

STUDENT NUMBER: _____

MATHEMATICAL METHODS

Written Examination 1

xxx xxxx xx 2020

Reading time: 15 minutes

Writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
10	10	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.
- Don't fucking use a calculator.

Materials supplied

- Question and answer book of 10 pages
- Working space is provided throughout the book.

Instructions

- Write your **name** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Question 1 (2 marks)

Find the derivative of $y = \frac{2x^2}{\sin(3x)}$

2 marks

Question 2 (2 marks)

If $f(x) = 2xe^{\sqrt{x}}$, find $f'(4)$

2 marks

Question 3 (2 marks)

Find $\int_{-1}^2 x^2 + 6x + 19 \, dx$

2 marks

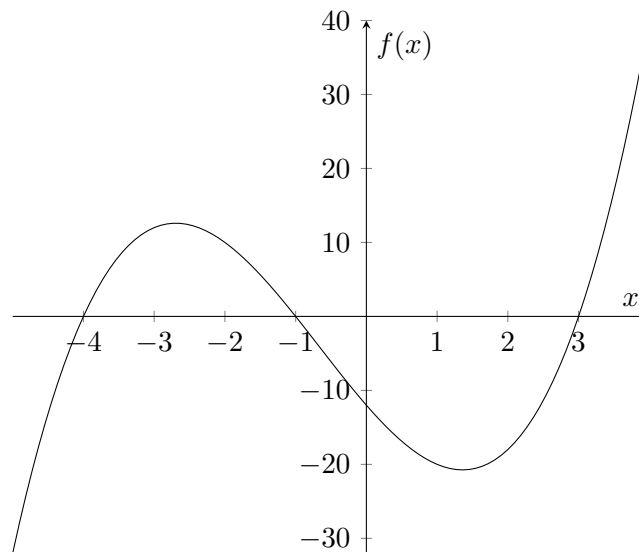
Question 4 (3 marks)

Solve $\sin(2x) = \frac{1}{2\sin(2x)} - \frac{1}{2}$ for x , given $x \in [-\pi, \pi]$

3 marks

Question 5 (4 marks)

The function $f(x) = (x - 3)(x + 1)(x + 4)$ is graphed below.



a.) Find and sketch the tangent to the graph at the point where $x = 1$.

3 marks

b.) State the other point(s) of intersection between $f(x)$ and the tangent found in part a.

1 mark

Question 6 (3 marks)

Let $g(x) = \frac{x+1}{x-2}$

a.) Find the domain and range of g .

1 mark

b.) Find the inverse function $g^{-1}(x)$.

2 marks

Question 7 (6 marks)

Luke decided to start a boys club called Luke's Lads.

a.) Luke holds his boys to the highest standards and therefore he inspects his little lads daily. The probability that one of his lads passes an inspection is p . In a sample of 3 lads, find a fully simplified expression for the probability that only one or two of them pass their inspection in terms of p .

2 marks

b.) Hence, determine the value of p , if the probability of one or two of the lads passing their inspection is a maximum.

2 marks

c.) Using your value for p , and a sample of 6 lads, find the probability that exactly 6 lads fail their inspections, given more than 4 of them fail.

2 marks

Question 8 (3 marks)

The amount of sleep you get every night is normally distributed with a mean of 4 hours and variance of 2 hours.

a.) Find the probability that you get less than 5 hours of sleep in terms of Z , the standard normal distribution.

1 mark

b.) Using the fact that $\Pr(Z < \sqrt{2})=0.92$, find the probability that you get between 2 and 6 hours of sleep.

2 marks

Question 9 (6 marks)

Samantha is a heroin addict. When she takes a hit, the level of heroin in her system in arbitrary units can be modelled by the function:

$$h : [0, 6] \rightarrow R, h(t) = -(x - 3)^2 + 9$$

a.) State the maximum value of h .

1 mark

b.) Samantha takes another hit m minutes later. State the translation applied to the function h in terms of m , and write a new function h_m .

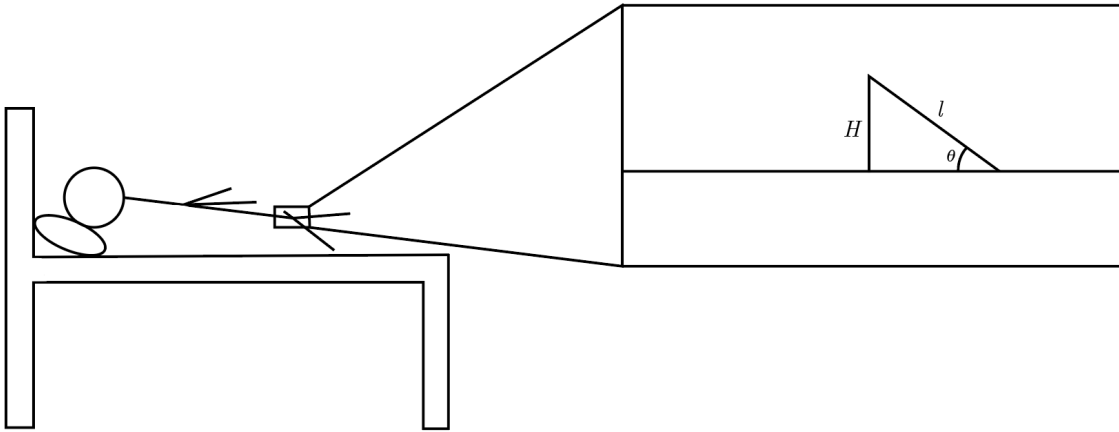
2 marks

c.) It is known that if the total heroin levels in Samantha's system exceed 10 units, she will die. Find the largest value for m that she will die.

3 marks

Question 10 (9 marks)

James lies in bed horizontally as shown below:



James is bored and decides to pull out an adjustable rod to play with. The length of this adjustable rod can be modelled by the function:

$$l = 2e^{-(t-2)^2} + 2$$

Where l is the length in inches, and t is time in minutes.

a.) Use calculus to find the maximum value of l and state at what time this value occurs.

3 marks

b.) Find an expression that can be used to determine the vertical height, H , above his body that rod reaches, in terms of t and θ .

1 mark

It is known that the angle the rod makes with his body is dependent upon its length, according to the ratio: $\theta = \frac{4}{l}$

c.) Show that:

$$\frac{dH}{dt} = 4(t-2)e^{-(t-2)^2} \left[4(2e^{-(t-2)^2} + 2) \cos\left(\frac{4}{2e^{-(t-2)^2} + 2}\right) - \sin\left(\frac{4}{2e^{-(t-2)^2} + 2}\right) \right]$$

3 marks

d.) Therefore, find the value for x at which the maximum vertical height occurs and give the maximum value in the form $a \sin b$ where a and b are both integers.

2 marks

[END OF EXAMINATION]