

St Leonard's College
Melbourne

Year 10A MATHEMATICS EXAMINATION 2016

Paper 2

Question and Answer Booklet

STUDENT NAME: SOLUTIONS

TEACHER(S):

TIME ALLOWED: Reading time 15 minutes

Writing time 60 minutes

INSTRUCTIONS:

A CAS calculator is permitted.

A book of notes is permitted.

- Section A: Multiple Choice Questions
Circle the letter corresponding to the correct alternative on the separate Answer Sheet.
- Section B: Extended Response Questions
Answer in the spaces provided on the exam paper.
Quote answers in simplified form where applicable.

STRUCTURE OF BOOKLET / MARKINGSCHEME

	Number of questions to be answered	Total marks
Section A	13	13
Section B	4	32
TOTAL	15	45

Section A: Multiple Choice

Circle the correct answer.

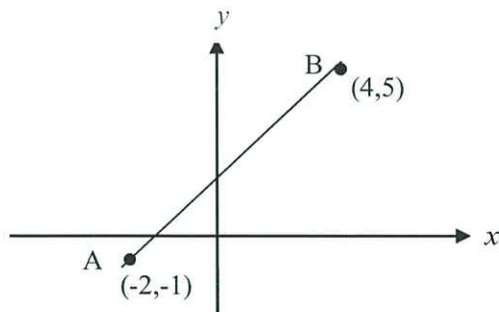
1. The distance between the points $(-5, 1)$ and $(10, 9)$ is:

A. ☒ 17 $\sqrt{15^2 + 8^2}$
 B. ☐ $\sqrt{89}$
 C. ☐ $\sqrt{104}$
 D. ☐ 13
 E. ☐ 23

2. The x intercept of $y - 12 = 3x$ is:

A. ☐ $(4, 0)$
 B. ☐ $(12, 0)$
 C. ☐ $(3, 0)$
 D. ☒ $(-4, 0)$
 E. ☐ $(-12, 0)$

Questions 3 and 4 refer to the following diagram.



3. The gradient of the line AB is:

A. ☐ 6
 B. ☐ 3
 C. ☐ -1
 D. ☒ 1
 E. ☐ 2

4. The equation of line AB is:

A. ☐ $y + x = 1$
 B. ☐ $2y - 2x = -1$
 C. ☒ $y = x + 1$
 D. ☐ $y + x + 1 = 0$
 E. ☐ $y = 2x + 1$

$$\begin{aligned} y &= x + c \\ 5 &= 4 + c \\ c &= 1 \end{aligned}$$

5. The solution to the pair of simultaneous equations $3y + 2x = -8$ and $y - 7x = 2$ is closest to

A. ☐ $(-2.3, -0.6)$
 B. ☒ $(-0.6, -2.3)$
 C. ☐ $(-0.7, -3.2)$
 D. ☐ $(-0.6, -2.4)$
 E. ☐ $(-2.4, -0.6)$

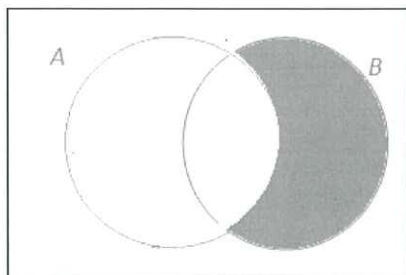
CAS

6. Two fair six-sided dice are rolled. The probability that one shows a 3 and the other shows a 5 is

A. ☒ $\frac{1}{18}$
 B. ☐ $\frac{1}{36}$
 C. ☐ $\frac{1}{8}$
 D. ☐ $\frac{1}{2}$
 E. ☐ $\frac{1}{9}$

$$\frac{2}{36}$$

7. The following venn diagram shows the universal set $\mathcal{E}$ and the sets A and B .



The shaded area of the diagram represents :

- A. $A \cap B$
 B. $A \cup B$
 C. $A \cap B'$
 D. $A' \cap B$
 E. A'

8. From the partly completed Karnaugh map below, $\Pr(A \cap B)$ is

$$\frac{5}{20}$$

- A. 10%
 B. 90%
 C. 20%
 D. 25%
 E. 50%

	A	A'	Total
B	5		
B'	7	2	9
Total	12	8	20

9. For two particular events A and B ,
 $\Pr(A) = 0.2$, $\Pr(B) = p$ and
 $\Pr(A \cup B) = 0.7$

$$\Pr(A \cap B) = \Pr(A) \times \Pr(B)$$

Given that A and B are independent events, then the value of p would be

- A. 0.14
 B. 0.625
 C. 0.9
 D. 0.36
 E. 0.28

$$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$$

$$0.7 = 0.2 + p - 0.2p$$

$$0.5 = 0.8p$$

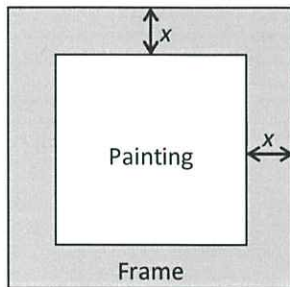
$$p = \frac{5}{8}$$

10. Which **one** of the following equations has no solutions?

- A. $x^2 - 3x + 1 = 0$
 B. $x^2 - 4x + 1 = 0$
 C. $x^2 + x + 1 = 0$
 D. $x^2 + 2x + 1 = 0$
 E. $x^2 + 3x + 1 = 0$

$$b^2 - 4ac < 0$$

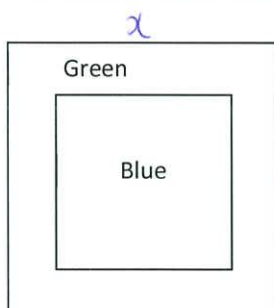
11. A square painting of width 70 cm has a wooden frame of width x around it, as shown below. A fully expanded expression that gives the area of the **wooden frame** in cm^2 is



$$(70 + 2x)^2 - 70 \times 70$$

- A. $4x^2 + 280x + 4900$
 B. $9x^2 + 560x + 9800$
☒ C. $4x^2 + 280x$
 D. $9x^2 + 560x + 4900$
 E. $4x^2 + 70x$

12. Another flag is shown below; a square flag with green and blue on it. The area of the blue part is 100 cm^2 and the area of the green part is 100 cm^2 . The overall width of the flag is therefore exactly



$$x^2 = 200$$

$$x = \sqrt{200}$$

- A. 100 cm
 B. 10 cm
☒ C. $10\sqrt{2} \text{ cm}$
 D. 20 cm
 E. $100\sqrt{2} \text{ cm}$

13. Two numbers a and b are chosen so that no x values make the equation $ax^2 + bx + 3 = 0$ true. A consequence of this is

$$b^2 - 4(a)(3) < 0$$

$$b^2 - 12a < 0$$

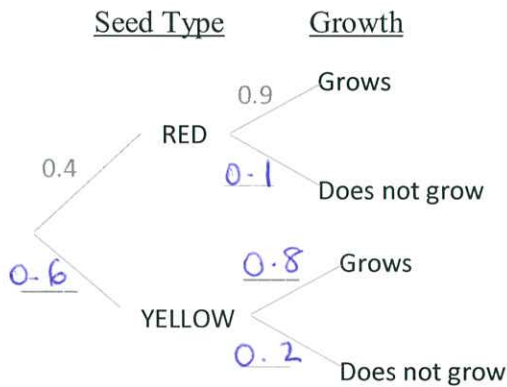
- A. $a^2 - 4b = 0$
 B. $\sqrt{b^2 - a} < 0$
☒ C. $b^2 - 12a < 0$
 D. $b^2 - 4a < 0$
 E. $b^2 - 12a = 0$

Section B: Extended Response:

Show all necessary working and quote answers in simplified form.

1. A packet of seeds contains 40% red seeds and 60% yellow seeds. The probability that a red seed grows is 0.9 and the probability that a yellow seed grows is 0.8. A seed is chosen at random from the packet.

(a) Complete the probability tree diagram



[2 marks]

(b)

- (i) Calculate the probability that the chosen seed is red and grows.

$$0.4 \times 0.9 = 0.36$$

[1 mark]

- (ii) Calculate the probability that the chosen seed grows.

$$0.4 \times 0.9 + 0.6 \times 0.8 = 0.84$$

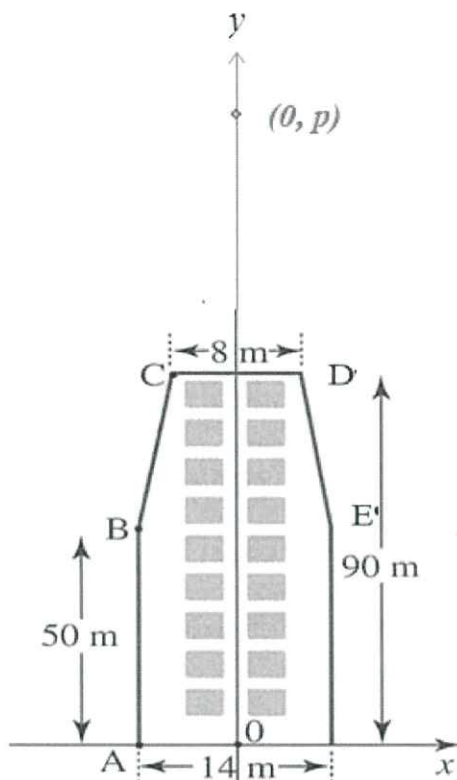
[2 marks]

- (iii) Given that the chosen seed grows, calculate the probability that it is red.

$$P(\text{red} | \text{grows}) = \frac{P(\text{red and grows})}{P(\text{grows})} = \frac{0.36}{0.84} = \frac{3}{7}$$

[2 marks]

2. An architect designed a wonderful new building. A cross-section of the building is shown below with $O, (0,0)$, representing the centre of the base.



- (a) What is the equation of the vertical line connecting A and B?

$$x = -7$$

[1 mark]

- (b) What are the coordinates of:

(i) point B $(-7, 50)$ and

(ii) point C $(-4, 90)$

[2 marks]

- (c) What is the gradient of the line BC?

$$m = \frac{\text{rise}}{\text{run}} = \frac{40}{3}$$

[2 marks]

- (d) The owners of the building decided they would prefer that the external walls, BC and DE, were extended upwards in a straight line to meet at a point, $(0, p)$, as shown on the diagram. Find the value of p , correct to 2 decimal places.

$$\begin{aligned} \text{line BC: } y &= \frac{40}{3}x + c \\ \text{sub } (-4, 90): 90 &= \frac{40}{3}(-4) + c \\ c &= 90 + \frac{160}{3} \\ &= \frac{430}{3} \end{aligned}$$

$$\therefore p = 143.33 \quad [2 \text{ marks}]$$

(to 2 dp)

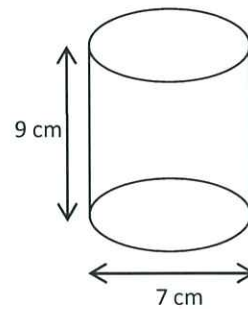
- (e) Hence or otherwise, find the equation of the line DE.

$$\begin{aligned} y &= -\frac{40}{3}x + \frac{430}{3} \\ \text{or } y &= -\frac{40}{3}x + 143.33 \end{aligned}$$

[1 mark]

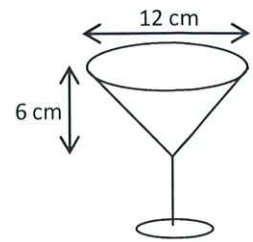
3. Chris and Stewart have gone into the catering business and have been looking at some different glass designs to serve drinks in. To the right are two such designs – a cylindrical glass and a cone-shaped glass.

The glass is of negligible width so the capacity of a design is the same as its volume. (Recall $1 \text{ mL} = 1 \text{ cm}^3$).



Design 1

“Tumbler”



Design 2

“Cocktail glass”

a. Calculate the capacity of each design, correct to the nearest mL.

$$\begin{aligned}
 \text{Design 1: } V &= \pi r^2 h \\
 &= \pi (3.5)^2 (9) \\
 &= 346.36 \dots \text{ cm}^3 \\
 &= \underline{346 \text{ mL}} \text{ to nearest mL}
 \end{aligned}$$

$$\begin{aligned}
 \text{Design 2: } V &= \frac{1}{3} \pi r^2 h \\
 &= \frac{1}{3} \pi (6)^2 (6) \\
 &= 226.19 \dots \text{ cm}^3 \\
 &= \underline{226 \text{ mL}} \text{ to nearest mL}
 \end{aligned}$$

[4 marks]

Whenever an ice block is placed inside a drink the volume of the drink is increased by the volume of the block. For instance, if the glass contained 300 mL of water and an ice block of volume 20 cm^3 was placed in it, the new volume would be 320 mL and the water level in the glass rises accordingly. (You should assume the ice block sinks and does not melt.)

b. (i) The cylindrical tumbler (design 1) above is initially filled to the 5 cm mark. Find the volume of the water in cm^3 (round to two decimal places).

$$\begin{aligned}
 V &= \pi (3.5)^2 \times 5 \\
 &= \underline{192.42 \text{ cm}^3} \text{ (to 2 dec. pl)}
 \end{aligned}$$

[1 mark]

(ii) A cuboid shaped ice block, with dimensions $2 \text{ cm} \times 3 \text{ cm} \times 5 \text{ cm}$, is placed in the drink. Find the combined volume of the drink and ice block in cm^3 (round to two decimal places).

$$\begin{aligned}
 \text{ice block vol} &= 2 \times 3 \times 5 \\
 &= 30 \text{ cm}^3
 \end{aligned}$$

$$\therefore \text{combined volume} = \underline{222.42 \text{ cm}^3} \text{ (to 2 dec. pl)}$$

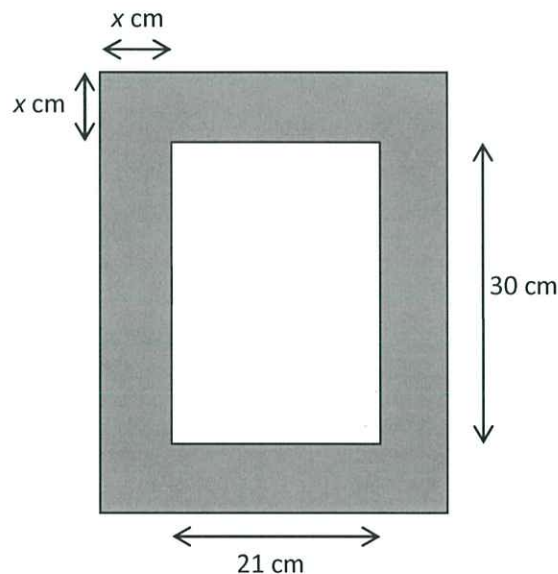
[1 mark]

(iii) Hence, find the new water level in the glass, correct to two decimal places.

$$\begin{aligned}
 222.42 &= \pi (3.5)^2 h \\
 h &= \frac{222.42}{\pi (3.5)^2} \quad \text{or solve on CAS} \\
 &= \underline{5.78 \text{ cm}} \quad (\text{to 2 dec. pl.})
 \end{aligned}$$

[2 marks]

4. Zeke has drawn a picture in child care that his parents wish to frame. The picture itself is on A4 paper with dimensions 21 cm by 30 cm and the framing company allows customers to choose the width (x cm) of the wooden frame.



a. Show clearly that the frame's area $A \text{ cm}^2$ is given by $A = 4x^2 + 102x$.

$$\begin{aligned}
 A_{\text{frame}} &= (30 + 2x)(21 + 2x) - 30 \times 21 \\
 &= 630 + 60x + 42x + 4x^2 - 630 \\
 &= 4x^2 + 102x \quad \text{as required}
 \end{aligned}$$

[3 marks]

b. Initially his parents have ordered a 2 centimetre wide frame. Find the area of this frame.

$$\begin{aligned}
 A_{\text{frame}} &= 4 \times 2^2 + 102 \times 2 \\
 &= \underline{220 \text{ cm}^2}
 \end{aligned}$$

[1 mark]

c. Realising that a 2 cm frame does not do justice to this artwork, they decide that the frame's area should be the same as the picture's area. Find the width of such a frame correct to the nearest millimetre.

$$\text{Solve } 4x^2 + 102x = 30 \times 21$$

$$x = -\frac{30 \times 64}{\text{reject}} \dots \text{ or } x = 5.140 \dots$$

$$\therefore \underline{5.1 \text{ cm}} \text{ to nearest mm}$$

[2 marks]

The framing company charges a fixed cost of \$40 for labour for every framing job, plus an additional 10 cents per square centimetre of wood.

d. (i) How much would the original frame have cost Zeke's parents (with $x = 2$)?

$$40 + 0.1 \times 220 \\ = \underline{\$62}$$

[1 mark]

(ii) To the nearest millimetre, what is the widest frame that Zeke's parents could afford if they are willing to spend up to \$100?

$$\text{Solve } 40 + 0.1(4x^2 + 102x) = 100$$

$$x = -\frac{30 \times 42}{\text{reject}} \dots \text{ or } x = 4.929 \dots$$

$$\therefore \underline{4.9 \text{ cm}} \text{ to nearest mm}$$

[2 marks]

End of Examination Paper 2