

Student Name: _____

Home Group: _____

Teacher's name: (please circle): Ms Nation Ms O'Rielly



Mathematical Methods

Unit 2

Wednesday 8th November 2017

Part I

Total 57 marks

- Topics covered:
- Combinatorics
 - Circular Functions
 - Rates of Change
 - Differential Calculus
 - Integral Calculus
 - Exponential Functions and Logarithms

Complete working must be shown and simplified wherever possible in order to gain full marks.

Reading Time: 15 minutes

Writing Time: 60 minutes

Students are NOT permitted to use any calculators or reference books for this section.

No paper or electronic dictionaries may be used.

Useful formulae:

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$${}^nC_r = \frac{n!}{(n-r)!r!} \qquad {}^nP_r = \frac{n!}{(n-r)!}$$

Newton's Iterative formula for approximating roots of a polynomial:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Any question worth more than 1 mark **must have** the appropriate working shown to justify the extra marks.

- 1) A menu offers a choice of five entrees, four mains and three desserts.

Find the number of meal choices possible

a) if one of each must be chosen for a 3 course meal.

b) if you have the choice of not having the dessert if you prefer.

(2 marks)

- 2) The digits 0, 1, 2, 3 and 4 are used to make a 3-digit number. No digit is repeated.

a) How many different 3-digit numbers are possible, if 0 cannot be the first digit?

b) If any of the 3-digit numbers in part a is equally likely to have been made, find the probability that number made is greater than or equal to 230.

(1 + 2 = 3 marks)

- 3) Evaluate $^{100}C_2$

(1 mark)

- 4) In how many ways can four girls be selected for a table tennis team, if seven girls try out?

(2 mark)

5) Find the exact values of:

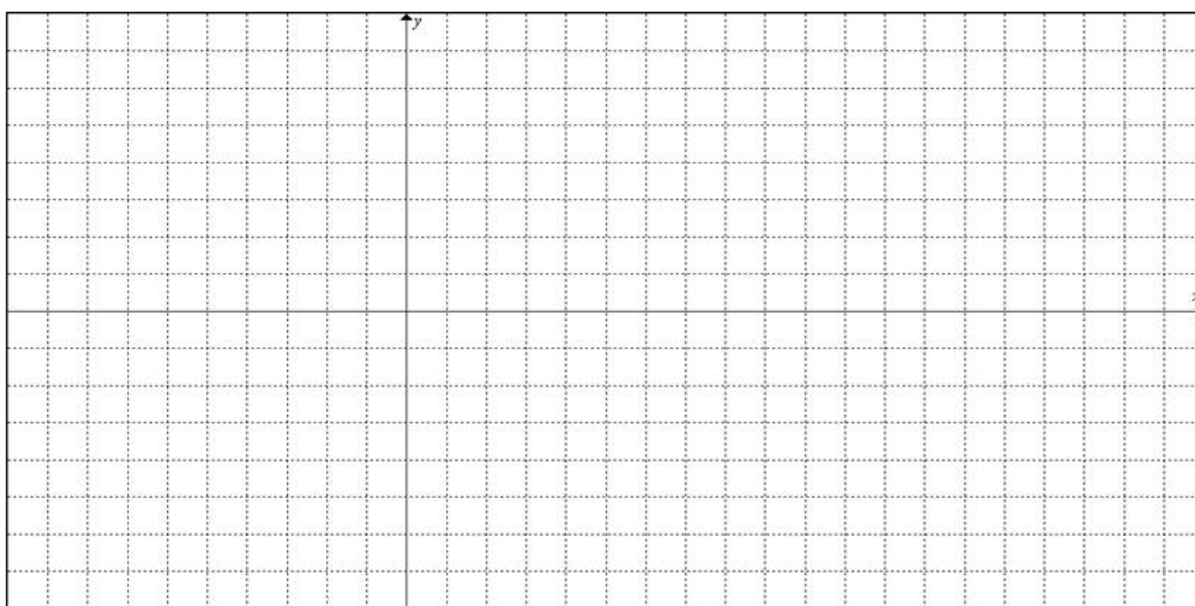
a) $\sin 120^\circ$	b) $\tan \frac{4\pi}{3}$
c) $\sin\left(-\frac{\pi}{6}\right)$	d) $\cos \frac{9\pi}{4}$

(4 marks)

6) a) What is the period and the amplitude of the graph of

$$y = -3\sin 2x?$$

b) Sketch the graph, showing one complete cycle. Clearly label key points.



(2 + 3 = 5 marks)

7) Solve the following equation, giving your answer(s) as exact values:

$$\sqrt{2} \sin x + 1 = 0, \quad 0 \leq x \leq 4\pi$$

(3 marks)

8) If $\sin \theta = 0.66$, $\cos \theta = 0.75$ and $\tan \theta = 0.87$, write down the value of:

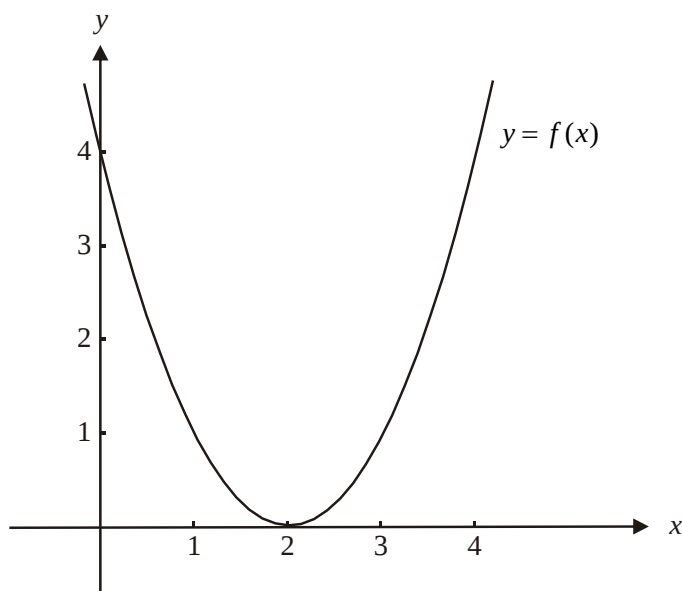
$\sin (2\pi - \theta)$	$\tan (\pi + \theta)$
$\cos (-\theta)$	$\cos (\pi - \theta)$

(4 marks)

9) If Evie drives at 60 km/h for 2 hours and 110 km/h for 3 hours, what is her average speed for the entire journey?

(1 mark)

10) The graph of the function $f: \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = (x-2)^2$ is shown below.



a) Find the average rate of change of $y = f(x)$ with respect to x , between $x = 1$ and $x = 4$.

b) Find the instantaneous rate of change of $y = f(x)$ with respect to x at the point where

i) $x = 2$

ii) $x = 4$

(1 + 1 + 1 = 3 marks)

11) If $f(x) = (x+2)(x+3)$, find $f'(3)$.

(2 marks)

- 12) Find, using first principles, the derivative of
 $f(x) = 3x^2 + x - 2$

(3 marks)

- 13) Find $\lim_{x \rightarrow 3} \frac{x^2 - x - 12}{x^2 - 16}$

(2 marks)

- 14) Evaluate:

a) $\int (4x^3 - x^2 + 9) dx$	b) $\int \frac{3x^4 + 5x^3}{2x} dx$
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(1 + 2 = 3 marks)

15) A particle moves in a straight line with velocity of $v(t) = 6t^2 - 4t$ (m/s) at time t seconds ($t \geq 0$). The particle has an initial position $x(t)$ of 3m left of the origin, O.

a) Find the equation of the position of the particle, $x(t)$

b) Find the acceleration of the particle at $t = 2$ seconds

(2 + 2 = 4 marks)

16) Simplify these expressions using appropriate index or logarithm laws:

a) $\frac{25^{x+3} \times 5^{6x}}{125^{2x-1}}$	b) $\frac{(2x^4y^{-3})^3}{2(x^{-3}y^2)^2}$	c) $2\log_{10} 5 + \log_{10} 4$
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(3 x 2 = 6 marks)

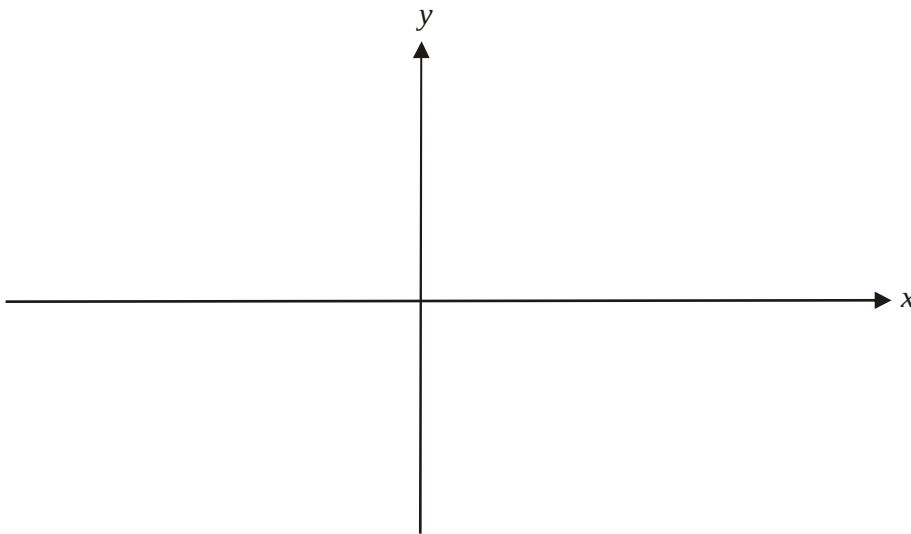
17) Solve the following equations for x :

a) $9^{2x} = 27^{2x-4}$

b) $\log_2(3x - 5) = 4$

(2 x 2 = 4 marks)

18) a) Sketch the graph of the function $y = 10^{-x} + 1$, $x \in \mathbb{R}$ on the set of axes below. Indicate clearly on the graph any intercepts or asymptotes.



b) What is the range of this function?

(4 + 1 = 5 marks)

END OF EXAM