

**THE
HEFFERNAN
GROUP**

P.O. Box 1180
Surrey Hills North VIC 3127
ABN 20 607 374 020
Phone 9836 5021
Fax 9836 5025

Student Name.....

SPECIALIST MATHEMATICS

TRIAL EXAMINATION 2

2000

Reading Time: 15 minutes
Writing Time: 90 minutes

Instructions to Students

This exam consists of 5 questions and all of these 5 questions should be answered.
There is a total of **60** marks available.
Students may bring up to two A4 pages of pre-written notes into the exam.

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Question 1

Let $z_1 = 3 - 4i$ and $z_2 = 2 - i$

- a.** **i.** Express $\frac{1}{z_1}$ in Cartesian form.

1 mark

- ii.** Find the value of $\text{Arg } z_1$ to the nearest minute.

1 mark

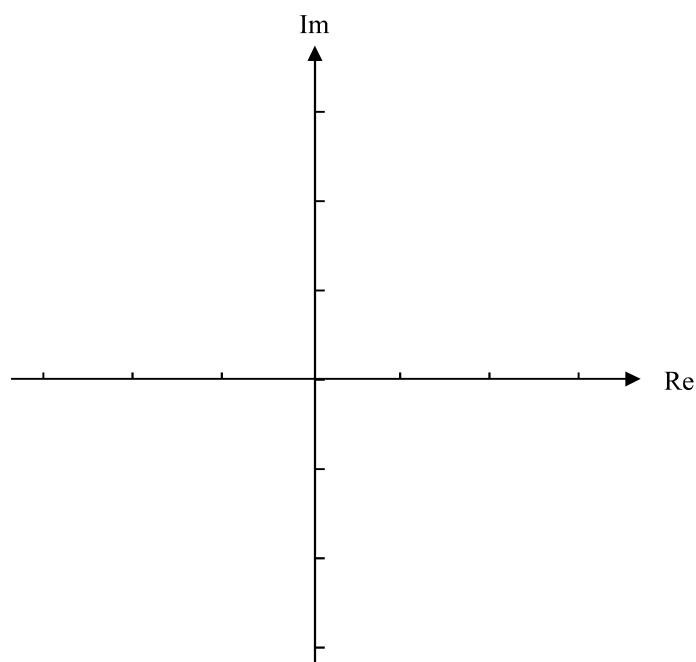
- iii.** Verify that $\text{Im } z_1 + \bar{z}_2 + z_1 z_2 = -10i$

1 mark

- iv.** Find algebraically the square roots of z_1 and express them in Cartesian form.

3 marks

- b. i.** On the complex plane below sketch S where $S = \{z : |z - 2 + i| = |z + 2 - i|\}$



1 mark

- ii.** Find algebraically the Cartesian equation of S .

1 mark

- iii.** Given that $z_4 = 3cis \frac{\pi}{3}$, state with reasons whether or not $z_4 \in S$.

1 mark

- iv.** Write down that subset of the complex plane for which the points are equidistant from the points z_4 and z_1

2 marks

Total 11 marks

Question 2

Consider the function $f : [1, 4] \rightarrow \mathbb{R}$ where $f(x) = \frac{16}{x^2} - 1$

- a.** Find the coordinates of the x intercept of the graph of f .

1 mark

- b.** The point (m, n) lies on f and the gradient of f at this point is -1. Find the values of m and n correct to 1 decimal place.

2 marks

A sketch of a vase is made. In a cross sectional view, the sides of the vase have the shape of the graph of $f(x)$ and of its reflection in the y axis. The cross sectional area of the vase is shaded in Figure 1 below.

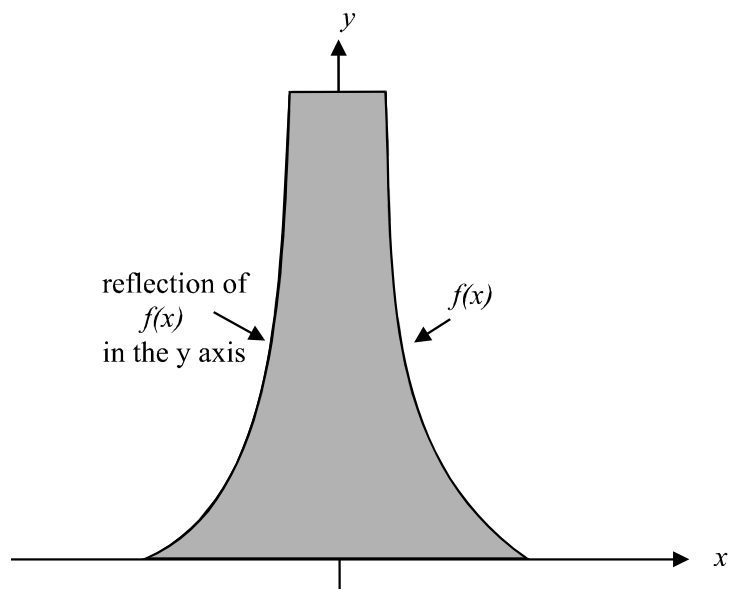


Figure 1

- c. Find this shaded area.

3 marks

Question 3

Particle A moves so that its position $\vec{r}$ (metres) from the origin at time t (secs) is given by

$$\vec{r}_A = (\sqrt{2} \sin(2t) + 1)\vec{i} + \sqrt{2} \cos(2t)\vec{j} \quad t \geq 0$$

- a.** **i.** Find the distance of particle A from the origin at time t .

1 mark

- ii.** Find the furthest distance that particle A can be from the origin and state the value(s) of t when this occurs.

2 marks

- b.** Show that speed of particle A is constant.

2 marks

A second particle B moves so that its position $\vec{r}$ (metres) from the origin at time t (secs) is given by

$$\vec{r}_B = \left(\frac{4}{\sqrt{3}} \cos(2t)\right) \vec{i} + (4 \sin t \cos t) \vec{j} \quad t \geq 0$$

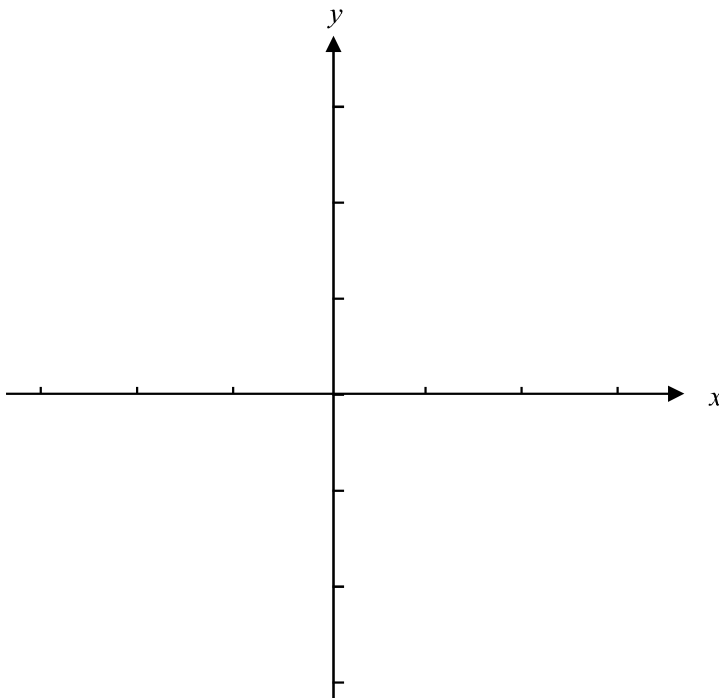
- c. i. Find the Cartesian equation of the path of particle B.

2 marks

- ii. Write down the domain and range of the Cartesian relation found in part i.

2 marks

- iii. Sketch the path of particle B on the set of axes below.



1 mark

- d.** Show that the acceleration and the position of particle B are always parallel.

2 marks

- e.** Show that the particle A and particle B do not collide in the interval $t \in [0, 0.1]$

2 marks

Total 14 marks

Question 4

A suitcase on wheels has a mass of 20 kg and rests on the carpeted floor of an airport. The coefficient of friction between the suitcase and the carpet is 0.5

The suitcase has a handle which extends from one of its corners and locks into position at an angle of θ with the horizontal as indicated in Figure 1 below.

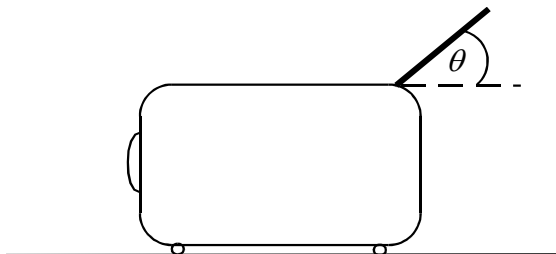
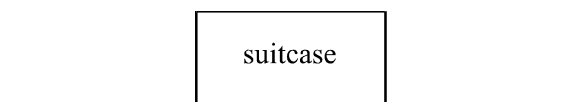


Figure 1

A man exerts a pulling force of P newton on the handle.

- a. Show on the diagram below the 4 forces, including the normal force N , acting on the suitcase.

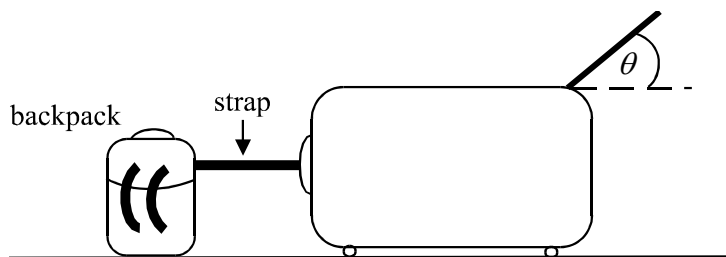


1 mark

- b. When the pulling force P is equal to $\frac{20\sqrt{2}g}{3}$ newton, the suitcase is on the point of moving. Show that θ equals 45° .

2 marks

The man decides to take off his backpack which has a mass of 8 kg and is uncomfortable and attach it to the suitcase using a strap. The coefficient of friction between the backpack and the carpet is 0.6



- c. Suggest why the coefficient of friction between the backpack and the carpet is greater than the coefficient of friction between the suitcase and the carpet.

1 mark

- d. i. The man exerts a pulling force on the handle of the suitcase of P newton. Show the forces acting now on the suitcase and the backpack on Figure 2 below.

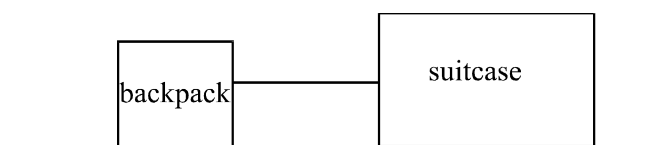


Figure 2

2 marks

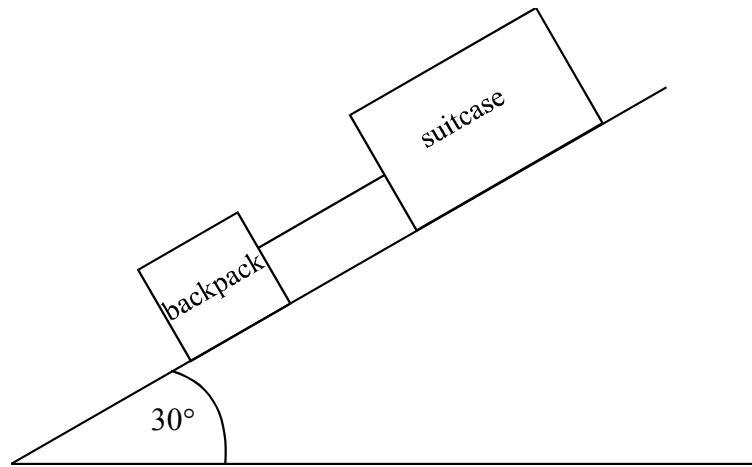
- ii. Show that if the pulling force P , is 90 newton and the tension in the strap connecting the suitcase and the backpack is 20 newton, that the suitcase and the backpack are not about to move.

1 mark

The man comes to a short ramp which has the same carpet as that in the other parts of the airport. The ramp is inclined at an angle of 30° .

The man changes the position of the handle of his suitcase. He locks it into a position which is parallel to the slope of the ramp. The man exerts a pulling force of P newton on the handle.

- e. On Figure 3 below, show the forces acting on the suitcase and the backpack now.



2 marks

Figure 3

- c.** Find how many degrees below -2° Celsius the temperature of the pathology sample is at $t = 4$ minutes. Express your answer to 1 significant figure.

1 mark

- d.** Graph the function obtained in part **b.** and use this to explain what happens to the temperature of the pathology sample once it is placed in the special container.

2 marks

- e.** Show algebraically that the temperature of the pathology sample approaches but never reaches -2° Celsius.

2 marks

Total 10 marks