

ALGEBRA

TECH-FREE TEST 1

Writing time: 30 minutes

Structure of test

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
5	5	20

Question 1

For the simultaneous linear equations

$$\begin{aligned}2mx + 3y &= 8 \\ x + my &= m\end{aligned}$$

- a. Find the value(s) of m for which these two lines have the same gradient.

2 marks

- b. Hence, determine the value(s) of m for which the equations have;
- i. no solutions.

- ii. a unique solution.

2 + 2 = 4 marks

Question 2

Solve the following equation for $x \in [\pi, 2\pi]$.

$$4\sin^2(x) + (2 - 2\sqrt{3})\sin(x) = \sqrt{3}$$

4 marks

Question 3

Consider the functions $f(x) = \tan^2(x)$ and $g(x) = \tan(x)$.

Show that $f(x)g(x) = \frac{1}{g(x)} - g(x)$. State the values of x for which this is true.

4 marks

Question 4

Solve the following equation for $x \in \mathbb{R}$.

$$|\log_e(2x + 1)| = 9$$

2 marks

Question 5

Let $f(x) = e^{x^2}$ and $g(x) = 3x^2 + 1$.

- a. Write down the rule for $g(f(x))$.

1 marks

- b. Find the rule for the inverse relation of the result found in part a, $g(f(x))^{-1}$.

2 marks

- c. State the domain of the inverse function found in part b.

1 mark

END OF TEST

ALGEBRA

TECH-FREE TEST 2

Writing time: 30 minutes

Structure of test

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
5	5	20

Question 1

Consider the function $f(x) = \frac{x-3}{x+1}$.

- a. Find the rule for the inverse of this function, f^{-1} .

3 marks

- b. State the domain of this inverse function.

1 marks

Question 2

Solve the equation $\log_3(x) = 4\log_x(3)$ for x . Comment on the validity of all solution(s) found.

4 marks

Question 3

Consider the functions $f(x) = \frac{1}{x} - 3$ and $g(x) = -ax$.

Find the value(s) of a for which $f(x)$ and $g(x)$ have no intersection.

4 marks

Question 4

Show that $(a + 3b)^2 - 12ab \geq 0$ for all $a, b \in \mathbb{R}$.

3 marks

Question 5

Find the values of x that satisfy $|x - 1|^2 - 2|x - 3| \geq 3$.

5 marks

END OF TEST

ALGEBRA

TECH-ACTIVE TEST 1

Writing time: 60 minutes

Structure of test

<i>Section</i>	<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
1	11	11	11
2	3	3	29
			40

SECTION 1 - Multiple-Choice Questions

Question 1

The number of solutions to the equation $f : \mathbb{N} \rightarrow \mathbb{R}, f(x) = 4x^5 - 6x^4 + 3x^3 - 2x + 6 = 0$ is

- A. 5
- B. 4
- C. 3
- D. 1
- E. 0

Question 2

Which of the following is a solution to the following functional equation for all $x, y \in \mathbb{R}$:

$$\frac{f(x) + f(y)}{f(x) - f(y)} = 1$$

- A. $f(x) = -2x + 1$
- B. $f(x) = 0$
- C. $f(x) = x^2$
- D. $f(x) = e^{2x}$
- E. none of these

Question 3

The solution(s) to the equation $\log_e(x+5) + \log_e(x^2) = -4\log_e(x)$, correct to two decimal places, is/are

- A. nonexistent
- B. $x = -5.00, -0.78, 0.74$
- C. $x = -5.00, 0.00$
- D. $x = 0.74$ only
- E. $x = -0.78, 0.74$

Question 4

Consider the following system of simultaneous linear equations:

$$\begin{aligned}4x - my &= 6 \\ nx + 3y &= 7\end{aligned}$$

For what value of m does there exist an n such that these simultaneous equations have an infinite number of solutions?

- A. -3
- B. 3
- C. -12
- D. $-\frac{7}{6}$
- E. $-\frac{18}{7}$

Question 5

A possible domain of the function $f(x) = \log_e(\sqrt{3} + 2\sin(3x - 2))$ is

- A. $\left[\frac{2}{3}, \frac{\pi+2}{3}\right)$
- B. $\left(-\frac{\pi}{3}, \frac{\pi}{3}\right)$
- C. $\left(0, \frac{2\pi}{3}\right)$
- D. $(2, \pi+3)$
- E. $\left[0, \frac{2\pi}{3}\right]$

Question 6

The function $f: \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = 2x^3 + 18x^2 + ax + 27$ will have an inverse exactly when

- A. $a < 54$
- B. $a \in \mathbb{R}$
- C. $a > 324$
- D. $a \geq 54$
- E. $0 < a < 54$

Question 7

Correct to two decimal places, the solutions to the equation $2\sin(3x - 2) - x = x^2 - 3x - 3$ are

- A. nonexistent
- B. $x = -1.25, 3.42$
- C. $x = 1.25, 3.42$
- D. $x = -1.12, 3.80$
- E. $x = -1.24, 3.42$

Question 8

Consider the following equation:

$$2x = e^y + e^{-y}$$

Which of the following equations is equivalent to the above?

- A. $x = 2(e^y + e^{-y})$
- B. $\log_e(2x) = 0$
- C. $y = \log_e(x \pm \sqrt{x^2 - 1})$
- D. $y = \log_e(x^2 + x - 1)$
- E. none of the above

Question 9

The general solution to the equation $2\tan(2x) = 1$ is

- A. $x = \frac{1}{2}n\pi + \frac{1}{2}\tan^{-1}\left(\frac{1}{2}\right), n \in \mathbb{N}$
- B. $\frac{1}{2}n\pi + \frac{\pi}{6}, n \in \mathbb{Z}$
- C. $x = \frac{1}{2}n\pi + \frac{1}{2}\tan^{-1}\left(\frac{1}{2}\right), n \in \mathbb{Z}$
- D. $\frac{1}{2}n\pi \pm \frac{\pi}{6}, n \in \mathbb{N}$
- E. none of these

Question 10

Which of the following functional equations is it possible for $f(x) = 2x^n$ to satisfy for at least one positive value of n ?

- A. $f(f(x)) = x$
- B. $f(xy) = f(x) + f(y)$
- C. $f(x + y) = f(x) + 2f(y)$
- D. $f(f(f(f(x)))) = 2^{40}x^{81}$
- E. none of these

Question 11

The number of real solutions to the equation $\sin^3(x) - 3\sin^2(x) + 3\sin(x) = 1$ is

- A. 0
- B. 1
- C. 2
- D. 3
- E. none of these

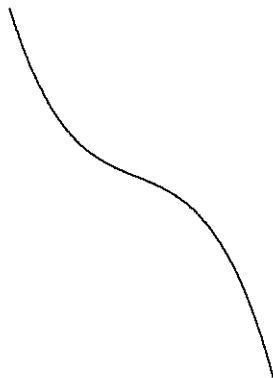
Question 1

a. Determine the minimum number of stationary points in this function and explain your answer.

2 marks

4 marks

Now, let us consider the function $g : \mathbb{R} \rightarrow \mathbb{R}$, $g(x) = ax^3 + 6x^2 - cx + d$. One possible shape of the graph is below:



- c. Given that the shape of $y = g(x)$ is as shown above, determine all values that a , c and d can take. Also determine a relationship between a and c , with c as the subject.

4 marks

Question 2

Consider the following functions:

$$f : \mathbb{R} \rightarrow \mathbb{R}, f(x) = 2^{x+4}$$

$$g : \mathbb{R} \rightarrow \mathbb{R}, g(x) = 2e^{x+3} - 5$$

- a. If $p(x) = 5^x$ and $q(x) = 6^x$, show that $p(x) = q((\log_6(5))x)$.

2 marks

- b. Determine a series of transformations to transform $y = f(x)$ to $y = g(x)$.

4 marks

Let us now consider the following functions:

$$r : \mathbb{R} \rightarrow \mathbb{R}, r(x) = e^{x+4}$$

$$s : \mathbb{R} \rightarrow \mathbb{R}, s(x) = 2e^{x-3}$$

- c. Deduce a translation to transform $y = r(x)$ to $y = s(x)$.

2 marks

Consider the following functions:

$$a : (0, \infty) \rightarrow \mathbb{R}, a(x) = \log_e(x)$$

$$b : S \rightarrow \mathbb{R}, b(x) = \log_e\left(\frac{x^6}{3}\right)$$

- d. Deduce a possible transformation to transform $y = a(x)$ to $y = b(x)$, involving at least one translation.

2 marks

- e. Determine the domain, S , of b , given your answer to part d. is valid.

1 mark

Question 3

- a. Using algebra, find the general solution to the equation $|\sin(\log_e(2x+5))| = \frac{\sqrt{3}}{2}$ for $x \in \mathbb{R}$.

4 marks

- b. Determine the domain of the function $f(x) = |\sin(\log_e(2x+5))|$.

2 marks

- c. Consider two adjacent x -intercepts of the graph of $f(x) = |\sin(\log_e(2x + 5))|$, $(x_1, 0)$ and $(x_2, 0)$. Explain, in words, whether the stationary point between those two x -intercepts is to the left, to the right, or the same as the point $\left(\frac{x_1 + x_2}{2}, f\left(\frac{x_1 + x_2}{2}\right)\right)$.

2 marks

ALGEBRA

TECH-ACTIVE TEST 2

Writing time: 60 minutes

Structure of test

<i>Section</i>	<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
1	11	11	11
2	2	2	29
			40

SECTION 1 - Multiple-Choice Questions

Question 1

Which of the following is an exact solution to the equation $5 \times 2^{3x} = 1$?

- A. $x = \frac{1}{5} \log_2 \left(\frac{1}{3} \right)$
- B. $x = 3 \log_2 \left(\frac{1}{5} \right)$
- C. $x = -\frac{1}{3}$
- D. $x = \frac{1}{3} \times 2^{\frac{1}{5}}$
- E. $x = \frac{1}{3} \log_2 \left(\frac{1}{5} \right)$

Question 2

The sum of the solutions to the equation $2 \cos(2x) = 1$ for $x \in [0, 2\pi]$ is

- A. $\frac{2\pi}{3}$
- B. π
- C. 2π
- D. 4π
- E. $\frac{\pi}{3}$

Question 3

The polynomial $P(x) = x^3 + ax^2 + bx + 6$ has a factor $(x + 1)$ and a remainder of 4 when divided by $(x - 1)$. The values of a and b , respectively, are

- A. -1 and 1
- B. 4 and 1
- C. -2 and -1
- D. 3 and -1
- E. -4 and 1

Question 4

The solution to the inequality $(0.6)^{-x} \leq 3$ is approximately

- A. $x \geq 0$
- B. $x \geq 2.15$
- C. $x \leq 2.15$
- D. $x \leq -0.86$
- E. $x \leq -2.15$

Question 5

The number of real solutions to the equation $(x^2 - 3x - 4)(x^2 - 6x + 9) = 0$ is

- A. 0
- B. 1
- C. 2
- D. 3
- E. 4

Question 6

The solution(s) to the equation $\log_3(x - 2) - \log_3(x) = 1$ are

- A. $x = -3, x = 1$
- B. $x = -1$
- C. $x = 3$
- D. $x = -1, x = 3$
- E. There are no real solutions for x

Question 7

The coefficient of x^6 in the expansion of $(a + x^3)^5$ is

- A. $-a^3$
- B. $10a^3$
- C. $-10a^3$
- D. a^3
- E. $10a^2$

Question 8

If $\cos(x) = 0.7$, the value of $\sin\left(\frac{\pi}{2} - x\right) + \cos(x + \pi)$ is

- A. -1
- B. 0.7
- C. $\frac{\pi}{2}$
- D. 1.4
- E. 0

Question 9

The solution to the equation $2\log_e(x) - \log_e(2x) = y$, in terms of y , is

- A. $x = 2e^y$
- B. $x = y$
- C. $x = \log_e(2y)$
- D. $x = 2$
- E. $x = \frac{1}{2}$

Question 10

If $\sin(2x) + \cos(2x) = 0$ for $0 \leq x \leq 2\pi$, the difference between the largest and smallest solution for x is

- A. π
- B. 2π
- C. $\frac{3\pi}{2}$
- D. $\frac{7\pi}{4}$
- E. 4π

Question 11

For which of the following conditions does the equation $2^{2x} + 2^x + b = 0$ have exactly one real solution?

- A. $b < 0$
- B. $b > 0$
- C. $b \leq 0$
- D. $b = \frac{1}{4}$
- E. $b = 4$

SECTION 2 - Extended-Response Questions

Question 1

Consider the following function, f , where c is a real constant:

$$f(x) = \frac{1}{3}x^3 - x^2 - 3x + c$$

a. Assuming that $c = 0$, show that

i. the function $f(x)$ has three x -intercepts.

ii. the x -coordinates of the stationary points are $x = -1$ and $x = 3$.

b. Find the maximal range of values for c such that the equation $f(x) = 0$ has

2 + 3 = 5 marks

i. one real solution.

ii. three distinct real solutions.

iii. two positive distinct real solutions.

2 + 2 + 2 = 6 marks

c. Consider the function $g(x) = \frac{1}{3} \times 2^{3x} - 2^{2x} - 3 \times 2^x + d$.

i. Find all real value(s) of x which satisfy the equation $g(x) = d$.

ii. Find the value(s) of d such that the equation $g(x) = 0$ has two real solutions.

3 + 2 = 5 marks

Question 2

Consider the following system of simultaneous equations, with variables x , y and z :

$$x - y - 2z = -3$$

$$tx + y - z = 3t$$

$$x + 3y + tz = 13$$

- a. Using algebra, show that $x = \frac{3t^2 - 21t + 30}{t^2 - 5t + 6}$.

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3 marks

- b. Find the two values of t such that the given system of simultaneous equations will have no unique solutions. (Hint: You need only analyse a 2-variable set of simultaneous equations. Consider the values of t for which x cannot be defined, and analyse your working in part a. to see what happens to your working for those values of t .)

- c. For which value of t does the system have no real solutions? 3 marks

- d. If $t = 0$, solve the system of simultaneous equations for x , y and z . 3 marks

4 marks

END OF TEST