BILLANOOK COLLEGE

NAME: **Student Number:**

MATHEMATICAL METHODS (CAS) UNITS 3 & 4

Practice July Exam

Test B TECHNOLOGY ACTIVE

Thursday 19th July, 2018
Reading time: 15 minutes 8:30am – 8:45am
Writing time: 1.5 hours 8:45am – 10:15am

QUESTION AND ANSWER BOOK

Structure of book

Section	*Number of Questions*	*Number of questions to be answered*	*Number of marks*
1	15	15	15
2	4	4	39

Students are permitted to bring into the SAC room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved CAS calculator (memory DOES NOT need to be cleared) and, if desired, one scientific calculator. For approved computer-based CAS, their full functionality maybe used.

Students are NOT permitted to bring into the SAC blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book, Multiple Choice answer sheet, Formula sheet

Instructions

- Write your **name** in the space provided above on this page.
- All written responses must be in English.

SECTION A – Multiple-choice questions

Question 1

Let $f: R \to R, f(x) = 3 - 2\cos\left(\frac{\pi x}{4}\right)$.

The period and range of this function are respectively

A. 4 and $[-2, 2]$

B. 8 and $[1, 5]$

C. 8π and $[1, 5]$

D. 8π and $[-2, 2]$

E. $\frac{1}{2}$ and $[-1, 5]$

Question 2

Let $f: R \to R, f(x) = 5\sin(2x) - 1$.

The period and range of this function are respectively

A. π and $[-1, 4]$

B. 2π and $[-1, 5]$

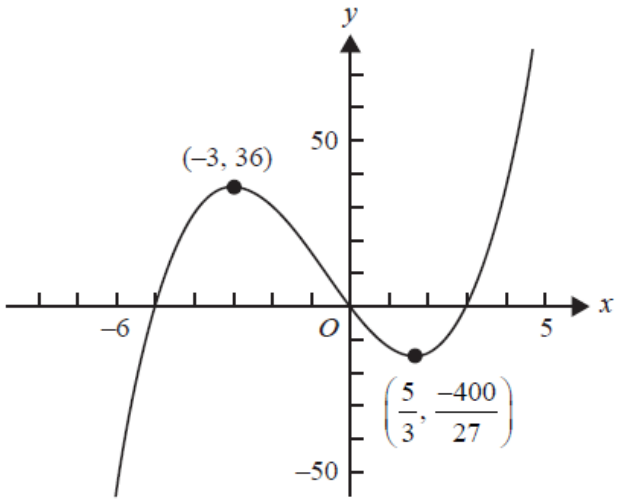
C. π and $[-6, 4]$

D. 2π and $[-6, 4]$

E. 4π and $[-6, 4]$

Question 3

Part of the graph of a cubic polynomial function f and the coordinates of its stationary points are shown below.



$f'(x) < 0$ for the interval

A. $(0, 3)$

B. $(-\infty, -5) \cup (0, 3)$

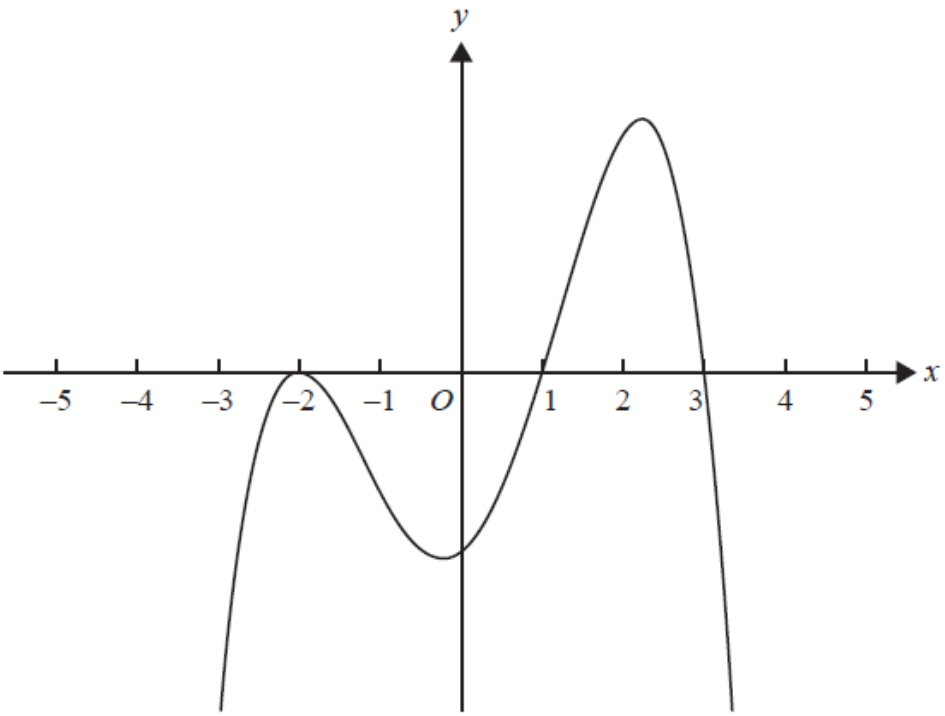
C. $(-\infty, -3) \cup \left(\frac{5}{3}, \infty\right)$

D. $\left(-3, \frac{5}{3}\right)$

E. $\left(\frac{-400}{27}, 36\right)$

Question 4

The diagram below shows part of the graph of a polynomial function.



A possible rule for this function is

A. $y = (x + 2)(x - 1)(x - 3)$

B. $y = (x + 2)^2(x - 1)(x - 3)$

C. $y = (x + 2)^2(x - 1)(3 - x)$

D. $y = -(x - 2)^2(x - 1)(3 - x)$

E. $y = -(x + 2)(x - 1)(x - 3)$

Question 5

A set of three numbers that could be the solutions of $x^3 + ax^2 + 16x + 84 = 0$ is

A. $\{3, 4, 7\}$

B. $\{-4, -3, 7\}$

C. $\{-2, -1, 21\}$

D. $\{-2, 6, 7\}$

E. $\{2, 6, 7\}$

Question 6

Let f and g be functions such that $f(2) = 5$, $f(3) = 4$, $g(2) = 5$, $g(3) = 2$ and $g(4) = 1$.

The value of $f(g(3))$ is

A. 1

B. 2

C. 3

D. 4

E. 5

Question 7

The average rate of change of the function with the rule $f(x) = x^2 - 2x$ over the interval $[1, a]$, where $a > 1$, is 8.

The value of a is

A. 9

B. 8

C. 7

D. 4

E. $1 + \sqrt{2}$

Question 8

The sum of the solutions to the equation $\sqrt{3}\sin(2x) = -3\cos(2x)$ for $x \in [0, 2\pi]$ is equal to

A. $\frac{\pi}{3}$

B. $\frac{7\pi}{6}$

C. $\frac{11\pi}{3}$

D. $\frac{13\pi}{3}$

E. $\frac{14\pi}{3}$

Question 9

The range of the function $f:\left(\frac{-1}{\sqrt{2}},\sqrt{2}\right]\to R,\ f(x)=2x^3-3x+4$ is

A. $\left(4-\sqrt{2},4+\sqrt{2}\right)$

B. $\left(\frac{-1}{\sqrt{2}},\sqrt{2}\right)$

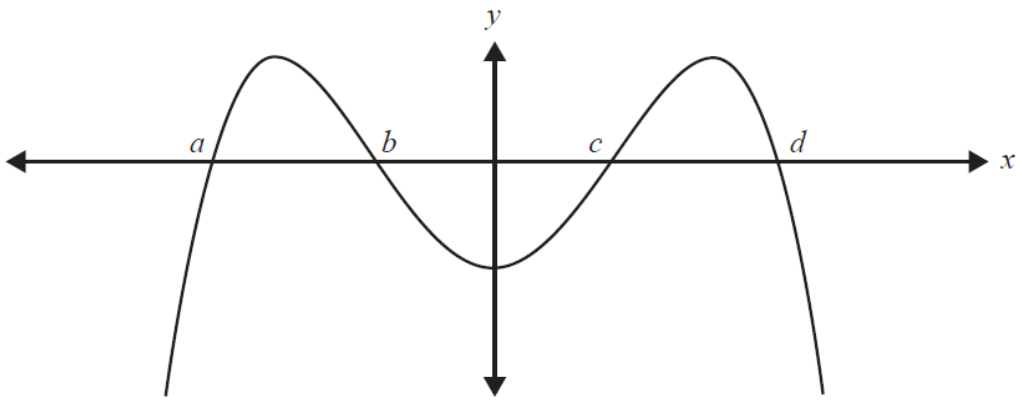
C. $\left(4-\sqrt{2},4+\sqrt{2}\right]$

D. $\left(\frac{-1}{\sqrt{2}},\sqrt{2}\right]$

E. $\left[4-\sqrt{2},\ 4+\sqrt{2}\right]$

Question 10

The graph of a function f, where $f(-x)=f(x)$, is shown below.



The graph has x-intercepts at $(a, 0)$, $(b, 0)$, $(c, 0)$ and $(d, 0)$ only.
The area bound by the curve and the x-axis on the interval $[a, d]$ is

A. $\int_a^d f(x)\,dx$

B. $\int_a^b f(x)\,dx-\int_c^b f(x)\,dx+\int_c^d f(x)\,dx$

C. $2\int_a^b f(x)\,dx+\int_b^c f(x)\,dx$

D. $2\int_a^b f(x)\,dx-2\int_b^{b+c} f(x)\,dx$

E. $\int_a^b f(x)\,dx+\int_c^b f(x)\,dx+\int_d^c f(x)\,dx$

Question 11

The graph of the function f is obtained from the graph of the function g with rule $g(x) = 3\cos\left(x - \frac{\pi}{6}\right)$ by a dilation of a factor of $\frac{1}{2}$ from the x-axis, a reflection in the y-axis, a translation of $\frac{\pi}{6}$ units in the negative x direction and a translation of 4 units in the negative y direction, in that order.
The rule of f is

A. $f(x) = \frac{3}{2}\cos\left(-x - \frac{\pi}{3}\right) - 4$

B. $f(x) = \frac{3}{2}\cos(-x) - 4$

C. $f(x) = -\frac{3}{2}\cos(x) - 4$

D. $f(x) = -3\cos\left(\frac{x}{2} - \frac{\pi}{3}\right) - 4$

E. $f(x) = \frac{3}{2}\cos\left(-x + \frac{\pi}{3}\right) - 4$

Question 12

Let $h : (-1, 1) \to R$, $h(x) = \frac{1}{x-1}$.
Which one of the following statements about h is **not** true?

A. $h(x)h(-x) = -h(x^2)$

B. $h(x) + h(-x) = 2h(x^2)$

C. $h(x) - h(0) = xh(x)$

D. $h(x) - h(-x) = 2xh(x^2)$

E. $(h(x))^2 = h(x^2)$

Question 13

The simultaneous linear equations $mx + 7y = 12$ and $7x + my = m$ have a unique solution only for

A. $m = 7$ or $m = -7$

B. $m = 12$ or $m = 3$

C. $m \in R\backslash\{-7, 7\}$

D. $m = 4$ or $m = 3$

E. $m \in R\backslash\{12, 1\}$

Question 14

The equation $(p-1)x^2 + 4x = 5 - p$ has no real roots when

A. $p^2 - 6p + 6 < 0$

B. $p^2 - 6p + 1 > 0$

C. $p^2 - 6p - 6 < 0$

D. $p^2 - 6p + 1 < 0$

E. $p^2 - 6p + 6 > 0$

Question 15

Question 10

A transformation $T: R^2 \to R^2$ with rule $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 2 & 0 \\ 0 & \frac{1}{3} \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$ maps the graph of $y = 3\sin\left(2\left(x + \frac{\pi}{4}\right)\right)$ onto the graph of

A. $y = \sin(x + \pi)$

B. $y = \sin\left(x - \frac{\pi}{2}\right)$

C. $y = \cos(x + \pi)$

D. $y = \cos(x)$

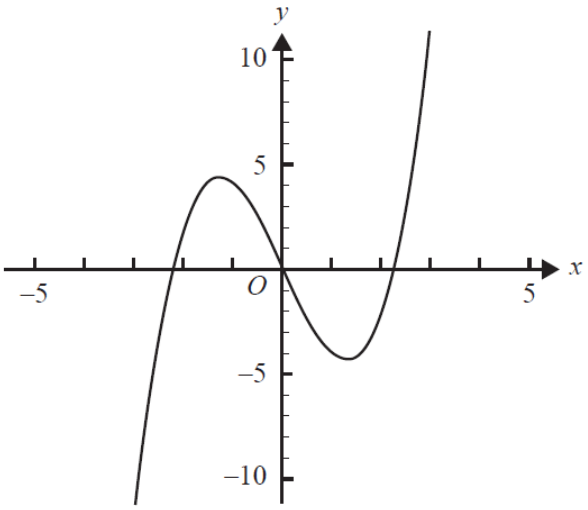
E. $y = \cos\left(x - \frac{\pi}{2}\right)$

SECTION B

Answer all questions in this section.

Question 1 (11 marks)

Let $f : R \to R,\ f(x) = x^3 - 5x$. Part of the graph of f is shown below.



a. Find the coordinates of the turning points. 2 marks

b. $A(-1, f(-1))$ and $B(1, f(1))$ are two points on the graph of f.

i. Find the equation of the straight line through A and B. 2 marks

ii. Find the distance AB. 1 mark

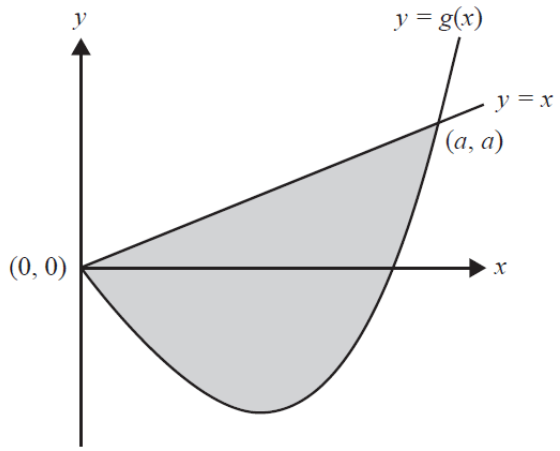
Let $g : R \to R, g(x) = x^3 - kx, k \in R^+$.

c. Let $C(-1, g(-1))$ and $D(1, g(1))$ be two points on the graph of g.

i. Find the distance CD in terms of k. 2 marks

ii. Find the values of k such that the distance CD is equal to $k + 1$. 1 mark

d. The diagram below shows part of the graphs of g and $y = x$. These graphs intersect at the points with the coordinates $(0, 0)$ and (a, a).



i. Find the value of a in terms of k. 1 mark

ii. Find the area of the shaded region in terms of k. 2 marks

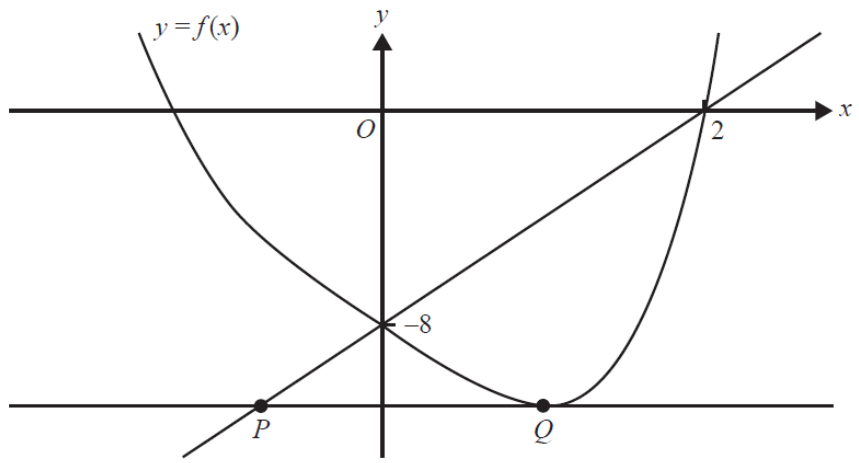
Question 2 (9 marks)

Let $f: R \to R$, $f(x) = x^4 - 4x - 8$.

a. Given $f(x) = (x - 2)(x^3 + ax^2 + bx + c)$, find a, b and c. 1 mark

b. Find two consecutive integers m and n such that a solution to $f(x) = 0$ is in the interval (m, n), where $m < n < 0$. 2 marks

The diagram below shows part of the graph of f and a straight line drawn through the points $(0, -8)$ and $(2, 0)$. A second straight line is drawn parallel to the horizontal axis and it touches the graph of f at the point Q. The two straight lines intersect at the point P.



c. **i.** Find the equation of the line through $(0, -8)$ and $(2, 0)$. 1 mark

ii. State the equation of the line through the points P and Q. 1 mark

iii. State the coordinates of the points P and Q. 2 marks

d. A transformation $T : R^2 \to R^2,\ T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} d \\ 0 \end{bmatrix}$ is applied to the graph of f.

i. Find the value of d for which P is the image of Q. 1 mark

ii. Let $(m', 0)$ and $(n', 0)$ be the images of $(m, 0)$ and $(n, 0)$ respectively, under the transformation T, where m and n are defined in **part b.**

Find the values of m' and n'. 1 mark

Question 3 (11 marks)

The temperature, T °C, in an office is controlled. For a particular weekday, the temperature at time t, where t is the number of hours after midnight, is given by the function

$T(t) = 19 + 6\sin\left(\frac{\pi}{12}(t-8)\right), 0 \le t \le 24.$

a. What are the maximum and minimum temperatures in the office? 2 marks

b. What is the temperature in the office at 6.00 am? 1 mark

c. Most of the people working in the office arrive at 8.00 am.

What is the temperature in the office when they arrive? 1 mark

d. For how many hours of the day is the temperature greater than or equal to 19 °C? 2 marks

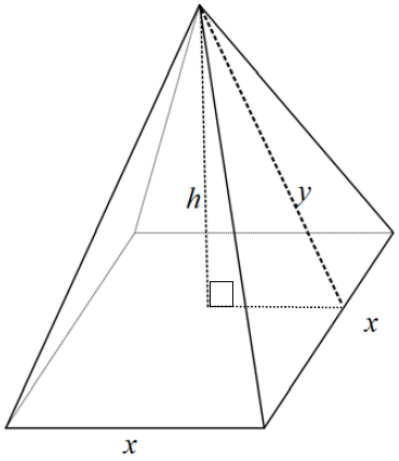
e. What is the average rate of change of the temperature in the office between 8.00 am and noon? 2 marks

f. **i.** Find $T'(t)$. 1 mark

ii. At what time of the day is the temperature in the office decreasing most rapidly? 2 marks

Question 4 (8 marks)

A group of architects, the *Pyramid Group*, are planning a cast iron installation in Federation Square in Melbourne. The first design is a simple right square based pyramid with vertical height, h metres, and length of the sides of the base, x metres.



The *Pyramid Group* know that the total cast iron required for the four sloping sides as well as the square base is 60 m^2.

a. Show that the slant height, y m, of the pyramid can be expressed as $y = \sqrt{h^2 + \frac{x^2}{4}}$. 1 mark

b. Find an expression for the Total Surface Area (TSA) of the pyramid in terms of both x and h. 2 marks

c. Hence show that the volume, V m^3, of the square based pyramid can be expressed in terms of x only as

$$V = \frac{1}{3}x\sqrt{900 - 30x^2}\ .$$

2 marks

d. State the implied domain for V. 1 mark

e. Find the maximum volume, in m^3, that the *Pyramid Group* can have for this square based pyramid. State the value of x, in m, that gives this maximum volume. 2 marks

END OF EXAMINATION

Mathematical Methods formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}\left(x^n\right)=nx^{n-1}$		$\int x^n dx=\frac{1}{n+1}x^{n+1}+c,\ n\neq -1$	
$\frac{d}{dx}\left((ax+b)^n\right)=an(ax+b)^{n-1}$		$\int (ax+b)^n dx=\frac{1}{a(n+1)}(ax+b)^{n+1}+c,\ n\neq -1$	
$\frac{d}{dx}\left(e^{ax}\right)=ae^{ax}$		$\int e^{ax}dx=\frac{1}{a}e^{ax}+c$	
$\frac{d}{dx}\left(\log_e(x)\right)=\frac{1}{x}$		$\int \frac{1}{x}dx=\log_e(x)+c,\ x>0$	
$\frac{d}{dx}\left(\sin(ax)\right)=a\cos(ax)$		$\int \sin(ax)dx=-\frac{1}{a}\cos(ax)+c$	
$\frac{d}{dx}\left(\cos(ax)\right)=-a\sin(ax)$		$\int \cos(ax)\,dx=\frac{1}{a}\sin(ax)+c$	
$\frac{d}{dx}\left(\tan(ax)\right)=\frac{a}{\cos^2(ax)}=a\sec^2(ax)$			
product rule	$\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}$	quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$		

Probability

$\Pr(A) = 1 - \Pr(A')$		$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$	
$\Pr(A \mid B) = \dfrac{\Pr(A \cap B)}{\Pr(B)}$			
mean	$\mu = \mathrm{E}(X)$	variance	$\mathrm{var}(X) = \sigma^2 = \mathrm{E}((X-\mu)^2) = \mathrm{E}(X^2) - \mu^2$

Probability distribution		**Mean**	**Variance**
discrete	$\Pr(X = x) = p(x)$	$\mu = \Sigma\, x\, p(x)$	$\sigma^2 = \Sigma\,(x-\mu)^2\, p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x)dx$	$\mu = \int_{-\infty}^{\infty} x\, f(x)dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2\, f(x)\,dx$

Sample proportions

$\hat{P} = \dfrac{X}{n}$		mean	$\mathrm{E}(\hat{P}) = p$
standard deviation	$\mathrm{sd}(\hat{P}) = \sqrt{\dfrac{p(1-p)}{n}}$	approximate confidence interval	$\left(\hat{p} - z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}},\ \hat{p} + z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}\right)$

MATHEMATICAL METHODS

TRIAL EXAMINATION 2

MULTIPLE - CHOICE ANSWER SHEET

STUDENT NAME:.....

INSTRUCTIONS

Fill in the letter that corresponds to your choice. Example: (A) (■) (C) (D) (E)

The answer selected is B. Only one answer should be selected.

1. (A) (B) (C) (D) (E)
2. (A) (B) (C) (D) (E)
3. (A) (B) (C) (D) (E)
4. (A) (B) (C) (D) (E)
5. (A) (B) (C) (D) (E)
6. (A) (B) (C) (D) (E)
7. (A) (B) (C) (D) (E)
8. (A) (B) (C) (D) (E)
9. (A) (B) (C) (D) (E)
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14. (A) (B) (C) (D) (E)
15. (A) (B) (C) (D) (E)
16. (A) (B) (C) (D) (E)
17. (A) (B) (C) (D) (E)
18. (A) (B) (C) (D) (E)
19. (A) (B) (C) (D) (E)
20. (A) (B) (C) (D) (E)