BILLANOOK COLLEGE

NAME: **Student Number:**

MATHEMATICAL METHODS (CAS) UNITS 3 & 4

Practice July Exam

Exam 1 TECHNOLOGY FREE

Tuesday 17th July, 2018
Reading time: 15 minutes 2:30pm- 2:45pm
Writing time: 1 hour 2:45pm – 3:45pm

QUESTION AND ANSWER BOOKLET

Structure of Booklet

Number of Questions	*Number of questions to be answered*	*Number of marks*
9	9	44

- Students are permitted to bring into the test room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the test room: notes of any kind, a calculator of any type, blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book with a detachable sheet of miscellaneous formulas.

Instructions

- Write your **name** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the test room.

Instructions

Answer **all** questions in the space provided.
In all questions where a numerical answer is required an exact value must be given unless otherwise specified.
In questions where more than one mark is available, appropriate working **must** be shown.

Question 1 (4 marks)

a. Let $f : (-2, \infty) \to R, f(x) = \dfrac{x}{x+2}$.

Differentiate f with respect to x. 2 marks

b. Let $g(x) = (2 - x^3)^3$.

Evaluate $g'(1)$. 2 marks

Question 2 (4 marks)

Let $y = x \log_e(3x)$.

a. Find $\frac{dy}{dx}$. 2 marks

b. Hence, calculate $\int_1^2 (\log_e(3x)+1)dx$. Express your answer in the form $\log_e(a)$, where a is a positive integer. 2 marks

Question 3 (4 marks)

a. Let $y = e^{2x}\cos\left(\frac{x}{2}\right)$.

Find $\frac{dy}{dx}$. 2 marks

b. Let $f:(0, \pi) \to R$, where $f(x) = \log_e(\sin(x))$.

Evaluate $f'\left(\frac{\pi}{3}\right)$. 2 marks

Question 4 (5 marks)

a. Find an antiderivative of $\cos(1-x)$ with respect to x. 1 mark

b. Evaluate $\int_1^2 \left(3x^2 + \frac{4}{x^2}\right) dx$. 2 marks

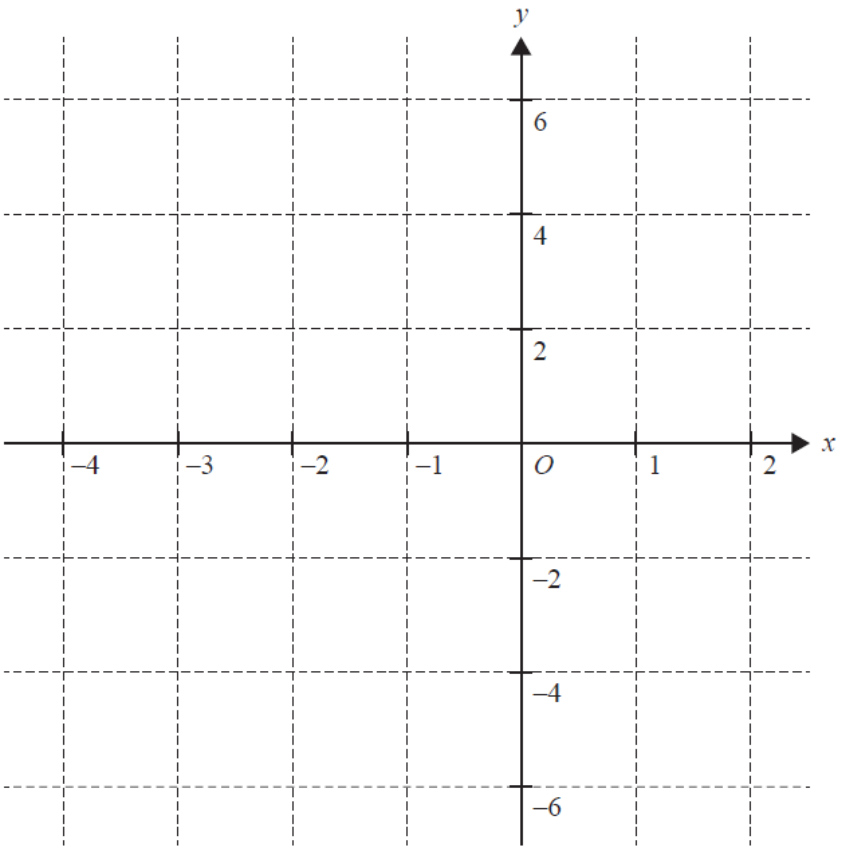
c. Find $f(x)$ given that $f(4) = 25$ and $f'(x) = \frac{3}{8}x^2 - 10x^{-\frac{1}{2}} + 1, x > 0$. 2 marks

Question 5 (4 marks)

Let $f: [-3, 0] \rightarrow R, f(x) = (x + 2)^2(x - 1)$.

a. Show that $(x + 2)^2(x - 1) = x^3 + 3x^2 - 4$. 1 mark

b. Sketch the graph of f on the axes below. Label the axis intercepts and any stationary points with their coordinates. 3 marks



Question 6 (5 marks)

Let $f: [0,\infty) \to R, f(x) = \sqrt{x+1}$.

a. State the range of f. 1 mark

b. Let $g: (-\infty, c] \to R, g(x) = x^2 + 4x + 3$, where $c < 0$.

i. Find the largest possible value of c such that the range of g is a subset of the domain of f. 2 marks

ii. For the value of c found in **part b.i.**, state the range of $f(g(x))$. 1 mark

c. Let $h: R \to R, h(x) = x^2 + 3$.

State the range of $f(h(x))$. 1 mark

Question 7 (3 marks)

a. State the smallest positive value of k such that $x = \frac{3\pi}{4}$ is a solution of $\tan(x) = \cos(kx)$. 1 mark

b. Solve $2\sin^2(x) + 3\sin(x) - 2 = 0$, where $0 \leq x \leq 2\pi$. 2 marks

Question 8 (6 marks)

Let $f:R \to R$, where $f(x) = 2x^3 + 1$, and let $g:R \to R$, where $g(x) = 4 - 2x$.

a. **i.** Find $g(f(x))$. 1 mark

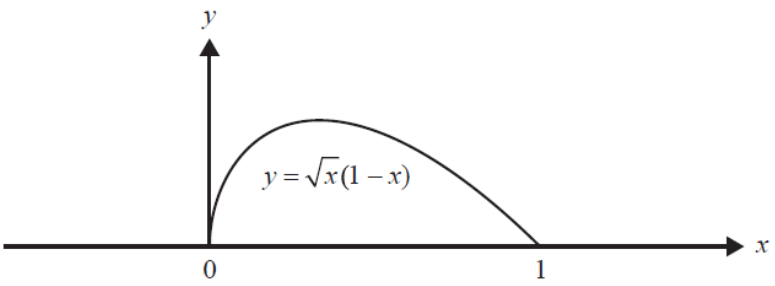
ii. Find $f(g(x))$ and express it in the form $k - m(x - d)^3$, where m, k and d are integers. 2 marks

b. The transformation $T:R^2 \to R^2$ with rule $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 \\ 0 & a \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} b \\ c \end{bmatrix}$, where a, b and c are integers, maps the graph of $y = g(f(x))$ onto the graph of $y = f(g(x))$.

Find the values of a, b and c. 3 marks

Question 9 (9 marks)

The graph of $f : [0,\ 1] \to R, f(x) = \sqrt{x}(1-x)$ is shown below.

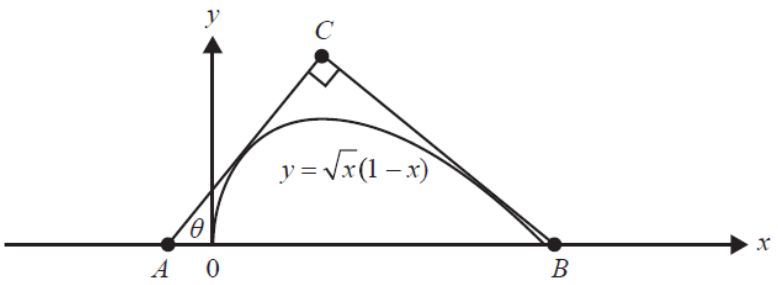


a. Calculate the area between the graph of f and the x-axis. 2 marks

b. For x in the interval (0, 1), show that the gradient of the tangent to the graph of f is $\frac{1-3x}{2\sqrt{x}}$. 1 mark

The edges of the **right-angled** triangle ABC are the line segments AC and BC, which are tangent to the graph of f, and the line segment AB, which is part of the horizontal axis, as shown below.

Let θ be the angle that AC makes with the positive direction of the horizontal axis, where $45° \leq \theta < 90°$.



c. Find the equation of the line through B and C in the form $y = mx + c$, for $\theta = 45°$. 2 marks

d. Find the coordinates of C when $\theta = 45°$. 4 marks

END OF QUESTION BOOKLET