



The Mathematical Association of Victoria

# Specialist Mathematics

## 2001 Written Examinations

### Solutions

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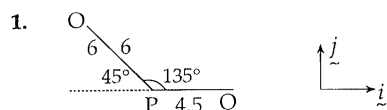
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**2001 Specialist Mathematics**  
**Written Examination 2 (Analysis task)**  
**Suggested answers and solutions**



a.  $\overline{OQ}^2 = \overline{OP}^2 + \overline{PQ}^2 - 2\overline{OP} \cdot \overline{PQ} \cos \theta$   
 $\overline{OP} = 6 \quad \overline{PQ} = 4.5 \quad \theta = 135^\circ$   
 $\overline{OQ}^2 = 6^2 + 4.5^2 - 2 \times 6 \times 4.5 \times \cos 135^\circ$   
 $= 94.434$   
 $\overline{OQ} = 9.7177$   
 $\approx 9.72 \text{ km}$

b. i.  $\overrightarrow{OP} = 6 \sin 45^\circ \underline{i} - 6 \cos 45^\circ \underline{j}$

$$= \frac{6}{\sqrt{2}} \underline{i} - \frac{6}{\sqrt{2}} \underline{j}$$

$$= 3\sqrt{2} \underline{i} - 3\sqrt{2} \underline{j}$$

$$\overrightarrow{PQ} = 4.5 \underline{i}$$

$$\overrightarrow{OQ} = \overrightarrow{OP} + \overrightarrow{PQ}$$

$$= (4.5 + 3\sqrt{2}) \underline{i} - 3\sqrt{2} \underline{j}$$

ii. Let  $\alpha = \angle POQ$

$$|\overrightarrow{OP}| = 6 \quad |\overrightarrow{OQ}| = 9.72 \quad (\text{from Q1a})$$

$$\overrightarrow{OP} = 3\sqrt{2} \underline{i} - 3\sqrt{2} \underline{j}$$

$$\overrightarrow{OQ} = (4.5 + 3\sqrt{2}) \underline{i} - 3\sqrt{2} \underline{j}$$

$$\overrightarrow{OP} \cdot \overrightarrow{OQ} = 13.5\sqrt{2} + 18 + 18$$

$$= 36 + 13.5\sqrt{2}$$

$$\cos \alpha = \frac{\overrightarrow{OP} \cdot \overrightarrow{OQ}}{|\overrightarrow{OP}| |\overrightarrow{OQ}|}$$

$$= \frac{36 + 13.5\sqrt{2}}{6 \times 9.7177}$$

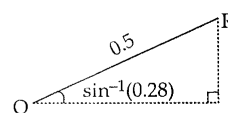
$$= 0.9449 \dots$$

$$\alpha = \cos^{-1}(0.9449 \dots)$$

$$= 19.1^\circ$$

(correct to the nearest 10<sup>th</sup> of a degree)  
 $\angle POQ = 19.1^\circ$ . Bearing of Q from O can be expressed as S(45° + 19.1°)E, i.e. S64.1°E.

c. i.



$$\overrightarrow{RQ} = 0.5 \left( \cos(\sin^{-1} 0.28) \underline{i} + \sin(\sin^{-1} 0.28) \underline{j} \right)$$

$$= 0.5 \times 0.96 \underline{i} + 0.5 \times 0.28 \underline{j}$$

$$= 0.48 \underline{i} + 0.14 \underline{j}$$

$$\overrightarrow{OR} = \overrightarrow{OQ} + \overrightarrow{QR} = 8.74 \underline{i} - 4.24 \underline{j} + 0.48 \underline{i} + 0.14 \underline{j}$$

$$= 9.22 \underline{i} - 4.24 \underline{j} + 0.14 \underline{j}$$

c. ii.  $|\overrightarrow{OR}| = \sqrt{9.22^2 + 4.24^2 + 0.14^2}$   
 $= \sqrt{103.01}$   
 $= 10.15$

Her transmitter has insufficient range.

2. a.  $\frac{ds}{dt} = \frac{kt}{12 + t^4}$   
at  $t = 4$

$$\frac{ds}{dt} = \frac{4 \times 10^6}{12 \times 256}$$

$$= 1.49 \times 10^4$$

b. i.  $\frac{ds}{dt} = \frac{kt}{12+t^4}$

$$\frac{d^2s}{dt^2} = \frac{k(12+t^4) - kt \times 4t^3}{(12+t^4)^2}$$

max value occurs when  $\frac{d^2s}{dt^2} = 0$

$$\Rightarrow k(12+t^4) - 4kt^4 = 0$$

$$12+t^4 - 4t^4 = 0$$

$$3t^4 = 12$$

$$t^4 = 4$$

$$t = \sqrt{2}$$

$\therefore a = \sqrt{2}$

b. ii. Maximum rate occurs at  $t = \sqrt{2}$

$$\frac{ds}{dt} = \frac{kt}{12+t^4}$$

at  $t = \sqrt{2}$

$$\frac{ds}{dt} = \frac{\sqrt{2} \times 10^6}{16} = 8.84 \times 10^4 \text{ litres per day}$$

c. i.  $v = \int \frac{t}{12+t^4} dt$

Let  $u = t^2 \quad t = \sqrt{u}$

$$\frac{du}{dt} = 2t$$

$$\frac{dt}{du} \cdot du = \frac{du}{2t}$$

Substituting  $\sqrt{u}$  for  $t$  and  $\frac{du}{2t}$  for  $dt$

$$v = \int \frac{1}{12+u^2} \frac{du}{2}$$

$$v = \frac{1}{2\sqrt{12}} \int \frac{\sqrt{12}}{12+u^2} du$$

$$v = \frac{1}{2\sqrt{12}} \tan^{-1}\left(\frac{u}{\sqrt{12}}\right)$$

substituting  $t^2$  for  $u$

$$v = \frac{1}{2\sqrt{12}} \tan^{-1}\left(\frac{t^2}{\sqrt{12}}\right)$$

$$\text{OR } = \frac{1}{4\sqrt{3}} \tan^{-1}\left(\frac{t^2}{2\sqrt{3}}\right)$$

c. ii.  $v = \int \frac{kt}{12+t^4} dt$

$$v = k \left( \frac{1}{2\sqrt{12}} \tan^{-1}\left(\frac{t^2}{\sqrt{12}}\right) \right) + c$$

at  $t = 0 \quad v = 0$

$$0 = k \left( \frac{1}{2\sqrt{12}} \tan^{-1}(0) \right) + c$$

$$0 = c$$

$$\therefore v = \frac{k}{2\sqrt{12}} \tan^{-1}\left(\frac{t^2}{\sqrt{12}}\right)$$

c. iii. For  $v = \frac{k}{2\sqrt{12}} \tan^{-1}\left(\frac{t^2}{\sqrt{12}}\right)$

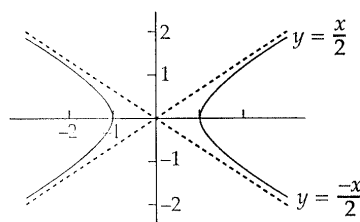
Since  $\tan^{-1}(t)$  is less than  $\frac{\pi}{2}$  for all  $t$

$$v < \frac{10^6}{4\sqrt{3}} \times \frac{\pi}{2}$$

$$\text{i.e. } v < 226725$$

This suggests that less than 300,000 litres will spill into the sea in the long term.

3. a.



b.  $v = \pi \int_0^h x^2 dy$

$$x^2 = 1 + y^2$$

$$v = \pi \int_0^h 1 + y^2 dy$$

$$= \pi \left[ y + \frac{y^3}{3} \right]_0^h$$

$$= \pi \left( \left( h + \frac{4h^3}{3} \right) - 0 \right)$$

$$= \pi \left( \frac{4h^3}{3} + h \right)$$

c. IN :  $\frac{dv}{dt} = 0.003 \text{ m}^3/\text{s}$

OUT :  $\frac{dv}{dt} = 0.004\sqrt{h}$

$$\frac{dv}{dt} = 0.003 - 0.004\sqrt{h}$$

$$\frac{dv}{dh} = \pi(4h^2 + 1)$$

$$\begin{aligned} \frac{dh}{dt} &= \frac{dv}{dt} \times \frac{dh}{dv} \\ &= \frac{0.003 - 0.004\sqrt{h}}{\pi(4h^2 + 1)} \end{aligned}$$

at  $h = 0.25$

$$\frac{dh}{dt} = \frac{0.001}{3.927}$$

$$\approx 2.5 \times 10^{-4} \text{ m/s}$$

d.  $\frac{dt}{dh} = \frac{\pi(4h^2 + 1)}{0.003 - 0.004\sqrt{h}}$

$$t = \int_0^{0.5} \frac{\pi(4h^2 + 1)}{0.003 - 0.004\sqrt{h}} dh$$

$$= 3525 \text{ sec}$$

- e. Depth will stabilise when inflow and outflow are the same.

i.e. when

$$0.003 - 0.004\sqrt{h} = 0$$

$$\Rightarrow \sqrt{h} = \frac{3}{4}$$

$$h = \frac{9}{16} \text{ metres}$$

$$\approx 0.5625 \text{ metres}$$

4. a.  $z^2 - 6z + 25 = 0$

$$z^2 - 6z + 9 - 9 + 25 = 0$$

$$(z - 3)^2 + 16 = 0$$

$$(z - 3 - 4i)(z - 3 + 4i) = 0$$

$$z_1 = 3 + 4i \quad z_2 = 3 - 4i$$

$$z_1 + z_2 = 3 + 4i + 3 - 4i$$

$$= 6$$

$$z_1 \times z_2 = (3 + 4i)(3 - 4i)$$

$$= 9 + 16 = 25$$

- b. i. If  $u$  and  $v$  are roots

$$\text{then } (z - u)(z - v) = 0$$

$$z^2 - uz - vz + uv = 0$$

$$z^2 - (u + v)z + uv = 0$$

$$\text{given } z^2 + bz + c = 0$$

equating co-efficients

$$b = -(u + v)$$

$$\Rightarrow u + v = -b \quad uv = c$$

- b. ii.  $u = p + qi$

$$v = \bar{u} = p - qi$$

$$u + v = p + qi + p - qi$$

$$= 2p$$

From 4. b. i  $u + v = -b$

Given  $p \in \mathbb{R}$

then  $v \in \mathbb{R}$

$$uv = (p + qi)(p - qi)$$

$$= p^2 + q^2$$

From 4. b. i.  $uv = c$

Given  $p + q \in \mathbb{R}$

then  $p^2 + q^2 \in \mathbb{R}$

$\therefore c \in \mathbb{R}$

- c. Let  $a = 2 + \sqrt{5}i$  and  $b = -2 + \sqrt{5}i$

$$(z - a)(z - b) = 0$$

$$z - (a + b)z + ab = 0$$

$$a + b = 2\sqrt{5}i$$

$$ab = (+2 + \sqrt{5}i)(-2 + \sqrt{5}i)$$

$$= -4 + 2\sqrt{5}i + 5i^2 - 2\sqrt{5}i$$

$$= -4 - 5 = -9$$

Quadratic equation with roots

$$2 + \sqrt{5}i \quad \text{and} \quad -2 + \sqrt{5}i$$

$$\text{is } z^2 - 2\sqrt{5}iz - 9 = 0$$

d.  $u + v = -3$      $uv = 4$

From 4. b. ii. we know

$$\begin{aligned} u &= p + qi & v &= p - qi \\ \& \ 2p &= -3 & p^2 + q^2 &= 4 \\ p &= \frac{-3}{2} & \frac{9}{4} + q^2 &= 4 \\ q^2 &= \frac{16-9}{4} = \frac{7}{4} \\ q &= \frac{\pm\sqrt{7}}{2} \end{aligned}$$

[OR  $u = \frac{-3}{2} + \frac{\sqrt{7}i}{2}$      $v = \frac{-3}{2} - \frac{\sqrt{7}i}{2}$  ]

taking the first set of values

$$\begin{aligned} u + v &= \frac{-3}{2} - \frac{3}{2} & u - v &= \frac{\sqrt{7}i}{2} + \frac{\sqrt{7}i}{2} \\ &= -3 & &= \sqrt{7}i \end{aligned}$$

$$(z + 3)(z - \sqrt{7}i) = 0$$

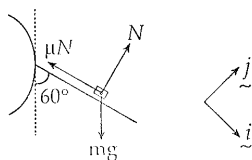
$$z^2 - \sqrt{7}iz + 3z - 3\sqrt{7}i$$

$$z^2 + (3 - \sqrt{7}i)z - 3\sqrt{7}i$$

[the second set of values gives:

$$z^2 + (3 + \sqrt{7}i)z + 3\sqrt{7}i = 0] \text{ only one equation is required}$$

5. a.



b.  $\underline{i}$  :  $F = mg \cos 60^\circ - \mu N$

$\underline{j}$  :  $mg \sin 60^\circ = N$

$$F = mg \cos 60^\circ - \mu mg \sin 60^\circ$$

$$Ma = mg \cos 60^\circ - \frac{1}{5} mg \sin 60^\circ$$

$$a = g \left( \cos 60^\circ - \frac{1}{5} \sin 60^\circ \right)$$

$$= g \left( \frac{1}{2} - \frac{1}{5} \times \frac{\sqrt{3}}{2} \right)$$

$$= \frac{g}{2} \left( 1 - \frac{\sqrt{3}}{5} \right) \quad \text{Note: acceleration is constant}$$

c. i.  $u = 0$      $a = \frac{g}{2} \left( 1 - \frac{\sqrt{3}}{2} \right)$      $s = 6$

$$s = ut + \frac{1}{2} at^2$$

Note: acceleration is constant

$$6 = 0 + \frac{g}{2} \left( 1 - \frac{\sqrt{3}}{5} \right) t^2$$

$$6 = 1.6013t^2$$

$$t^2 = 3.747$$

$$t = 1.94 \text{ sec (correct to 2 decimal places)}$$

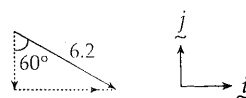
c. ii.  $v = u + at$

$$v = 0 + \frac{g}{2} \left( 1 - \frac{\sqrt{3}}{5} \right) \times 1.94$$

$$= 6.199$$

$$\approx 6.2 \text{ m/s}$$

d.



$$\underline{v}(t) = 6.2 \sin 60^\circ \underline{j} - 6.2 \cos 60^\circ \underline{j} - gt \underline{j}$$

$$= 3.1\sqrt{3} \underline{j} - (3.1 + gt) \underline{j}$$

$$\underline{r}(t) = 3.1\sqrt{3}t \underline{i} - \left( 3.1t + \frac{1}{2}gt^2 \right) \underline{j} + c$$

consider vertical distance

$$3.1t + \frac{1}{2}gt^2 = 2$$

$$4.9t^2 + 3.1t - 2 = 0$$

$$t = 0.3966 \text{ (positive value)}$$

horizontal distance at  $t = 0.3966$

$$\text{given by } r = 3.1\sqrt{3} \times 0.3966$$

$$= 2.13$$

$$\approx 2.1 \text{ metres}$$

e.  $\underline{p} = m \underline{v}$

$$\underline{v}(t) = 3.1\sqrt{3} \underline{i} - (3.1 + gt) \underline{j}$$

$$\underline{v}(0.3966) = 3.1\sqrt{3} \underline{i} - (3.1 + 9.8 \times 0.3966) \underline{j}$$

$$= 5.369 \underline{i} - 6.987 \underline{j}$$

$$|\underline{v}(0.3966)| = 8.812$$

$$p = mv \qquad m = 75 \text{ kg}$$

$$p = 75 \times 8.812$$

$$= 661 \text{ kg m/s}$$