



St Leonard's College

Mathematical Methods $\frac{3}{4}$

Calculus – Are you Exam Ready?

Instructions: There are 4 questions in each section:

- Part 1
- Part 2 Multiple Choice
- Part 2 Extended Response

Groups of 3.

The whole group collaboratively does Q1 in each section. Then, Person A does Q2 in each section, Person B does Q3 in each section and Person C does Q4 in each section.

Part 1 – No Calculator Allowed. No reference material.

Question 1

Let $f(x) = \tan\left(\frac{\sqrt{x}}{2}\right)$.

- a. If $f(x) = g(h(x))$ write down, the rules for the functions $g(x)$ and $h(x)$.

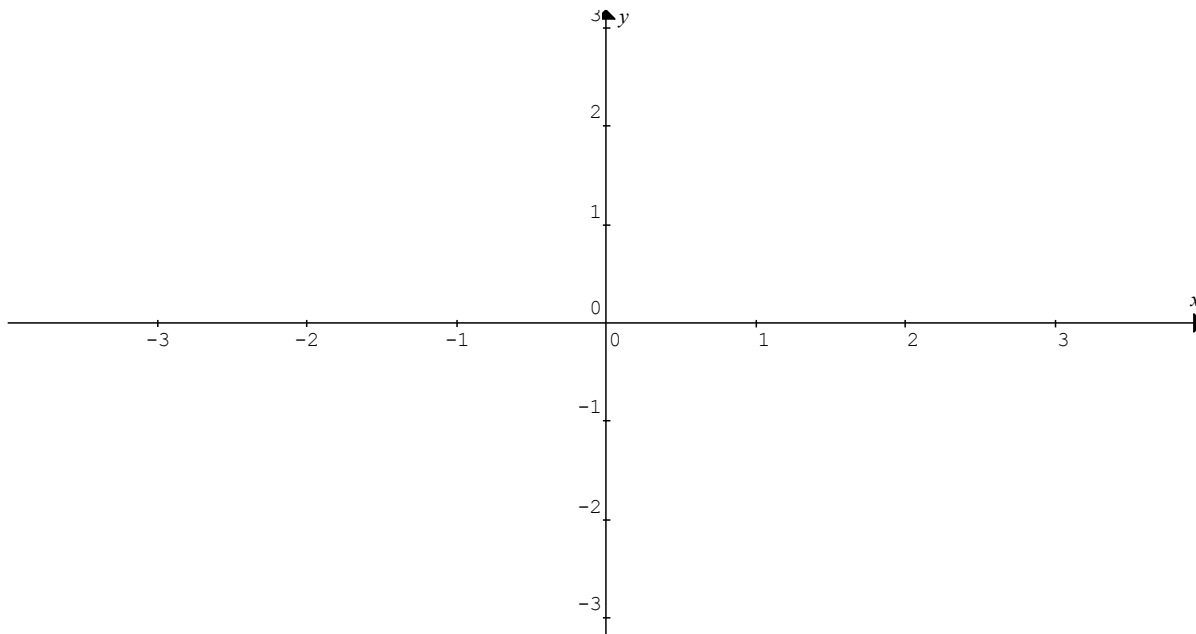
1 mark

- b. Evaluate $f'\left(\frac{\pi^2}{9}\right)$.

3 marks

Question 2

- a.** Sketch the graph of function $y = 1 - \frac{4}{(2x+3)^2}$ on the axes below, clearly indicating all axial cuts and equation of any asymptotes.



2 marks

- b.** Find the area bounded by the curve $y = 1 - \frac{4}{(2x+3)^2}$, the co-ordinates axes and $x = 1$.

3 marks

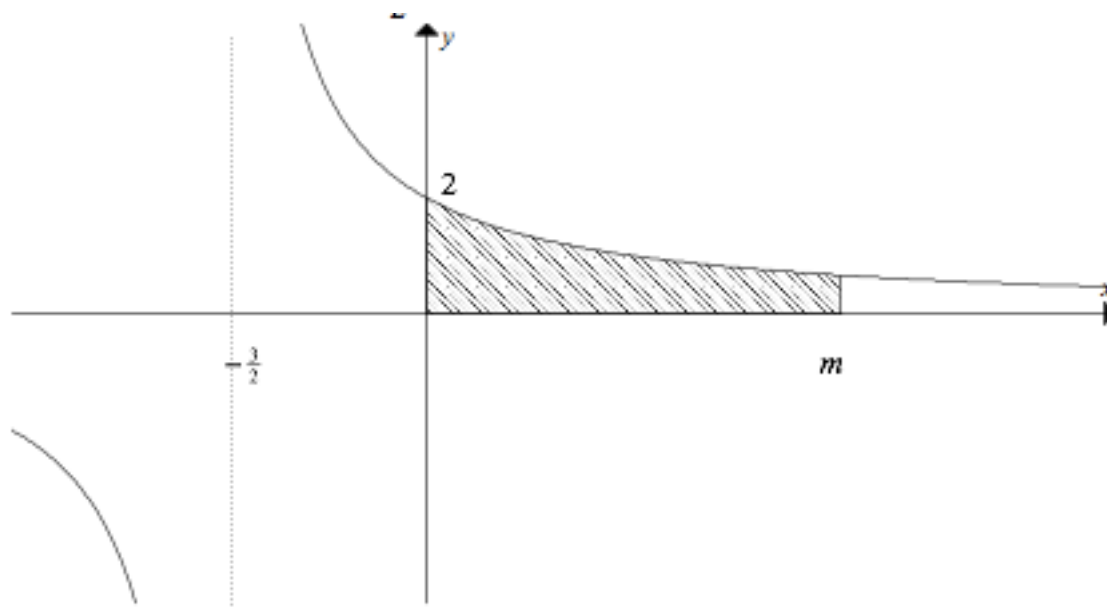
Question 3

Differentiate xe^{-3x} and, hence, find $\int xe^{-3x} dx$.

3 marks

Question 4

Consider the graph of the function $f: R \setminus \{-\frac{3}{2}\} \rightarrow R$, $f(x) = \frac{b}{2x+a}$. The graph has a vertical asymptote at $x = -\frac{3}{2}$ and crosses the y -axis at $y = 2$, as shown below.



The shaded area is the area bounded by the graph of $y = \frac{b}{2x+a}$, the coordinate axes and the line $x = m$. If the shaded area is equal to $\log_e(27)$ square units, find the values of a , b and m .

Part 2 – Calculator Allowed. Reference Material Allowed.

Question 1

If $f(x)$ and $g(x)$ are two differentiable functions with $f'(x) = \frac{d}{dx}(f(x))$ and $g'(x) = \frac{d}{dx}(g(x))$, then $\frac{d}{dx}(f(g(x)))$ is equal to

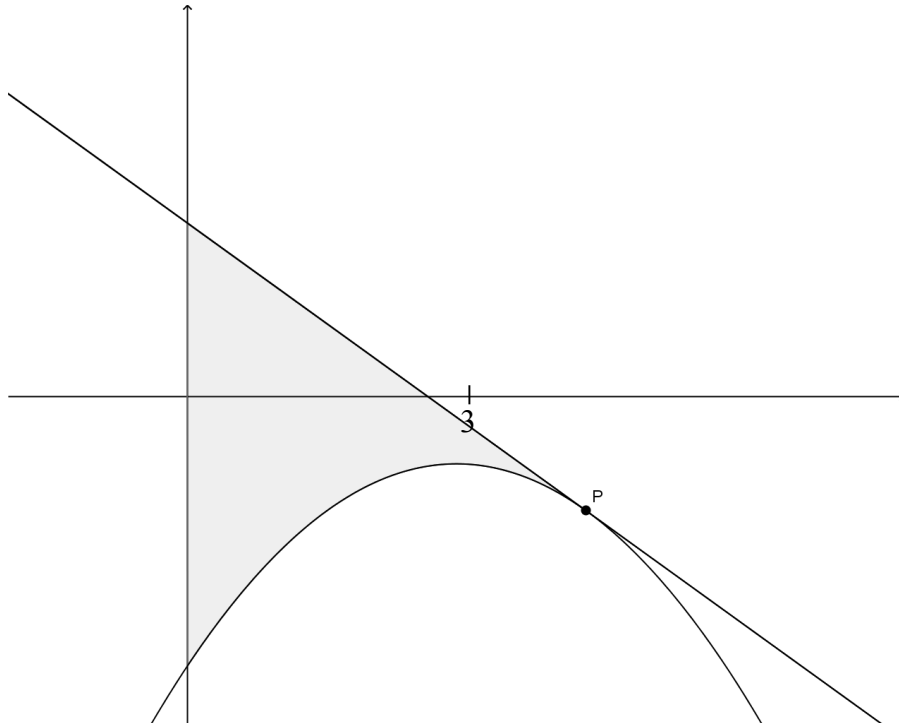
- A. $f'(g(x))$
- B. $f'(g'(x))$
- C. $f'(g(x)) + f'(g'(x))$
- D. $g'(x)f'(g(x))$
- E. $f'(x)g'(x)$

Question 2

The average value of the function with the rule $f(x) = x^3 + e^{2x}$ between $x = 0$ and $x = 2$ is equal to

- A. $\frac{7 + e^4}{2}$
- B. $\frac{7 + e^4}{4}$
- C. $12 + 2e^4$
- D. $5 + e^4$
- E. $\frac{11 + 2e^4}{2}$

Question 3



The diagram above show the graph of $y = -x^2 + 3x - 3$ and the tangent to the graph at the point P , where $x = 3$. The tangent has the equation $y = mx + c$. The shaded area A is the area between the graph, the tangent and the y-axis. Which of the following is **FALSE**?

- A. $c = 6$
- B. $m = -3$
- C. $-3m + 6 = -3$
- D. $m < 0$ and $c > 0$
- E. $A = \int_0^3 (x^2 + (m-3)x + (c+3)) dx$

Question 4

If $f(x) = f(-x)$ and $\int_{-6}^6 f(x) dx = 10$, then $\int_0^6 (2f(x) - 1) dx$ is equal to

- A. 4
- B. 14
- C. 7
- D. $10 - x$
- E. $20 - x$

Extended Response Questions

Question 1

Consider the function $f: R \rightarrow R$, $f(x) = x^3 - 3x^2 + cx + d$, where c and d are real numbers.

- a. The coordinates of the turning point on the graph of $y = f(x)$ are $(-1, 5)$ and (u, v) .
- i. Show that in this case $c = -9$, $d = 0$, $v = -27$ and determine the value of u .

2 marks

- ii. For what values of d does the graph of $y = x^3 - 3x^2 - 9x + d$ cross the x -axis at three distinct points?

2 marks

- b.** For what values of c and d does the graph of $y = x^3 - 3x^2 + cx + d$ have two distinct turning points?

2 marks

- c.** If the graph of $y = f(x)$ is translated p units to the left away from the y -axis, it becomes the graph of $y = x^3$. Find the values of p , c and d in this case.

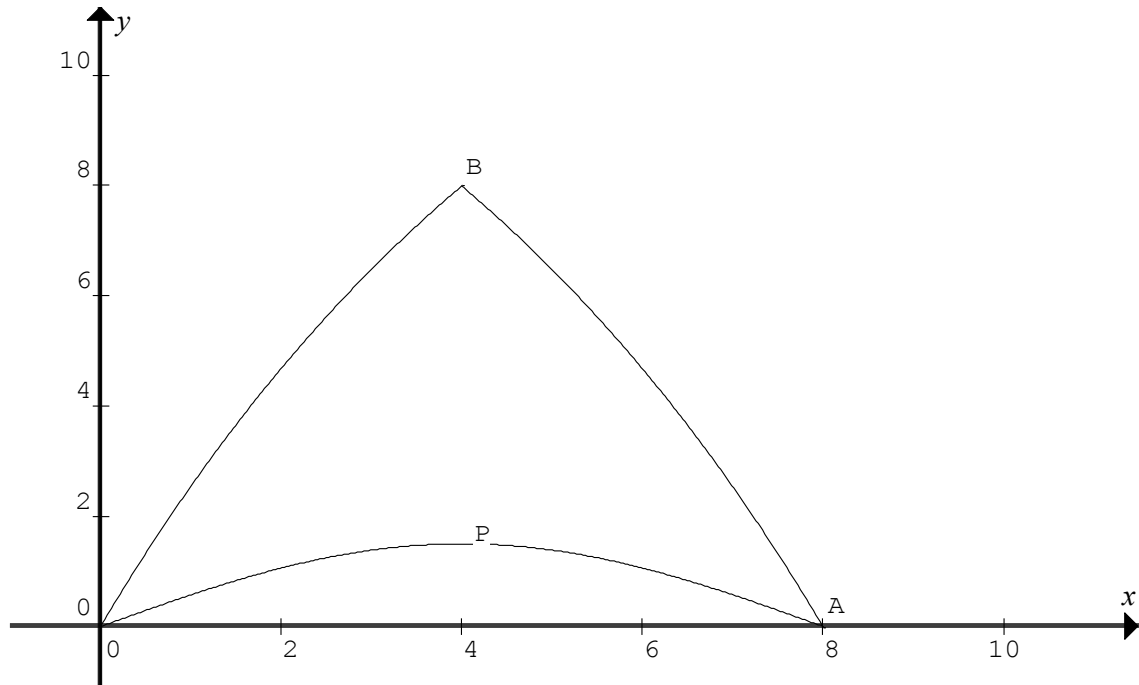
2 marks

- d. Let A be the area bounded by the graph of $y = x^3 - 3x^2 + cx + d$, the coordinate axes and $x = 2$, and that $y \geq 0$ for $0 \leq x \leq 2$. If this area is approximated by four equally spaced left rectangles, the area is 10 square units. If this area is approximated by four equally spaced right rectangles, the area is 6 square units. The exact area bounded by the graph of $y = x^3 - 3x^2 + cx + d$, the coordinate axes and $x = 2$ is 8 square units. Determine the values of c and d .

5 marks
Total 13 marks

Question 2

The diagram below shows a “triangular” shade cloth, which is designed to block the direct sunlight onto a children’s playground. The cloth lies in a horizontal plane and has vertical posts erected at points O, A and B. The point O is the origin and the co-ordinates of the points A, P and B are $(8,0)$, $(4,1.5)$ and $(4,8)$ respectively. The axes are shown on the drawing and the units are in metres.



The curve OPA has the form $y = a \sin(nx)$.

- a. Explain why $a = 1.5$ and $n = \frac{\pi}{8}$.

1 mark

The curve OB has the form $y = 16(1 - e^{-kx})$.

b. Find the value of k .

1 mark

c. The curve BA is the reflection of the curve OB in the line $x = 4$.

i. Write down **two** transformations, which take the curve OB into the curve BA.

1 mark

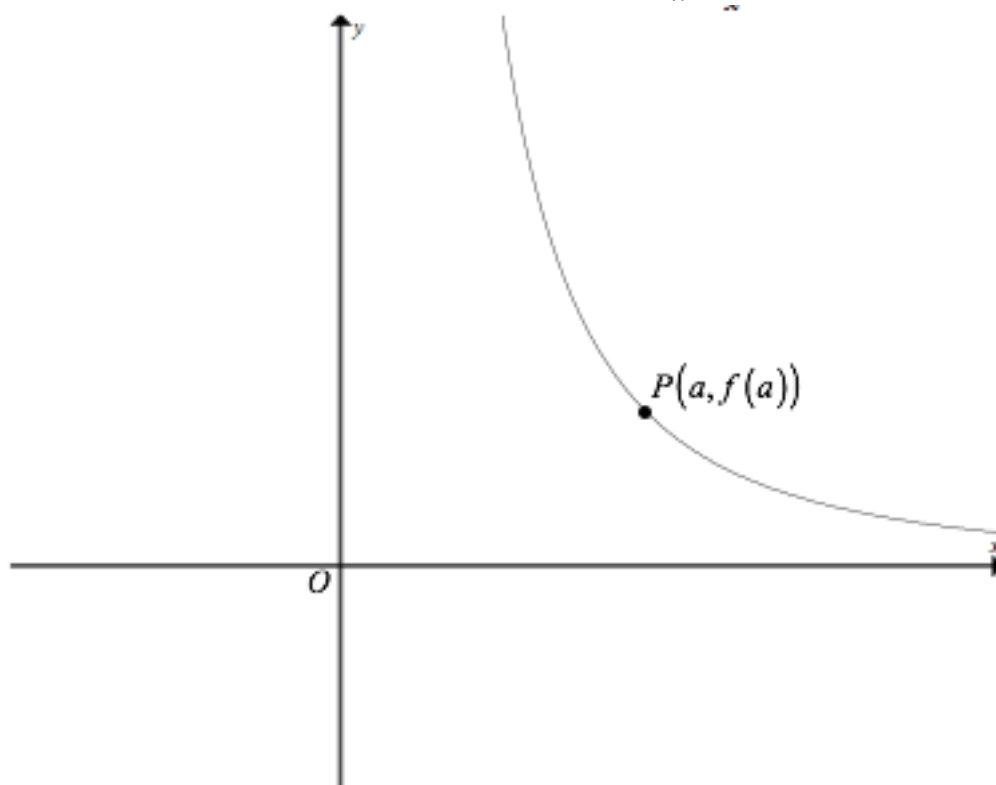
ii. Hence write down a **function** in terms of k , which describes the curve BA.

2 marks

Question 3

The diagram below shows part of the graph of the function $f : (0, \infty) \rightarrow \mathbb{R}$ where $f(x) = \frac{4}{x^2}$.

Let $P(a, f(a))$ where $a > 0$ be a point on the graph of $y = \frac{4}{x^2}$.



- a. Show that the distance s from the origin O to the point P is given by $s = \frac{\sqrt{16 + a^6}}{a^2}$.

2 marks

- b i.** Find, the exact value of a , for which the distance s is a minimum.
Verify that it is a minimum.

3 marks

- ii.** Find the minimum distance, give an exact answer.

1 mark

c.i. Find in terms of a , the equation of the normal to the curve $y = f(x)$ at the point P .

2 marks

ii. Find the value of a , for which the normal passes through the origin.

1 mark

Total 9 marks

Question 4

Given the function $g : \mathbb{R} \rightarrow \mathbb{R}$, $g(x) = \sqrt{3} \sin(2x) + \cos(2x)$

a. Find the general solution of $g(x) = 0$.

1 mark

b. State the smallest positive value of T for which $g(x+T) = g(x)$.

1 mark

Given the function $f : [0, 2\pi] \rightarrow \mathbb{R}$, $f(x) = \sqrt{3} \sin(2x) + \cos(2x)$.

c. Find $\{x : f(x) = 0\}$.

1 mark

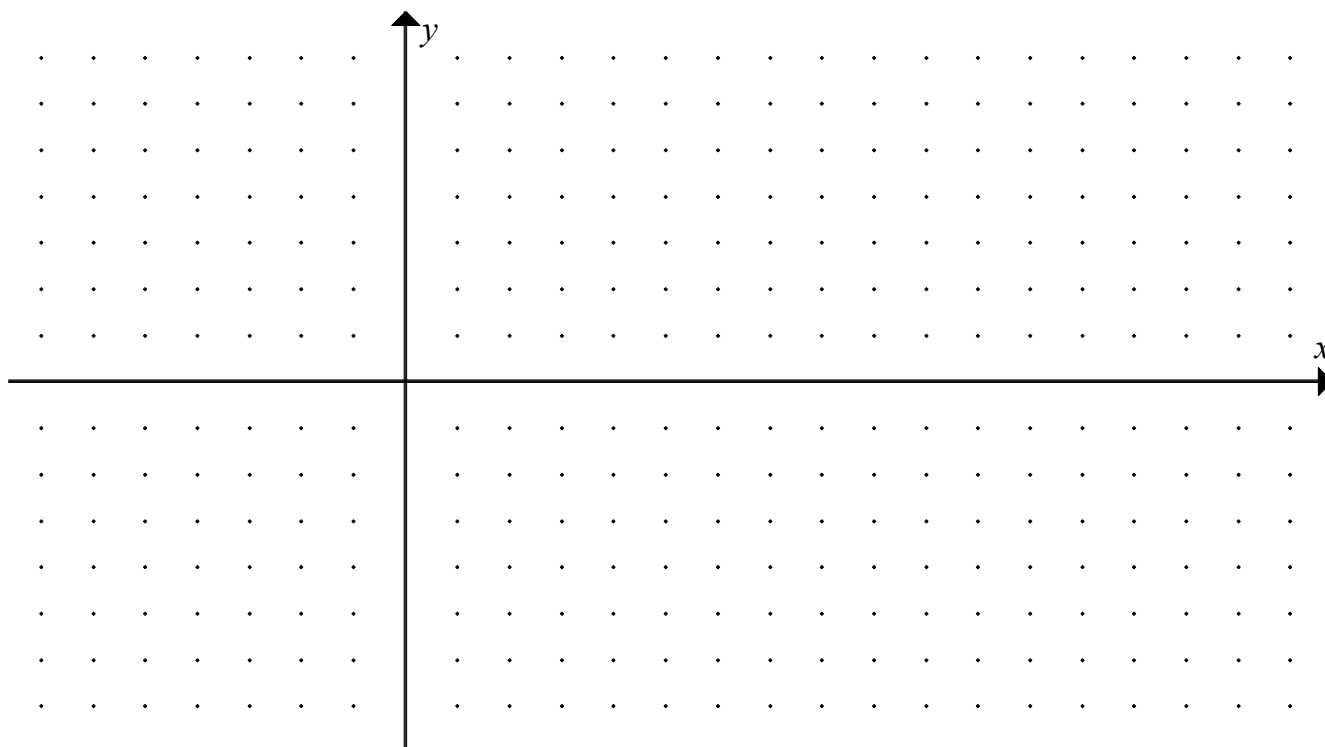
d.i. Find $\{x : f'(x) = 0\}$.

2 marks

- ii. Find the exact coordinates of the maximum and minimum turning points on the graph of $y = f(x)$.

1 mark

- e. Sketch the graph of $y = f(x)$ on the axes below, clearly labelling the scale.



2 marks

- f. If $f(x) = A\sin(2(x+\alpha))$ state the values of A and α .

2 marks
Total 10 marks