

MATHEMATICAL METHODS

Units 3 & 4 – Written examination 1



Test A
Tech Free .

(TSSM's 2015 trial exam updated for the current study design)

SOLUTIONS

Question 1

a. $3 - 5x \geq 0$

$$x \leq \frac{3}{5}$$

$$\text{Domain: } \left(-\infty, \frac{3}{5}\right]$$

A1
1 mark

b. $f'(x) = \frac{1}{2}(3 - 5x)^{-\frac{1}{2}} \times -5$

$$f'(x) = -\frac{5}{2\sqrt{3-5x}}$$

M1+A1
2 marks

c. $f'\left(\frac{1}{5}\right) = -\frac{5}{2\sqrt{3-1}} = -\frac{5}{2\sqrt{2}} = -\frac{5\sqrt{2}}{4}$

A1
1 mark

Question 2

a. $\int \sin(3x) dx = -\frac{\cos(3x)}{3} + c$

$$0 = -\frac{1}{3} + c \text{ which gives } c = \frac{1}{3}$$

$$F(x) = -\frac{\cos(3x)}{3} + \frac{1}{3}$$

M2+A1
3 marks

MATHMETH EXAM 1

b. $-\frac{\cos(3x)}{3} + \frac{1}{3} = \frac{1}{2}$
 $\cos(3x) = -\frac{1}{2}$
 $3x = \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}$
 $x = \frac{2\pi}{9}, \frac{4\pi}{9}, \frac{8\pi}{9}$

M2+A1
3 marks

Question 3

a. $y = \frac{x-2}{x+2}$
 $x = \frac{y-2}{y+2}$
 $yx + 2x = y - 2$
 $y(x - 1) = -2 - 2x$
 $y = \frac{2+2x}{1-x}$
 $f^{-1}(x) = \frac{2+2x}{1-x}$

M2+A1
3 marks

b. Domain: $\mathbb{R} \setminus \{1\}$
Range: $\mathbb{R} \setminus \{-2\}$

A2
2 marks

c. Using long division,

$$f^{-1}(x) = -2 + \frac{4}{1-x}$$

$$\int_0^{\frac{1}{2}} \left(-2 + \frac{4}{1-x}\right) dx = [-2x - 4\log_e(1-x)]_0^{\frac{1}{2}} = -1 - 4\ln\left(\frac{1}{2}\right) = -1 + 4\ln 2$$

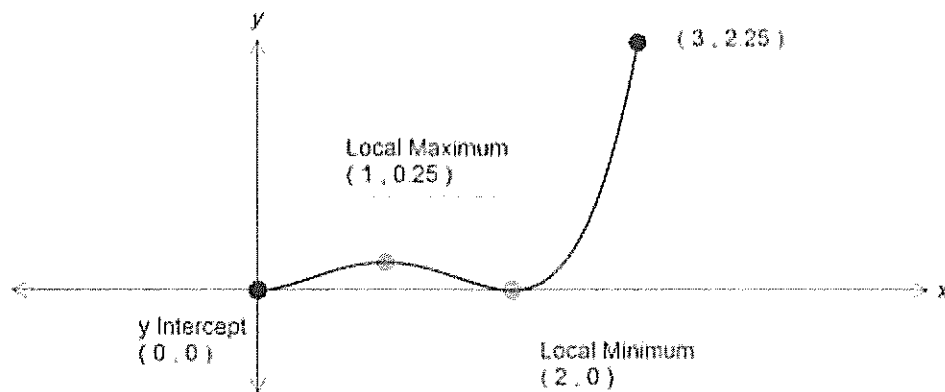
M3+A1
4 marks

Question 4

a. $f'(x) = x^3 - 3x^2 + 2x$
 $f'(x) = 0$ gives $x(x^2 - 3x + 2) = 0$
 $x(x-2)(x-1) = 0$ gives $x = 0, 1, 2$
 $(0, 0), \left(1, \frac{1}{4}\right), (2, 0)$

M2+A1
3 marks

b.



1 for shape, 1 for stationary points, 1 for end points.
3 marks

c.
$$\text{Area} = \int_0^2 \left(\frac{1}{4}x^4 - x^3 + x^2 \right) dx = \left(\frac{x^5}{20} - \frac{x^4}{4} + \frac{x^3}{3} \right)_0^2$$

$$\text{Area} = \frac{8}{5} - 4 + \frac{8}{3} = \frac{4}{15} \text{ square units}$$

M1+A1
2 marks

Question 5

a.
$$4000 = 5(2 + 7^{3x})$$

$$800 = 2 + 7^{3x}$$

$$798 = 7^{3x}$$

$$3x = \log_7(798)$$

$$x = \frac{1}{3} \log_7(798)$$

M1+A1
2 marks

b.
$$2 \times 2^{2x} + 2^x - 1 = 0$$

$$2y^2 + y - 1 = 0, \text{ where } y = 2^x$$

$$(2y - 1)(y + 1) = 0$$

$$y = \frac{1}{2}, -1$$

$$2^x = \frac{1}{2}, 2^x = -1$$

$$x = -1 \text{ } (2^x = -1 \text{ has no solution})$$

M2+A1
3 marks

Question 6

a. $\frac{1}{5} + \frac{1}{10} + \frac{1}{3} + k = 1$
 $k = \frac{11}{30}$

A1
1 mark

b. $\Pr(X < 2) = \frac{1}{5} + \frac{1}{3} = \frac{8}{15}$

A1
1 mark

c. Mean = $\sum x\Pr(X = x) = 0 + \frac{1}{3} + \frac{1}{5} + \frac{11}{10} = \frac{49}{30}$

M1+A1
2 marks

Question 7 (6)

$$\frac{dy}{dx} = -\frac{3}{x^2}$$

$$\text{grad of tangent} = -\frac{3}{a^2}$$

$$-\frac{3}{a^2} = -9$$

$$a = \pm \frac{1}{\sqrt{3}}$$

$$a = \frac{\sqrt{3}}{3}$$

M1+A1
2 marks

Question 8

a. $\hat{p} = 0.9$

A1
1 mark

b. $M = 1.96 \sqrt{\frac{0.9 \times 0.1}{r}}$

If you double r

$$M = 1.96 \sqrt{\frac{0.9 \times 0.1}{2r}}$$

Margin of error will decrease by a factor of $\sqrt{2}$

A1
1 mark

MATHEMATICAL METHODS

Units 3 & 4 – Written examination 2



Test B
Tech Active.

(TSSM's 2015 trial exam updated for the current study design)

SOLUTIONS

SECTION 1: Multiple-choice questions (1 mark each)

Question 1

Answer: A

Explanation:

Solve the two equations on CAS.

Question 2

Answer: C

Explanation:

It is negative cubic so either C or D. Check the x-intercept.

Question 3

Answer: E

Explanation:

Define the functions on CAS and find $f(g(x))$

MATHMETH EXAM 2

Question 4

Answer: D

Explanation:

$$f(x) = 2\left(\sqrt{x} + \frac{1}{2}\right)$$

$$g(x) = 2 \times \frac{1}{2}\left(\sqrt{x} + \frac{1}{2}\right)$$

Question 5

Answer: C

Explanation:

Domain: $4 - x \geq 0$ gives $x \leq 4$ and the graph is above the x-axis.

Question 6

Answer: A

Explanation:

$$Av ROC = \frac{f(8) - f(2)}{8 - 2}$$

Question 7

Answer: C

Explanation:

Note the shaded end-points.

Question 8

Answer: ~~C~~ B

Explanation:

$$f(g(x)) = \frac{3}{x+5}, x \neq -2 \quad \checkmark$$

$\therefore R \setminus \{-2\}$

MATHMETH EXAM 2

Question 9

Answer: E

Explanation:

Eliminate incorrect options

Question 10

Answer: D

Explanation:

$$\text{Amp} = 2, \text{ Period} = \frac{2\pi}{\frac{1}{5}}.$$

Question 11

Answer: E

Explanation:

$$\frac{dy}{dx} \text{ at } x = 4 \text{ on CAS.}$$

Question 12

Answer: B

Explanation:

$$A_1 = A_2$$

Question 13

Answer: B

Explanation:

$$\text{normalline}(f(x), x = 0) \text{ on CAS.}$$

MATHMETH EXAM 2

Question 14

Answer: C

Explanation:

$$(f(x))^2 \times (f(y))^2 = e^{2x} \times e^{2y} = e^{2x+2y} = f(2x+2y)$$

Question 15

Answer: A

Explanation:

$$\frac{1}{k} \int_0^k x^3 dx = 9 \text{ gives } k = 6^{\frac{2}{3}} \text{ on CAS.}$$

Question 16

Answer: B

Explanation:

$$\text{binompdf}\left(10, \frac{1}{5}, 6\right)$$

Question 17

Answer: C

Explanation:

$$\text{normcdf}(165, 170, 165, 7.62).$$

Question 18

Answer: A

Explanation:

$$\text{binomcdf}(6, 0.2, 5, 6) \text{ on CAS.}$$

MATHMETH EXAM 2

Question 19

Answer: D

Explanation:

50th percentile means she is on average, due to the symmetry of the normal distribution

Question 20

Answer: C

Explanation:

Sketch on CAS and read the maximum value.

Question 21

Answer: C

Explanation:

$$k = 0.2, E(X) = 3.9$$

Question 22

Answer: B

Explanation:

$$\frac{\pi}{n} = 3 \text{ gives } n = \frac{\pi}{3}$$

MATHMETH EXAM 2

SECTION 2: Analysis Questions

Question 1

a. $r = l \sin \alpha, h = l \cos \alpha$

A2
2 marks

b. $V = \frac{1}{3} \pi r^2 h = \frac{\pi}{3} (l \sin \alpha)^2 (l \cos \alpha) = \frac{\pi}{3} l^3 \sin^2 \alpha \cos \alpha$

M1
1 mark

c. $V'(\alpha) = \frac{\pi}{3} l^3 (\sin^2 \alpha \times -\sin \alpha + \cos \alpha \times 2 \sin \alpha \cos \alpha) = 0$

$\sin \alpha (-\sin^2 \alpha + 2 \cos^2 \alpha) = 0$

$\sin \alpha = 0, \tan^2 \alpha = 2$

$\alpha = 0, \alpha = \pm \tan^{-1} \sqrt{2}$

$\alpha = \tan^{-1} \sqrt{2}, V(\alpha) = \frac{2\sqrt{3}}{27} \pi l^3$

$(\tan^{-1} \sqrt{2}, \frac{2\sqrt{3}}{27} \pi l^3)$

Alternate form: $(\cos^{-1} \frac{\sqrt{3}}{3}, \frac{2\sqrt{3}}{27} \pi l^3)$ also correct

M3+A1
4 marks

~~d. $\alpha = \tan^{-1} \sqrt{2}$ is a point of maximum volume.~~

~~Max volume = $\frac{2\sqrt{3}}{27} \pi \times 6^3 = 16\sqrt{3} \pi \text{ cm}^3$.~~

~~M1+A1
2 marks~~

Question 2

a. Period = $\frac{2\pi}{\frac{\pi}{2.2}} = 4.4$ years and Amplitude = 300

A2
2 marks

b. Min = 200, Max = 800

A2
2 marks

MATHMETH EXAM 2

- c. Solve $P(t) = 800$ over $[0, 5]$
 $t = 0.7$. After 8.4 months

M1+A1
 2 marks

- d. Sketch the graph on CAS and read the domain when $P < 300$
 $2.3 < t < 3.5$ and $6.7 < t < 7.9$

M1+A2
 3 marks

- e. Strictly increasing for $t \in [0, 0.7] \cup [2.9, 5]$
 Note that we include endpoints for strictly increasing intervals.

A3
 3 marks

Question 3

- a. Sketch on CAS and read the max: $0.45 \mu\text{g/mL}$

A1
 1 mark

- b. 3.5 minutes

A1
 1 mark

- c. $C(10) = 0.32 \mu\text{g/mL}$

M1+A1
 2 marks

- d. $\frac{C(5) - C(\frac{3}{2})}{5 - \frac{3}{2}} = 0.0115 \frac{\mu\text{g}}{\text{mL}}/\text{minute}$

M1+A1
 2 marks

- e. Solve $\frac{dc}{dt} < 0$ on CAS
 $t > 3.53$ minutes

M1+A1
 2 marks