The Mathematical Association of Victoria

Specialist Mathematics

2000 Written Examinations

Solutions

These answers and solutions to the 2000 VCE Specialist Mathematics Written Examinations 1 and 2 have been written and published to assist teachers and students in their preparations for future Specialist Mathematics Examinations. They have been published without the relevant questions to avoid any breaches of copyright. They are suggested answers and solutions only and do not necessarily reflect the views of the Board of Studies Assessing Panels.

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2000 Specialist Mathematics Exam 1

Suggested Answers and Solutions

Part I (Multiple-choice) Answers

1. **B**	2. **A**	3. **B**	4. **D**	5. **E**
6. **E**	7. **A**	8. **B**	9. **E**	10. **C**
11. **C**	12. **E**	13. **E**	14. **D**	15. **D**
16. **B**	17. **B**	18. **D**	19. **C**	20. **A**
21. **A**	22. **C**	23. **E**	24. **A**	25. **D**
26. **B**	27. **C**	28. **D**	29. **C**	30 **A**

Solutions: Part 1

Question 1 [B]

$$f(x) = 3x^2 + 8x + 5$$
$$= (3x+5)(x+1)$$

$\frac{1}{f(x)}$ will have asymptotes when $f(x) = 0$,

for $x = \frac{-5}{3}$ and $x = -1$

Question 2 [A]

Consider first a hyperbola, with asymptotes passing through the origin, O.

It would cut the x–axis at (–3, 0) and (3, 0)

i.e: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

For $y = 0$, $x = \pm a = \pm 3$ hence $a = 3$

Equations of asymptotes are given by: $y = \pm\frac{b}{a}x$

From the diagram, gradient of asymptotes is ± 2,

hence $\frac{b}{a} = 2$

$b = 6$

Equation becomes: $\frac{x^2}{9} - \frac{y^2}{36} = 1$

Translate the centre to (1, 0) and so

$\frac{(x-1)^2}{9} - \frac{y^2}{36} = 1$

Question 3 [B]

Implied domain of

Cos^{-1} is $[-1, 1]$

that of $\text{Cos}^{-1} 3x$ is $\left[\frac{-1}{3}, \frac{1}{3}\right]$

that of $1+ \text{Cos}^{-1} 3x$ is also $\left[\frac{-1}{3}, \frac{1}{3}\right]$

Question 4 [D]

$$y = \text{Sin}^{-1}\frac{5}{x},\quad x > 5$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-\left(\frac{5}{x}\right)^2}}\cdot\frac{-5}{x^2} \quad \text{using the Chain Rule}$$

$$= -\frac{5}{x\sqrt{x^2-25}}$$

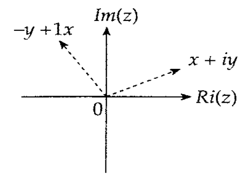
Question 5 [E]

$$w = 5 - 2i$$
$$2 - w = 2i - 3$$

$$= \frac{1}{2-w} = \frac{1}{2i-3}$$
$$= \frac{1}{2i-3} \times \frac{2i+3}{2i+3}$$
$$= \frac{-(2i+3)}{13}$$

Question 6 [E]

$$u = i^5 z$$
$$= i(i)^4 z$$
$$= iz$$
if $z = x + iy$
then $u = ix - y$



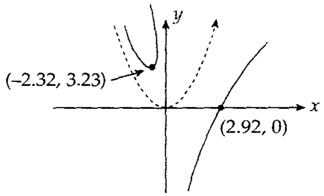
The effect is that of a rotation of $\frac{\pi}{2}$ in an anti-clockwise direction around the origin.

Exam 1 : Part 2

Question 1

$f(x) = \frac{x^3 - 25}{5x}$

$= \frac{x^2}{5} - \frac{5}{x}$



Question 2a

$\underset{\sim}{r} = \sqrt{t} - (t-2)\underset{\sim}{j}$

The distance of the particles from the origin is given by:

$r^2 = t + (t-2)^2$

$= t^2 - 3t + 4$

distance will be a minimum when

$\frac{d(r^2)}{dt} = 0$

$2t - 3 = 0$

$t = 1.5$

Alternatively

$r = \sqrt{t^2 - 3t + 4}$

$\frac{dr}{dt} = \frac{2t-3}{2\sqrt{t^2-3t+4}} = 0$

$2t + 3 = 0$

$t = \frac{3}{2}$

Question 2b

$\underset{\sim}{r} = x\underset{\sim}{i} + x\underset{\sim}{j}$

$= \sqrt{t}, y = -(t-2)$

ence $y = -t + 2$

$-2 = -t$

$-2 = -(x)^2$

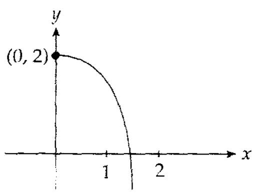
$y = 2 - x^2$ for $x \geq 0$, since $t \geq 0$

ne value of the intercept is given by

$-x^2 = 0$

$x = \sqrt{2},$

$x = 1.414$



Question 3

$\int_{\frac{\pi}{3}}^{\pi} \sin^3 \frac{x}{2}\, dx$

$\int_{\frac{\pi}{3}}^{\pi} \sin^2\left(\frac{x}{2}\right)\sin\left(\frac{x}{2}\right)dx$

let $u = \cos\frac{x}{2}, \frac{du}{dx} = -\frac{1}{2}\sin\frac{x}{2}$

$\int_{\frac{\sqrt{3}}{2}}^{0} \left(1-u^2\right) \cdot -2\frac{du}{dx} \cdot dx$

Since when $x = \frac{\pi}{3}, u = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

And when $x = \pi, u = \cos\frac{\pi}{2} = 0$

The integral can be rearranged as

$2\int_{0}^{\frac{\sqrt{3}}{2}} \left(1-u^2\right)du$

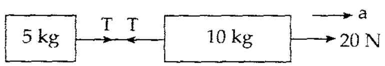
$2\left[u - \frac{u^3}{3}\right]_0^{\frac{\sqrt{3}}{2}}$

$2\left[\frac{\sqrt{3}}{2} - \frac{3\sqrt{3}}{24}\right]$

$2.\frac{9\sqrt{3}}{24}$

$\frac{3\sqrt{3}}{4}$

Question 4a



For the 10 kg mass

$20 - T = 10a$

For the 5 kg mass

$T = 5a$

$20 - 5a = 10a$

$15a = 20$

$a = \frac{4}{3}$

acceleration is $\frac{4}{3}$ m/s^2

Question 4b

$T = 5a$

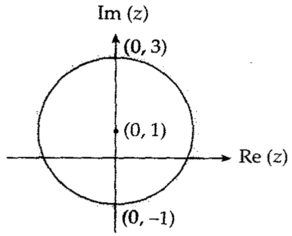
From 4a. $a = \frac{4}{3}$

$= \frac{20}{3}$

tension in string is $\frac{20}{3}$ N (or 6.67 N)

Question 5a

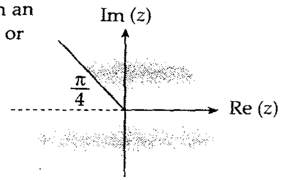
$\{z : |z - i| \geq 2\}$ defines all points on the circumference and outside the circle which has the centre at (0, 1) and with radius of 2 units.



Question 5b

$\left\{z : Arg(z) \leq \frac{3\pi}{4}\right\}$ *Note* $-\pi < ArgZ \leq \pi$

defines all points in the complex plane with an argument less than or equal to $\frac{3\pi}{4}$. Only those points are excluded with an argument greater than $\frac{3\pi}{4}$ and less than π, ie. The ray which gives Arg $Z = \frac{3\pi}{4}$ is a solid line and the negative x = axis is dotted.



Question 6a

$z = 2 - i$ Note $z - (2 - i) = 0$

$z^2 = 4 + i^2 - 4i$

$= 3 - 4i$

$z^3 = (3 - 4i)(2 - i)$

$= 6 + 4i^2 - 11i$

$= 2 - 11i$ Note $z^3 = (2 - i)z^2$

$z^3 - (2 - i)z^2 + z - 2 - i = 0$

Since the first two terms and the last two terms are equal to zero. (You could substitute for every term.)

An alternative

$z^3 - (2 - i)z^2 + z - 2 + i = 0$

$(2 - i)^3 - (2 - i)(2 - i)^2 + 2 - i - 2 + i$

$\Rightarrow (2 - i)^3 - (2 - i)^3 + (2 - i) - (2 - i) = 0$

Terms cancel out.

Question 6b

$z^3 - (2 - i)\ z^2 + z\ - 2 + i$ can be factorised

$(z - 2 + i)(z^2 + 1)$

so, if $z^3 - (2 - i)z^2 + z - z + i = 0$

$(z - 2 + i)(z^2 + 1) = 0$

Solutions are $z = 2 - i,\ z = \pm i$