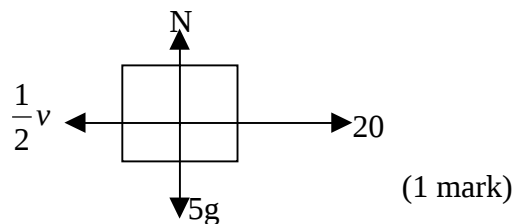
$$\therefore h = -0.002t + 4$$

Pool is empty when $h = 0$ (1 mark)

$$0.002t = 4$$

$$t = \frac{4}{0.002}$$

$$t = 2000 \text{ hrs. (1 mark)}$$

Question 5**a.****b.**

$$\vec{F}_R = 20 - \frac{1}{2}v$$

(1 mark)

Question 5 (continued)

c.

$$m \frac{dv}{dt} = 20 - \frac{1}{2}v$$

$$5 \frac{dv}{dt} = 20 - \frac{1}{2}v \quad (1 \text{ mark})$$

$$\frac{dv}{dt} = 4 - \frac{v}{10}$$

$$\frac{dv}{dt} = \frac{40 - v}{10}$$

Invert b.s.

$$\frac{dt}{dv} = \frac{10}{40 - v}$$

Antidiff b.s.w.r.t.v

$$t = -10 \int \frac{-dv}{40 - v} \quad (1 \text{ mark})$$

$$t = -10 \log_e(40 - v) + c \text{ where } c \text{ is a constant}$$

$$\text{When } t = 0, v = 0$$

$$0 = -10 \log_e 40 + c$$

$$c = 10 \log_e 40 \quad (1 \text{ mark})$$

$$t = 10 \log_e 40 - 10 \log_e(40 - v)$$

$$t = 10 \log_e \frac{40}{40 - v}$$

$$\frac{t}{10} = \log_e \frac{40}{40 - v}$$

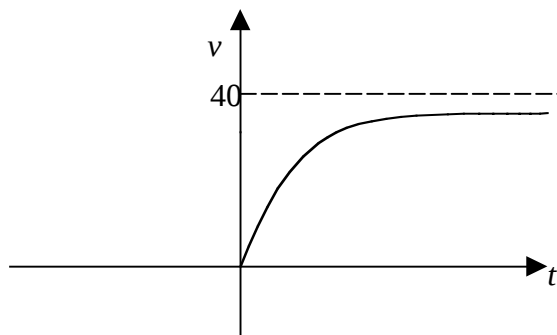
$$e^{\frac{t}{10}} = \frac{40}{40 - v}$$

$$e^{-\frac{t}{10}} = \frac{40 - v}{40}$$

$$40e^{-\frac{t}{10}} = 40 - v$$

$$v = 40(1 - e^{-\frac{t}{10}}) \quad (1 \text{ mark})$$

d.



- shape (1 mark)
- asymptote (1 mark)

e.

As time $\rightarrow \infty$ the boat approaches a speed of 40 m/sec (1 mark)

f.

$$\frac{dx}{dt} = 40 - 40e^{-\frac{t}{10}}$$

$$x = \int_0^{60} 40 - 40e^{-\frac{t}{10}} dt \quad (1 \text{ mark})$$

$$x = 40t + 400e^{-\frac{t}{10}} \Big|_0^{60}$$

$$x = 2400 + 400e^{-6} - 400$$

$$x = 2000.99 \text{ m}$$

$$x = 2.0 \text{ km} \quad (1 \text{ mark})$$

END OF SUGGESTED SOLUTIONS
2002 Specialist Mathematics Trial Examination 2

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