



BILLANOOK COLLEGE

NAME:

Student Number:

MATHEMATICAL METHODS (CAS) UNITS 3 & 4

Practice July Exam

Exam 1 TECHNOLOGY FREE

Friday 21st July, 2017

Reading time: 15 minutes 11:15am- 11:30am

Writing time: 1 hour 11:30am – 12:30pm

QUESTION AND ANSWER BOOKLET

Structure of Booklet

<i>Number of Questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
10	10	40

- Students are permitted to bring into the test room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the test room: notes of any kind, a calculator of any type, blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book with a detachable sheet of miscellaneous formulas.

Instructions

- Write your **name** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the test room.

Instructions

Answer **all** questions in the space provided.

In all questions where a numerical answer is required an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

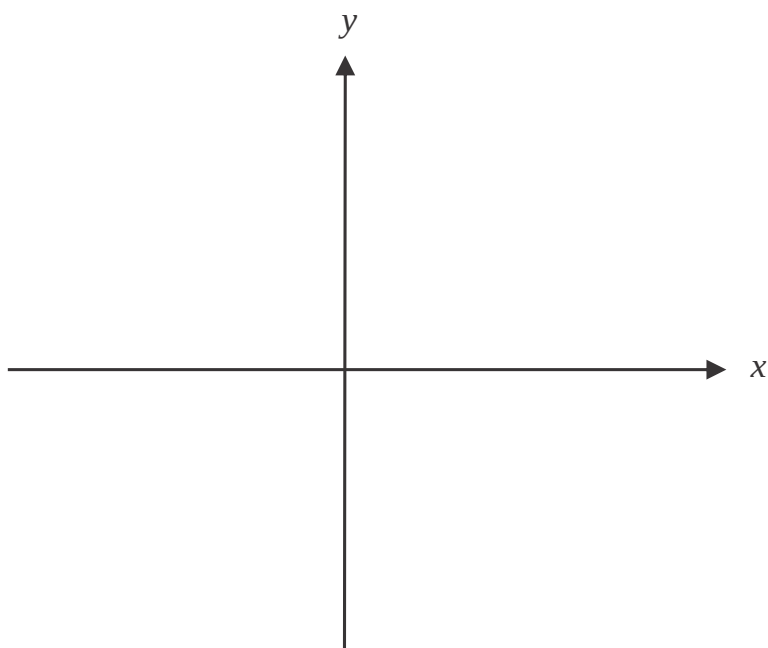
Question 1 (5 marks)

- a.** If $y = x^2 \sin(x)$, find $\frac{dy}{dx}$. 2 marks

- b.** If $f(x) = \sqrt{x^2 + 3}$, find $f'(1)$. 3 marks

Question 2 (3 marks)

Sketch the graph of $f: \mathbb{R} \setminus \{2\} \rightarrow \mathbb{R}$, $f(x) = -1 + \frac{3}{x-2}$ on the set of axes below. Label axes intercepts with their coordinates. Label asymptotes with their equations



Question 3 (2 marks)

Find $\int_1^3 \left(\frac{2}{x} + 1 \right) dx$.

Question 4 (3 marks)

Let $f(x) = \frac{1}{\sqrt{3}} \cos(x)$ and $g(x) = \sin(x)$.

a. Solve the equation $f(x) = g(x)$ for $x \in [0, 2\pi]$.

2 marks

b. Evaluate $f(g(0))$.

1 mark

Question 5 (2 marks)

Solve the following equation:

$$\frac{4000}{2 + 7^{3x}} = 5$$

Question 6 (2 marks)

The tangent to the curve $y = \frac{3}{x} - 2$ at the point $x = a$, where $a > 0$, has a gradient of -9 .

Find the value of a .

Question 7 (3 marks)

Solve the equation $\log_e(x) + \log_e(3x+2) = 2\log_e(x+1)$ for x , where $x > 0$.

Question 8 (9 marks)

Consider the function with the rule $f(x) = \frac{x-2}{x+2}$

- a. Find the rule, f^{-1} , for the inverse of f .

3 marks

- b.** Find the domain and range of the inverse of f .

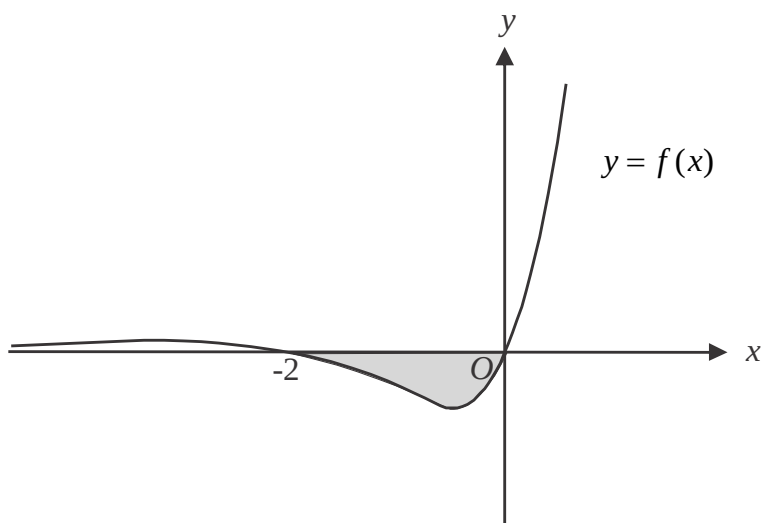
2 marks

- c.** Show that $f^{-1}(x)$ can be written in the form of $a + \frac{b}{1-x}$ and hence find $\int_0^{\frac{1}{2}} f^{-1}(x) dx$

4 marks

Question 9 (4 marks)

The graph of $f: \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = (x^2 + 2x)e^x$ is shown below.



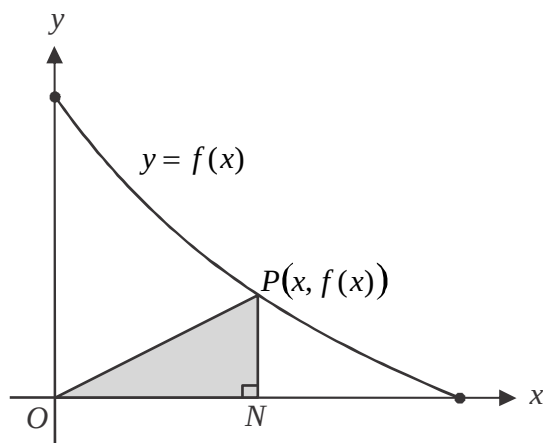
The region enclosed by the graph of f and the x -axis is shaded.

- a.** Find the derivative of $(3 - x^2)e^x$. Give your answer in the form $ae^x - f(x)$, where a is a positive constant. 1 mark

- b.** Use your answer to part **a.** to find the area of the shaded region. 3 marks

Question 10 (7 marks)

Let $f : [0, 1] \rightarrow \mathbb{R}$, $f(x) = 1 - x^{\frac{2}{3}}$. The graph of f is shown below.



The right-angled triangle NOP has vertex N on the x -axis, and vertex O at the origin. The vertex P lies on the graph of f and has coordinates $(x, f(x))$ as shown.

- a.** Find the area A , of triangle NOP in terms of x .

1 mark

b. Find

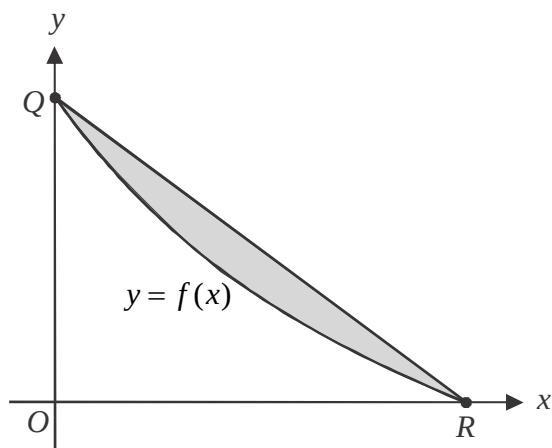
i. the value of x for which A is a maximum.

2 marks

ii. the maximum area of triangle NOP . Give your answer in the form $\frac{a\sqrt{b}}{c}$
where a , b and c are positive integers.

1 mark

- c. The point Q lies on the graph of f and on the y -axis. The point R lies on the graph of f and on the x -axis.



Find the area enclosed by the line segment QR and the graph of f .

3 marks

END OF QUESTION BOOKLET