

**THE
HEFFERNAN
GROUP**

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Student Name.....

SPECIALIST MATHEMATICS

TRIAL EXAMINATION 2

(ANALYSIS TASK)

2001

Reading Time: 15 minutes

Writing time: 90 minutes

Instructions to students

This exam consists of 6 questions.
All questions should be answered.
There is a total of 60 marks available.
The marks allocated to each of the six questions are indicated throughout.
Students may bring up to two A4 pages of pre-written notes into the exam.
The acceleration due to gravity should be taken to have magnitude $g \text{ m/s}^2$ where $g = 9.8$

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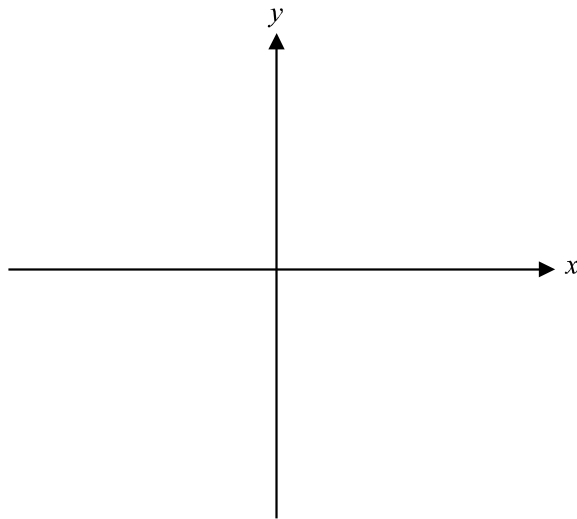
Question 1

Consider the function $f : (-\frac{\pi}{2}, \frac{\pi}{2}) \rightarrow \mathbb{R}$ where $f(x) = x \sec x$.

- a.** Find the y -intercept of the graph of $y = f(x)$.

1 mark

- b.** Sketch the graph of $y = f(x)$ on the set of axes below.



2 marks

- c.** Find $f'(x)$

1 mark

- d. Explain whether or not the graph of $y = f(x)$ has a stationary point at the point where $x = 0$.

1 mark

- e. Use the fact that $f''(x) = \frac{x + x \sin^2 x + 2 \sin x \cos x}{\cos^3 x}$ to find the coordinates of the point on the graph of $y = f(x)$ where the gradient is a minimum.

2 marks

- f. Verify that $f(x) = x \sec x$ is a solution to the differential equation

$$\frac{f''(x)}{\sec^3 x} - \frac{\cos x}{\operatorname{cosec}^2 x} f(x) = x + \sin(2x)$$

2 marks

Total 9 marks

Question 3

Let $u = 5i$.

- a.** Express u in polar form.

1 mark

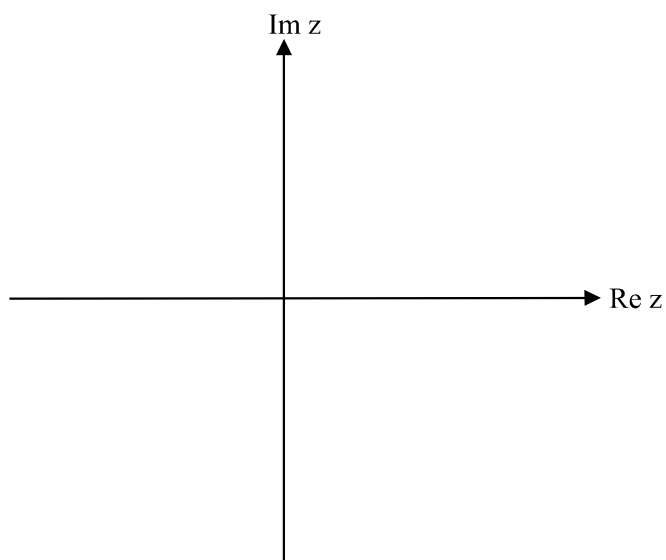
- b.** If $v = \bar{u} + |u| - 1 + 6i + \operatorname{Re} u$, find v in Cartesian form.

1 mark

- c.** If $z = x + yi$, where x and y are real and $|z - 5i| = |z - 4 - i|$, express y in terms of x .

3 marks

- d. Hence or otherwise, sketch on the Argand diagram below, S , where $S = \{z : |z - 5i| < |z - 4 - i|\}$ and $z \in \mathbb{C}$. Indicate clearly whether or not the boundary is included.



2 marks

- e. Write down, in Cartesian form, the cube root(s) of 8 that lie in S .

2 marks

Total 9 marks

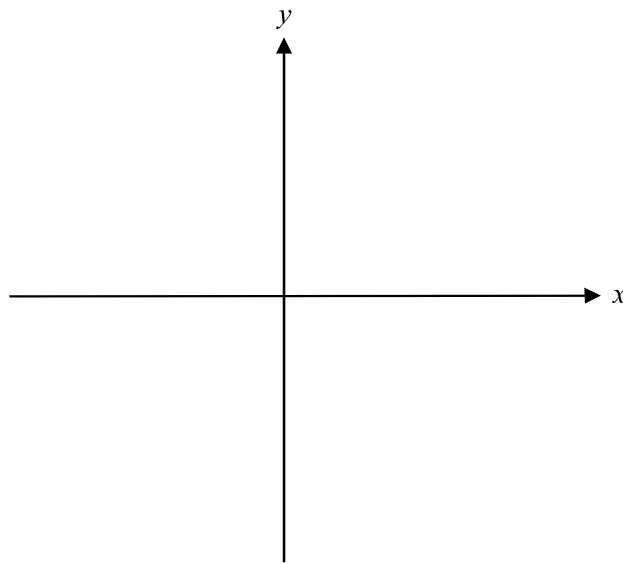
Question 4

Consider the function $f : [-a, a] \rightarrow \mathbb{R}$ where $f(x) = \sin^{-1}\left(\frac{x}{2}\right)$.

- a.** Write down the value of a given that f has a maximal domain.

1 mark

- b. i.** Sketch the graph of $y = f(x)$ on the set of axes below.



1 mark

- ii.** Show algebraically that the graph in part **i.** has no stationary points.

2 marks

c. Find $\frac{d}{dx}\left(x \sin^{-1} \frac{x}{2}\right)$

1 mark

d. i. Hence, use calculus to find an antiderivative of $\sin^{-1} \frac{x}{2}$.

2 marks

ii. Hence find the area enclosed by the graph of $y = f(x)$, the x -axis and the line with equation $x = 1$. Express your answer as an exact value.

2 marks

Question 5

A particle moves so that its position vector at time t , $t \geq 1$, is given by

$$\vec{r}(t) = \left(t + \frac{1}{t}\right) \vec{i} + \left(t - \frac{1}{t}\right) \vec{j}$$

- a.** Find an expression for the distance from the origin to the particle at time t .

1 mark

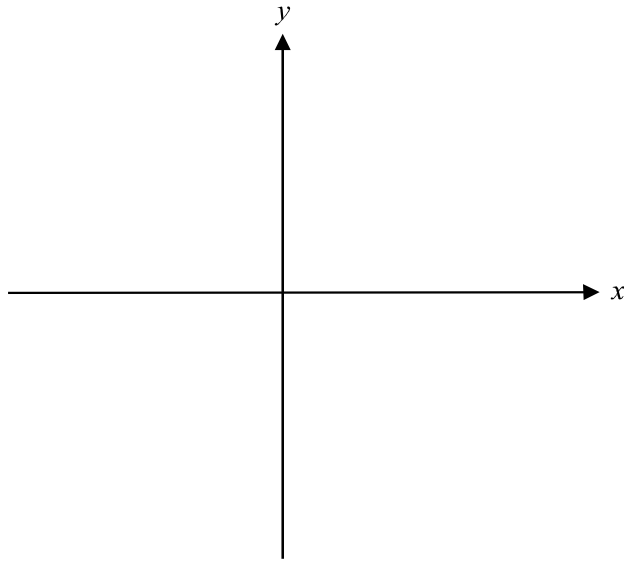
- b.** Find the speed of the particle at time $t = 5$.

2 marks

- c. i.** Find the Cartesian equation of the path of the particle and state the domain and range.

4 marks

- ii. Sketch the path of the particle on the diagram below.



1 mark

A second particle has a velocity vector given by

$$\vec{v}_B(t) = \left(2 - \frac{2}{t^2}\right) \vec{i} + \left(2 + \frac{2}{t^2}\right) \vec{j}, \quad t > 0$$

When $t = 1$, the position vector is given by $\vec{r}_B(t) = 4\vec{i}$.

- d. Show that the position vectors of this second particle and of the first particle at time t are parallel.

3 marks

Total 11 marks

Question 6

Julie gets onto a straight stretch of a freeway at time $t = 0$ and has a velocity in m/s at time t seconds of

$$v(t) = 0.03t^2 + 15 \quad t \in [0, 25]$$

- a.** What is Julie's entry speed onto the freeway?

1 mark

Julie's friend Tom is also traveling on the freeway in the same direction as Julie. Tom passes Julie the instant she gets onto the freeway. Tom is travelling at a constant speed of 20m/s.

The velocity-time graph for Tom's travel is shown below.



- b.** Sketch on the same set of axes the velocity-time graph for Julie.

1 mark

- ii. Hence show that $v = \frac{25(6e^t + 1)}{6e^t - 1}$

2 marks

- f. If Julie were to maintain this pattern of deceleration, find her limiting velocity.
Explain your answer.

2 marks

Total 11 marks