

SET 2 EXAM 1

Reading time: 15 minutes

Writing time: 60 minutes

Structure of examination

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
10	10	40

Note: Formula Sheet is NOT supplied. You will need to use your own!

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape or a calculator of any type.

Materials supplied

- Question and answer book
- Working space is provided throughout the book

Instructions

- Complete all responses in the spaces provided
- All responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Instructions

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working must be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

- a. Differentiate $\cos(x \log_e(x))$ with respect to x .

2 marks

- b. Find the equation of the normal to the curve $y = 2\sin^3(x) + 4x + 11$ at the point where $x = \pi$.

3 marks

Question 2

Find $g(x)$ given that $g'(x) = \cos(2x) - 2\sin(x)$ and $g(0) = 1$.

3 marks

Question 3

- a. Let f be a function defined over its maximal domain with the rule $f(x) = 2\log_e(5x) - 2$. Find the rule of the inverse function, f^{-1} .

2 marks

- b. State the domain of the function f^{-1} .

1 mark

- c. Let $g(x) = e^x$. List the transformations required in order to transform $g(x)$ into $f^{-1}(x)$.

3 marks

Question 4

Find the general solution to the equation $2\sin^2(x) - 1 = \cos(x)$.

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

3 marks

Question 5

Let X be a continuous random variable with the following probability density function:

$$f(x) = \begin{cases} a \sin(x) & 0 \leq x \leq \pi \\ 0 & \text{otherwise} \end{cases}$$

- a. Find the exact value of a .

3 marks

- b. Evaluate $\Pr\left(X < \frac{\pi}{3}\right)$.

2 marks

Question 6

Let $y = \sin(\log_e(x))$.

- a. Find the derivative, $\frac{dy}{dx}$.

1 mark

- b. Hence, evaluate $\int \left(\frac{1}{x} \sin \left(\frac{\pi}{2} - \log_e(x) \right) \right) dx$.

2 marks

- c. Find the average value of the function $\frac{1}{x} \sin \left(\frac{\pi}{2} - \log_e(x) \right)$ for the interval $x \in [1, \pi]$.

2 marks

Question 7

Two points, $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ lie on the curve $x^2 = 4ay$. If the chord PQ passes through the point $(0, a)$, where a is a non-zero constant, find the value of pq (the product of p and q).

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3 marks

Question 8

The Hettige Bakery prides itself on producing large loaves of bread. The mass of their loaves are normally distributed, with a mean of 760 g and a standard deviation of 7 g. A competing bakery, the Levy Bakery, also produces loaves with normally distributed masses. The Levy Bakery's masses have a mean of 768 g and a standard deviation of 14 g.

Tim wishes to buy a loaf of bread with a mass greater 750 g. Determine which bakery has the higher probability of producing a loaf with a mass greater than 750 g.

3 marks

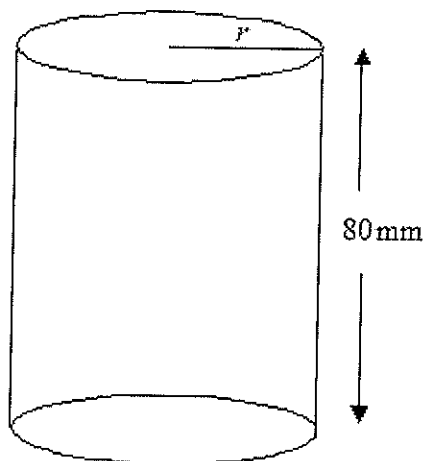
Question 9

Trevor either drives to university or catches the train. If he drives today, the probability that he will drive tomorrow is 80%. However, if he catches the train today, the probability he drives tomorrow is 75%. If Trevor drives to university on Monday, what is the probability that he drives to university exactly once on the next two days?

3 marks

Question 10

A small aluminium cylinder is placed in direct sunlight, where its radius increases at a constant rate whilst its height remains constant. The radius is increasing at a rate of 0.5 mm per hour.



- a. Find an expression for the surface area of the cylinder, $S \text{ mm}^2$, in terms of the radius, $r \text{ mm}$.

1 mark

- b. Find the rate at which the surface area is increasing when the radius is 10 mm, in mm^2 per hour.

3 marks

END OF EXAMINATION