

Question 1 C

A constant over the squared factor, plus a constant over this factor unsquared plus a constant over the other factor.

Question 2 E

$$\begin{aligned} Z^{-1} &= \frac{1}{Z} = \frac{1}{4-3i} \times \frac{4+3i}{4+3i} \\ &= \frac{4+3i}{16+9} \\ &= \frac{4+3i}{25} \\ Z^{-1} &= \frac{4}{25} + \frac{3}{25}i \end{aligned}$$

Question 3 A

$$\begin{aligned} Z^4 - Z^2 - 20 &= (Z^2 + 4)(Z^2 - 5) \\ Z^4 - Z^2 - 20 &= (Z - 2i)(Z + 2i)(Z - \sqrt{5})(Z + \sqrt{5}) \\ \therefore \text{ a factor is } Z + 2i \end{aligned}$$

Question 4 B

Represents all points which are less than or equal to 2 units from $Z = 2 - i$

Question 5 D

$$3(\operatorname{Re} Z)^2 + 6(\operatorname{Im} Z)^2 = 6$$

$$\text{If } Z = x + iy$$

$$3x^2 + 6y^2 = 6$$

$$\frac{x^2}{2} + \frac{y^2}{1} = 1$$

ellipse, $a = \sqrt{2}, b = 1$

Question 6 E

$$y = \frac{x^3 - 32}{x^2}$$

$$\begin{array}{r} x \\ x^2 \overline{) x^3 - 32} \\ \underline{x^3} \\ -32 \end{array}$$

$$y = x - \frac{32}{x^2}$$

$$\frac{dy}{dx} = 1 + 64x^{-3} = 0 \text{ for T.P.}$$

$$\frac{64}{x^3} = -1$$

$$x^3 = -64$$

$$x = -4$$

$$\text{When } x < -4 \frac{dy}{dx} > 0$$

$$\text{When } x > -4 \frac{dy}{dx} < 0$$

$\therefore$ local maximum when $x = -4$

Question 7 B

$$\int \frac{2x}{\sqrt{x^2 + 6}} dx$$

$$\text{Let } u = x^2 + 6$$

$$du = 2x dx$$

$$\int \frac{du}{\sqrt{u}} = \int u^{-\frac{1}{2}} du$$

$$= 2u^{\frac{1}{2}} + c = 2\sqrt{x^2 + 6} + c$$

An antiderivative is $2\sqrt{x^2 + 6}$

Question 8 A

$$\int_0^{\frac{\pi}{3}} \sin^2(5x) dx = \frac{1}{2} \int_0^{\frac{\pi}{3}} (1 - \cos 10x) dx$$

$$= \frac{1}{2} \left[x - \frac{1}{10} \sin 10x \right]_0^{\frac{\pi}{3}}$$

$$= \frac{1}{2} \left[\frac{\pi}{3} - \frac{1}{10} \sin \frac{10\pi}{3} \right] - [0]$$

$$= \frac{\pi}{6} - \frac{1}{20} \sin \frac{10\pi}{3}$$

$$= \frac{\pi}{6} + \frac{\sqrt{3}}{40}$$

$$= \frac{20\pi + 3\sqrt{3}}{120}$$

Question 9 C

$$V = \int_0^1 \pi y^2 dx$$

$$V = \pi \int_0^1 \frac{1}{4} (e^x + e^{-x})^2 dx$$

$$V = \frac{\pi}{4} \int_0^1 (e^{2x} + 2 + e^{-2x}) dx$$

$$V = \frac{\pi}{4} \left[\frac{1}{2} e^{2x} + 2x - \frac{1}{2} e^{-2x} \right]$$

$$V = \frac{\pi}{4} \left[\frac{1}{2} e^2 + 2 - \frac{1}{2} e^{-2} \right] - \frac{\pi}{4} \left[\frac{1}{2} - \frac{1}{2} \right]$$

$$V = 4.4$$

Question 10 E

$$y = \cos^{-1}(2x - 1)$$

$$\text{Let } u = 2x - 1$$

$$\frac{dy}{du} = \frac{-1}{\sqrt{1 - u^2}}$$

$$\frac{du}{dx} = 2$$

$$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$$

$$\frac{dy}{dx} = \frac{-1}{\sqrt{1 - u^2}} \times 2$$

$$\frac{dy}{dx} = \frac{-2}{\sqrt{1 - (2x - 1)^2}}$$

$$\frac{dy}{dx} = \frac{-2}{\sqrt{1 - [4x^2 - 4x + 1]}}$$

$$\frac{dy}{dx} = \frac{-2}{\sqrt{1 - 4x^2 + 4x - 1}}$$

$$\frac{dy}{dx} = \frac{-2}{\sqrt{-4x^2 + 4x}}$$

$$\frac{dy}{dx} = \frac{-2}{\sqrt{-4x(x - 1)}}$$

$$\frac{dy}{dx} = \frac{-2}{2\sqrt{(x - x^2)}} = \frac{-1}{\sqrt{(x - x^2)}}$$

Question 11 B

$$-1 \leq 4x + 1 \leq 1$$

$$-2 \leq 4x \leq 0$$

$$-\frac{1}{2} \leq x \leq 0$$

Question 12 C

$$f(x) = x \log_e x$$

$$\Delta x = \frac{1}{2}$$

$$\int_1^3 x \log_e x dx$$

$$\approx \frac{1}{4} (1 \log_e 1 + 2 \times \frac{3}{2} \log_e \frac{3}{2} + 2 \times 2 \log_e 2 + 2 \times \frac{5}{2} \log_e \frac{5}{2} + 3 \log_e 3)$$

$$= \frac{1}{4} (0 + 1.216 + 2.773 + 4.581 + 3.296)$$

$$= 2.97 \text{ to 2 decimal places.}$$

Question 13 A

$$y = \sin(\log_e x)$$

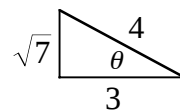
$$\frac{dy}{dx} = \cos u \times \frac{1}{x}$$

$$\frac{dy}{dx} = \frac{1}{x} \cos(\log_e x)$$

$$\frac{d^2y}{dx^2} = \frac{1}{x} \times -\sin(\log_e x) \times \frac{1}{x} + \cos(\log_e x) \left(-\frac{1}{x^2}\right)$$

$$\frac{d^2y}{dx^2} = \frac{-[\sin(\log_e x) + \cos(\log_e x)]}{x^2}$$

Question 14 E



θ is in the 4th quadrant $\therefore \sin \theta$ is negative

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$\sin 2\theta = 2 \times \frac{-\sqrt{7}}{4} \times \frac{3}{4}$$

$$\sin 2\theta = \frac{-3\sqrt{7}}{8}$$

Question 15 D

At $x = 4$ gradient of $f(x) = 0$. Passes from positive to positive gradient either side of $x = 4$
 $\therefore$ stationary point of inflexion.

At $x = 0$ gradient of $f(x) = 0$. Passes from positive to negative gradient either side of $x = 0$
 $\therefore$ local maximum.

At $x = 1$ gradient of $f(x) \neq 0 \therefore$ not a turning point.

At $x = 2$ gradient of $f(x) = 0$. Passes from negative to positive gradient either side of $x = 2$
 $\therefore$ local minimum.

At $x = 3$ gradient of $f(x) > 0$.

Question 16 D

$$(3\hat{i} - 2\hat{j} + \hat{k}) \cdot (\hat{i} + 2\hat{j} + 2\hat{k}) = 3 - 4 + 2 = 1$$

$$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos\theta = 1$$

$$3\sqrt{14}\cos\theta = 1$$

$$\cos\theta = \frac{1}{3\sqrt{14}} \times \frac{\sqrt{14}}{\sqrt{14}}$$

$$\cos\theta = \frac{\sqrt{14}}{42}$$

Question 17 A

$$a = -6v$$

$$\frac{dv}{dt} = -6v$$

$$\frac{dt}{dv} = -\frac{1}{6v}$$

$$t = -\frac{1}{6} \int \frac{dv}{v}$$

$$t = -\frac{1}{6} \log_e v + c \text{ where } c \text{ is a constant}$$

$$\log_e v = -6t + c_1$$

$$\text{When } t = 0, v = 3$$

$$\therefore c_1 = \log_e 3$$

$$\log_e v = -6t + \log_e 3$$

$$\log_e \frac{v}{3} = -6t$$

$$e^{-6t} = \frac{v}{3}$$

$$v = 3e^{-6t}$$

Question 18 C

Overtake occurs when car A and car B have gone the same distance at the same time, t .

$$\text{car A : } s = 25t$$

$$\text{car B : } s = ut + \frac{1}{2}at^2$$

$$a = \frac{5}{3}$$

$$\Rightarrow s = 0 + \frac{1}{2} \times \frac{5}{3} \times t^2$$

$$\Rightarrow s = \frac{5}{6}t^2$$

$$\Rightarrow \frac{5}{6}t^2 = 25t$$

$$\Rightarrow t^2 = 30t$$

$$\Rightarrow t^2 - 30t = 0$$

$$\Rightarrow t(t - 30) = 0$$

$$\Rightarrow t = 0 \text{ or } 30$$

$$\text{But } t > 0$$

$$\therefore t = 30 \text{ sec}$$

Suggested Solutions Part I

Question 19 C

$$3\vec{a} = 9\hat{i} - 6\hat{j} + 12\hat{k}$$

$$2\vec{b} = 4\hat{i} - 6\hat{j} - 2\hat{k}$$

$$3\vec{a} - 2\vec{b} = 9\hat{i} - 6\hat{j} + 12\hat{k} - 4\hat{i} + 6\hat{j} + 2\hat{k}$$

$$3\vec{a} - 2\vec{b} = 5\hat{i} + 14\hat{k}$$

Question 20 A

$(\vec{a} \cdot \hat{b})\hat{b}$ is the component of $\vec{a}$ parallel to $\vec{b}$

$$|\vec{b}| = \sqrt{9 + 4} = \sqrt{13}$$

$$(\vec{a} \cdot \hat{b})\hat{b} = \left[(6\hat{i} + 12\hat{j}) \cdot \frac{1}{\sqrt{13}}(-3\hat{i} + 2\hat{j}) \right] \hat{b}$$

$$(\vec{a} \cdot \hat{b})\hat{b} = \frac{1}{\sqrt{13}}(-18 + 24)\hat{b}$$

$$(\vec{a} \cdot \hat{b})\hat{b} = \frac{6}{\sqrt{13}}\hat{b}$$

$$(\vec{a} \cdot \hat{b})\hat{b} = \frac{6}{\sqrt{13}} \times \frac{1}{\sqrt{13}}(-3\hat{i} + 2\hat{j})$$

$$(\vec{a} \cdot \hat{b})\hat{b} = \frac{6}{13}(-3\hat{i} + 2\hat{j}) = -\frac{6}{13}(3\hat{i} - 2\hat{j})$$

Question 21 E

$$\frac{dy}{dt} = \frac{dy}{dx} \frac{dx}{dt}$$

$$\frac{dx}{dt} = 3$$

$$\frac{dy}{dt} = 2x \times 3 = 6x$$

$$x = 3t + c$$

$$\frac{dy}{dt} = 6(3t + c) = 18t + c_1$$

$$\frac{d^2y}{dt^2} = 18$$

Question 22 E

Either $\vec{a} = 0$ or $\frac{d\vec{a}}{dt} = 0$ or $\theta = 90^\circ$

If $\theta = 90^\circ$ then $\vec{a}$ is perpendicular to $\frac{d\vec{a}}{dt}$

This means that the tangent is perpendicular to the motion

Hence, $\vec{a}$ moves in a circle.

If $\frac{d\vec{a}}{dt} = 0$ then $\vec{a}$ is a constant vector.

Question 23 D

$$x - 1 = \cos t$$

$$(x - 1)^2 = \cos^2 t$$

$$\frac{y}{3} = \sin t$$

$$\frac{y^2}{9} = \sin^2 t$$

$$(x - 1)^2 + \frac{y^2}{9} = \sin^2 t + \cos^2 t$$

$$\frac{(x - 1)^2}{1} + \frac{y^2}{9} = 1$$

$\Rightarrow$ ellipse with centre (1,0),

with semi major axis = 1

and semi minor axis = 3

Question 24 E

$$a = -6$$

$$u = 10$$

$$v = 0$$

$$s = ?$$

$$v^2 = u^2 + 2as$$

$$0 = 100 - 12s$$

$$100 = 12s$$

$$s = \frac{100}{12}$$

$$s = 8.3m$$

Question 25 B

$$\begin{aligned}\text{Distance} &= \sqrt{\vec{r} \cdot \vec{r}} \\ &= \sqrt{(\cos 2t\hat{i} - \sin 2t\hat{j}) \cdot (\cos 2t\hat{i} - \sin 2t\hat{j})} \\ &= \sqrt{\cos^2 2t + \sin^2 2t} \\ &= 1 \text{ (for all values of } t\text{)}\end{aligned}$$

Question 26 A

$$\begin{aligned}\vec{F} &= m\vec{a} \\ 3\vec{a} &= 3\hat{j} - 6\hat{k} \\ \vec{a} &= \hat{j} - 2\hat{k} \\ \frac{d\vec{v}}{dt} &= \hat{j} - 2\hat{k} \\ \vec{v} &= t\hat{j} - 2t\hat{k} + c \\ \text{When } t = 0, v &= 0 \\ \therefore c &= 0 \\ \vec{v} &= t\hat{j} - 2t\hat{k} \\ \vec{x} &= \frac{t^2}{2}\hat{j} - t^2\hat{k} + c_1 \\ \text{When } t = 0, \vec{x} &= 3\hat{i} + \hat{j} - \hat{k} \\ \text{So } c_1 &= 3\hat{i} + \hat{j} - \hat{k} \\ \text{So } \vec{x} &= 3\hat{i} + (1 + \frac{t^2}{2})\hat{j} - (1 + t^2)\hat{k} \\ \text{When } t = 2 \\ \vec{x} &= 3\hat{i} + 3\hat{j} - 5\hat{k}\end{aligned}$$

Question 27 B



$$\vec{p}_{\text{before}} = \vec{p}_{\text{after}}$$

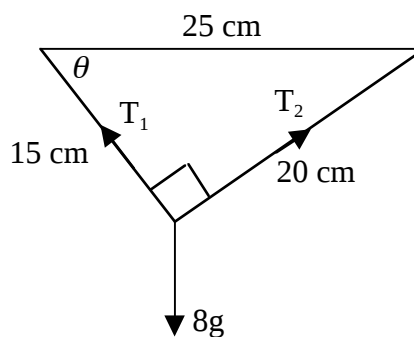
$$10 \times 40 + 30 \times 20 = 40v$$

$$400 + 600 = 40v$$

$$\frac{1000}{40} = v$$

$$v = 25 \text{ km/hr}$$

Question 28 D



15:20:25 = 3:4:5 so strings form a right angle with each other.

$$\sin \theta = \frac{20}{25} = \frac{4}{5}$$

$$\cos \theta = \frac{15}{25} = \frac{3}{5}$$

Horizontal Equilibrium $T_1 \cos \theta = T_2 \sin \theta$

$$\frac{3}{5} T_1 = \frac{4}{5} T_2 \text{ so } T_2 = \frac{3}{4} T_1$$

Vertical Equilibrium

$$T_1 \sin \theta + T_2 \cos \theta = 8g$$

$$\frac{4}{5} T_1 + \frac{3}{4} T_1 \times \frac{3}{5} = 8g$$

$$\frac{4}{5} T_1 + \frac{9}{20} T_1 = 8g$$

$$\frac{25}{20} T_1 = 8g$$

$$T_1 = 8g \times \frac{20}{25} = 62.72 \text{ N}$$

Question 29 B

$$\int_1^{e^2} \frac{\log_e x}{x} dx$$

Let $u = \log_e x$

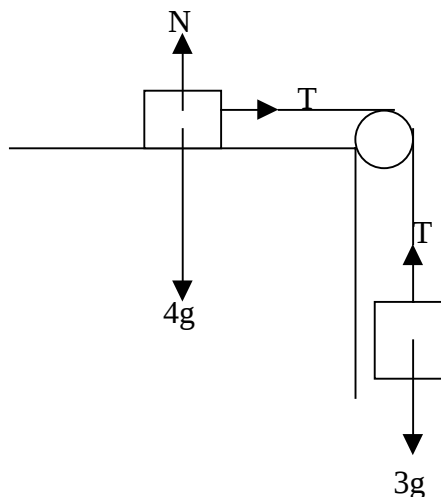
$$du = \frac{1}{x} dx$$

When $x = 1, u = \log_e 1 = 0$

When $x = e^2, u = \log_e e^2 = 2$

$$\int_0^2 u du = \left[\frac{u^2}{2} \right]_0^2 = 2$$

Question 30 B



For 4 kg mass

$$\vec{R} = m\vec{a}$$

$$\Rightarrow \vec{T} = 4\vec{a}(1)$$

For 3 kg mass

$$\vec{R} = m\vec{a}$$

$$3g - \vec{T} = 3\vec{a}(2)$$

Substituting (1) in (2)

$$3g - 4\vec{a} = 3\vec{a}$$

$$7\vec{a} = 3g$$

$$\vec{a} = \frac{3 \times 9.8}{7}$$

$$\vec{a} = 4.2 m/sec^2$$

**END OF PART I
MULTIPLE CHOICE QUESTION BOOK**

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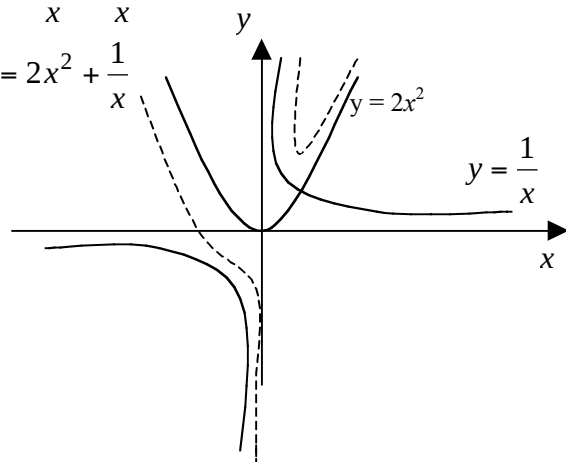
TEL: (03) 9817 5374
FAX: (03) 9817 4334
chemas@chemas.com
www.chemas.com

Question 1

a.

$$y = \frac{2x^3}{x} + \frac{1}{x}$$

$$y = 2x^2 + \frac{1}{x}$$



is the graph of $y = 2x^2 + \frac{1}{x}$

(1 mark)

b.

$$\frac{dy}{dx} = 4x - \frac{1}{x^2} = 0 \text{ for minimum}$$

$$4x = \frac{1}{x^2}$$

$$4x^3 = 1$$

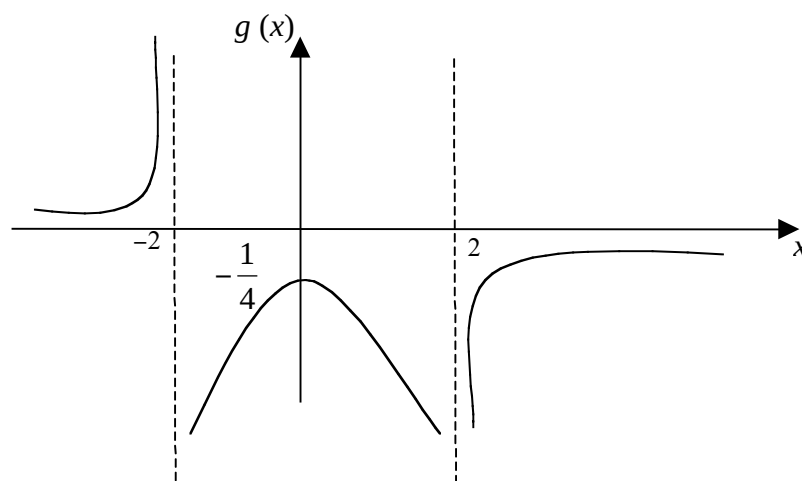
$$x^3 = \frac{1}{4}$$

$$x = \frac{1}{\sqrt[3]{4}} \quad (1 \text{ mark})$$

$$y = \frac{\frac{1}{\sqrt[3]{4}} + 1}{\frac{1}{\sqrt[3]{4}}} = \sqrt[3]{4} \left(\frac{3}{2} \right) = \frac{3\sqrt[3]{4}}{2} \quad (1 \text{ mark})$$

$$\left(\frac{1}{\sqrt[3]{4}}, \frac{3\sqrt[3]{4}}{2} \right)$$

c.



(1 mark)

Question 2

$$\tan\left(x + \frac{\pi}{3}\right) = \pm \frac{1}{\sqrt{3}} \quad \frac{\pi}{3} \leq x \leq \frac{7\pi}{3} \quad (1 \text{ mark})$$

$$x + \frac{\pi}{3} = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{13\pi}{6} \quad (1 \text{ mark})$$

$$x = \frac{3\pi}{6}, \frac{5\pi}{6}, \frac{9\pi}{6}, \frac{11\pi}{6}$$

$$x = \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6} \quad (1 \text{ mark})$$

Question 3

$$Z = 2\sqrt{2}(\cos 45^\circ + i \sin 45^\circ)$$

$$Z^2 = 8(\cos 90^\circ + i \sin 90^\circ) \quad (1 \text{ mark})$$

$$W = 2\sqrt{2}(\cos 135^\circ + i \sin 135^\circ)$$

$$W^5 = 128\sqrt{2}(\cos 675^\circ + i \sin 675^\circ)$$

$$W^5 = 128\sqrt{2}(\cos 315^\circ + i \sin 315^\circ) \quad (1 \text{ mark})$$

$$\frac{Z^2}{W^5} = \frac{8}{128\sqrt{2}} \left[\cos(90 - 315)^\circ + i \sin(90 - 315)^\circ \right]$$

$$\frac{Z^2}{W^5} = \frac{1}{16\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} \left[\cos(-225)^\circ + i \sin(-225)^\circ \right]$$

$$\frac{Z^2}{W^5} = \frac{\sqrt{2}}{32} \left[\cos 225^\circ - i \sin 225^\circ \right] \quad (1 \text{ mark})$$

$$\frac{Z^2}{W^5} = \frac{\sqrt{2}}{32} \left[-\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i \right]$$

$$\frac{Z^2}{W^5} = -\frac{1}{32} + \frac{1}{32}i \quad (1 \text{ mark})$$

Question 4

a.

$$\frac{dy}{dx} = \frac{1}{2}e^x [\cos x - \sin x] + [\sin x + \cos x] \frac{1}{2}e^x$$

$$\frac{dy}{dx} = \frac{1}{2}e^x [\cos x - \sin x + \sin x + \cos x]$$

$$\frac{dy}{dx} = \frac{1}{2}e^x \times 2 \cos x = e^x \cos x$$

(1 mark)

b.

$$\int (7e^x \cos x) dx - \int 7 dx + \int \frac{1}{\sqrt{e^2 - (2x)^2}} dx$$

$$= \frac{7}{2}e^x (\sin x + \cos x) - 7x + \frac{1}{2} \int \frac{1}{\sqrt{\frac{e^2}{4} - x^2}} dx$$

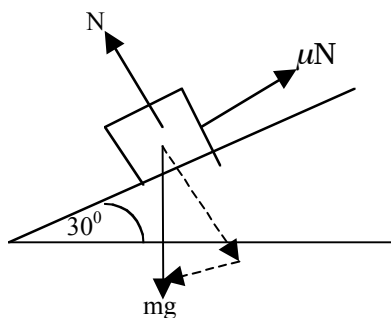
(1 mark)

$$= \frac{7}{2}e^x (\sin x + \cos x) - 7x + \frac{1}{2} \sin^{-1} \left(\frac{2x}{e} \right) + c$$

where c is a constant

(1 mark)

Question 5



Resolving forces parallel to plane:

$$\text{Resultant force} = mg \sin 30^\circ - \mu N$$

$$2m = mg \sin 30^\circ - \mu N \quad (1) \quad (1 \text{ mark})$$

Resolving forces perpendicular to the plane

$$N - mg \cos 30^\circ = 0$$

$$N = mg \cos 30^\circ \quad (2) \quad (1 \text{ mark})$$

Substituting (2) in (1)

$$2m = mg \sin 30^\circ - \mu \times mg \cos 30^\circ$$

$$2 = 4.9 - \mu \times 8.487$$

$$8.487\mu = 2.9$$

$$\mu = 0.34 \text{ to 2 decimal places} \quad (1 \text{ mark})$$

Question 6

Let $\vec{u} = x\hat{i} + y\hat{j} + z\hat{k}$

$$\vec{u} \cdot \vec{a} = 0$$

$$2x + y + 3z = 0 \quad (1) \quad (1 \text{ mark})$$

$$\vec{u} \cdot \vec{b} = 0$$

$$x - 2y - z = 0 \quad (2) \quad (1 \text{ mark})$$

$$(2) \times 2 \rightarrow 2x - 4y - 2z = 0 \quad (2a)$$

$$2x + y + 3z = 0 \quad (1)$$

$$\Rightarrow -5y - 5z = 0$$

$$\Rightarrow -5y = 5z$$

$$\Rightarrow y = -z$$

Substituting in (1)

$$2x - z + 3z = 0$$

$$2x = -2z$$

$$x = -z$$

$$\therefore \vec{u} = -z\hat{i} - z\hat{j} + z\hat{k}$$

$$\Rightarrow \vec{u} = z(-\hat{i} - \hat{j} + \hat{k})$$

But z is a constant

$$\text{So } \vec{u} = c(-\hat{i} - \hat{j} + \hat{k}) \quad (1 \text{ mark})$$

END OF SUGGESTED SOLUTIONS

2002 Specialist Mathematics Trial Examination 1 Part II

KILBAHA PTY LTD (Publishers in Education)
ABN 47 065 111 373
PO BOX 2227 KEW VIC 3101
AUSTRALIA

TEL: (03) 9817 5374
FAX: (03) 9817 4334
chemas@chemas.com
www.chemas.com