



Billanook College

NAME:

Answers

Student Number:

MATHEMATICAL METHODS UNIT 3

Thursday 8th March, 2018

Topic Test: Transition, Functions and Transformations and Exponentials
and Logarithmic Functions

Technology and Reference book permitted.

Expected Writing time for this section: 60 minutes

QUESTION AND ANSWER BOOK

Structure of book

Section	Number of Questions	Number of questions to be answered	Number of marks
1	10	10	10
2	7	7	26

- Students are permitted to bring into the TEST room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved CAS calculator (memory DOES NOT need to be cleared) and, if desired, one scientific calculator. For approved computer-based CAS, their full functionality may be used.
- Students are NOT permitted to bring into the SAC blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book.

Instructions

- Write your **name** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorized electronic devices into the test room.

Multiple Choice Answer Sheet

Use a **PENCIL** for **ALL** entries. For each question, shade the box which indicates your answer.

All answers must be completed like **THIS** example:

Marks will **NOT** be deducted for incorrect answers.

A	☒	C	D	E
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NO MARK will be given if more than **ONE** answer is completed for any question.

If you make a mistake, **ERASE** the incorrect answer – **DO NOT** cross it out.

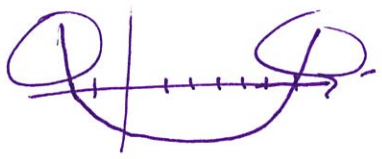
ONE ANSWER PER LINE					
1	☐ A	☒	☐ C	☐ D	☐ E
2	☐ A	☐ B	☐ C	☐ D	☒ E
3	☒ A	☐ B	☐ C	☐ D	☐ E
4	☐ A	☐ B	☐ C	☐ D	☒ E
5	☐ A	☐ B	☒ C	☐ D	☐ E
6	☒ A	☐ B	☐ C	☐ D	☐ E
7	☐ A	☒ B	☐ C	☐ D	☐ E
8	☐ A	☒ B	☐ C	☐ D	☐ E
9	☒ A	☐ B	☐ C	☐ D	☐ E
10	☐ A	☒ B	☐ C	☐ D	☐ E

Section B: Multiple Choice questions: (10 Marks)

Choose the correct answer and shade the relevant box provided on the multiple choice answer page.

1 The maximal domain D of the function $f: D \rightarrow R$ with the rule $f(x) = \log_e(2x + 1)$ is A $R \setminus \{-\frac{1}{2}\}$ B $(-\frac{1}{2}, \infty)$ C R D $(0, \infty)$ E $(-\infty, -\frac{1}{2})$ <i>Handwritten: $2x+1 > 0$ $x > -\frac{1}{2}$</i>	4 If $\log_m p = 3^2$ then p is equal to A m^6 B 9 C $9m$ D $\sqrt{m^3}$ E m^9 <i>Handwritten: $m^9 = p$</i>
2 The function f has rule $f(x) = 3\log_e(2x)$. If $f(5x) = \log_e(y)$ then y is equal to A $30x$ B $6x$ C $125x^3$ D $50x$ E $1000x^3$ <i>Handwritten: $f(5x) = 3\log_e(10x)$ $\log_e y = \log_e(10x)^3$ $y = 1000x^3$</i>	5 $\log_3 7$ could be evaluated as A $\frac{\log 3}{\log 7}$ B $\frac{\log_e 7}{\log_3 e}$ C $\frac{\log_e 7}{\log_e 3}$ D $\log_{10} 7 - \log_{10} 3$ E $\log_{10} 3^7$ <i>Handwritten: Change of base rule.</i>
3 The transformation that maps the graph of $y = \sqrt{8x^3 + 1}$ onto the graph of $y = \sqrt{x^3 + 1}$ is a A Dilation by a factor of 2 from the y axis B Dilation by a factor of 2 from the x axis C Dilation by a factor of $\frac{1}{2}$ from the x axis D Dilation by a factor of 8 from the y axis E Dilation by a factor of $\frac{1}{2}$ from the y axis <i>Handwritten: $y = \sqrt{8x^3 + 1} \rightarrow y = \sqrt{x^3 + 1}$</i>	6 $\frac{1}{2} \log_{10} 49 - \log_{10} \sqrt{7} + \log_{10} \frac{1}{7}$ simplifies to A $-\frac{1}{2} \log_{10} 7$ B $\log_{10} \sqrt{7}$ C $-\log_{10} \sqrt{7}$ D $\frac{3}{2} \log_{10} 7$ E $\frac{50}{7} - \sqrt{7}$ <i>Handwritten: $\log_{10} \sqrt{49} - \log_{10} \sqrt{7} + \log_{10} \frac{1}{7}$ $\log_{10} 7 - \log_{10} \sqrt{7} + \log_{10} \frac{1}{7}$ $\log_{10} (\frac{7}{\sqrt{7}} \times \frac{1}{7})$</i>

Handwritten:
 $y = \sqrt{(2x)^3 + 1} \rightarrow y = \sqrt{(\frac{2x}{2})^3 + 1} = \log_{10} (\frac{7}{\sqrt{7}})$
 $= \log_{10} 7^{-1/2} \rightarrow -\frac{1}{2} \log_{10} 7$

7	The maximum value of $y = 2.6e^{0.3t}$ for $t \in [-1, 4]$ is approximately: A 4 B 8.63 C 1.93 D 2.6 E 0.78 <i>Increasing function. Max occurs when $t = 4$. $y_{\max} = 2.6e^{0.3(4)} = 8.63$.</i>
8	The graph of $y = kx - 4$ intersects the graph of $y = x^2 + 2x$ at two distinct points for A $k = 6$ B $k > 6$ or $k < -2$ C $-2 \leq k \leq 6$ D $6 - 2\sqrt{3} \leq k \leq 6 + 2\sqrt{3}$ E $k = -2$ <i>$\Delta = b^2 - 4ac$ $x^2 + 2x = kx - 4$ $= (2-k)^2 - 16$ $x^2 + (2-k)x + 4 = 0$ $= k^2 - 4k - 12$ $= (k-6)(k+2)$ $\Delta > 0$</i> 
9	If $5^{x-1} = 10$ then x can be found by evaluating A $\frac{1}{\log_{10} 5} + 1$ B $\frac{1}{\log_e 5} + 1$ C $\log_5 10 - 1$ D $1 + \log_{10} 5$ E $\log_e 5 + \log_e 10$ <i>$\log_{10} 5^{x-1} = \log_{10} 10$ $x-1 \in (\log_{10} 5) = \log_{10} 10$ (1) $x-1 = \frac{1}{\log_{10} 5}$ $x = \frac{1}{\log_{10} 5} + 1$</i>
10	Let $g(x) = \log_2(x)$, $x > 0$. Which one of the following equations is true for all positive real values of x? A $2g(8x) = g(x^2) + 8$ B $2g(8x) = g(x^2) + 6$ C $2g(8x) = [g(x) + 8]^2$ D $2g(8x) = g(2x) + 6$ E $2g(8x) = g(2x) + 64$ <i>$2g(8x) = 2\log_2(8x)$ $= \log_2(2^6 x^2)$ $= 6 + \log_2 x^2$ $= g(x^2) + 6$.</i>

Short Answer Section:

Any question worth more than 1 mark must have the appropriate working shown to justify the extra marks.

1. Solve $2\log_e(x-2) - \log_e(x) = 0$, where $x > 2$

$$\log_e(x-2)^2 - \log_e x = 0$$
$$\log_e \left(\frac{(x-2)^2}{x} \right) = 0$$

$$\boxed{x=4}$$

1 marks

2. If $\log_{10}(y) = -2 + 4\log_{10}(x) - \log_{10}(y^2)$, **show** that $y = \sqrt[3]{\frac{x^4}{100}}$.

$$\log_{10}(y) = \cancel{\log} -2 + 4\log_{10}(x) - 2\log_{10}(y)$$

$$3\log_{10}(y) = -\log_{10}(100) + 4\log_{10}(x)$$

$$\log_{10} y^3 = \log_{10} \left(\frac{x^4}{100} \right)$$

$$y^3 = \frac{x^4}{100} \Rightarrow y = \sqrt[3]{\frac{x^4}{100}}$$

3 marks

3. State the gradient of a line perpendicular to the line that passes through the points (3,0) and (0,-6).

$$m = \frac{-6-0}{0-3} = \frac{-6}{-3} = 2$$

$$\boxed{m_{\perp} = -\frac{1}{2}}$$

2 marks

4. The transformation $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is defined by

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -1 \\ 4 \end{bmatrix}$$

The image of the curve $y = 2x^2 + 1$ under the transformation T has equation $y = ax^2 + bx + c$. Find the values of a , b and c .

$$x' = 3x - 1 \Rightarrow x = \frac{x' + 1}{3}$$

$$y' = 2y + 4 \Rightarrow y = \frac{y' - 4}{2}$$

$$y = 2x^2 + 1 \text{ becomes } \frac{y' - 4}{2} = 2\left(\frac{x' + 1}{3}\right)^2 + 1$$

$$y' = \frac{4}{9}(x')^2 + \frac{8}{9}(x') + \frac{4}{9}$$

$$a = \frac{4}{9} \quad b = \frac{8}{9} \quad c = \frac{58}{9}$$

3 marks

- 5 If $\log_e(y) = \log_e(x) - \log_e(p)$, write an equation relating x , y and p that **does not** involve logarithms.

$$\log_e(y) = \log_e(x) - \log_e(p)$$

$$\log_e(y) = \log_e\left(\frac{x}{p}\right)$$

$$\boxed{y = \frac{x}{p}}$$

2 marks

6. A species of small fish is being farmed in the sea to keep up supplies for the exporting of seafood to other countries. The number of fish, F , is increasing according to the rule: $F = 5000 \times 1.5^t$ where t is the number of years since the farm started

- a) How many fish were there initially?

$$\text{When } t=0 \quad F = 5000 \times 1.5^0 \\ = 5000 \text{ fish.}$$

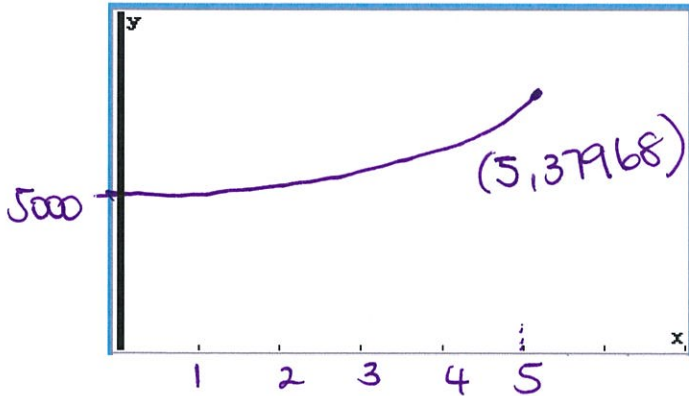
1 mark

- b) How many fish were there after 2 years?

$$t=2 \quad F = 5000 \times 1.5^2 \\ = 11250.$$

1 mark

- c) Sketch the graph of F against t for the first 5 years.



- d) How many years did it take for the number of fish to double? (to 1 decimal place) 3 marks

When $F = 10,000$

$$10000 = 5000 \times 1.5^t$$

$$2 = 1.5^t$$

$$t = 1.7 \text{ years.}$$

1 marks

- e) In which month between $t = 5$ and $t = 6$ did the number of fish reach 50000?

$$50,000 = 5000 \times 1.5^t$$

$$t = 5.6789$$

$\therefore$ in September.

2 marks

7. A go-kart racing track has 2 different tracks that can be modelled by the function

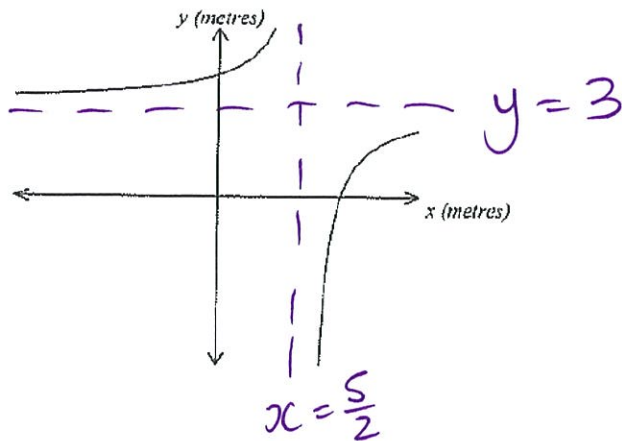
$$f: D \rightarrow R, f(x) = \frac{-5}{2x-5} + 3 \text{ where } x \text{ and } f(x) \text{ are in meters.}$$

- a) State the largest possible domain of D

$$\text{Dom } f = R \setminus \left\{ \frac{5}{2} \right\}$$

1 marks

The tracks are sketched below:



b) i) Give the equations of the asymptotes and sketch these above.

$$x = \frac{5}{2} \quad y = 3.$$

ii) Give the coordinates of axis intercepts of $f(x)$.

When $y = 0$.

$$0 = \frac{-5}{2x-5} + 3$$

$$-3 = \frac{-5}{2x-5}$$

$$2x-5 = \frac{5}{3}$$

$$2x = \frac{5}{3} + \frac{15}{3}$$

$$2x = \frac{20}{3}$$

$$x = \frac{20}{6}$$

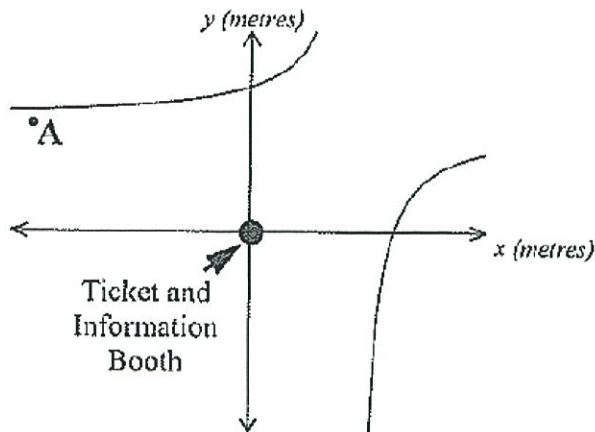
$$x = \frac{10}{3}$$

$$\begin{aligned} \underline{X=0} \\ y &= \frac{-5}{-5} + 3 \\ &= 1 + 3 \\ &= 4 \quad (0, 4) \end{aligned}$$

2 mark

$$\therefore \text{X int} \\ \left(\frac{10}{3}, 0\right)$$

Safety regulations require spectators to be at least one meter away from the track. A ticket and information booth is located at the origin of the graph. A spectator is standing at point A with co-ordinates $(-5,3)$ on the diagram shown.



c) Is the spectator breaking safety regulations? Justify your answer.

yes ↘

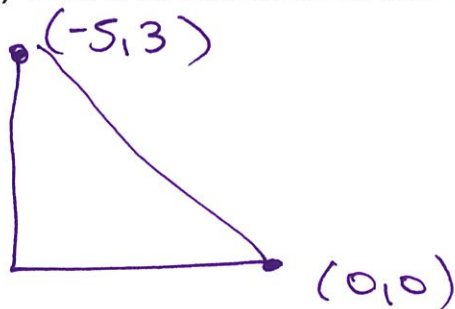
$$\text{When } x = -5 \quad y = \frac{-5}{-15} + 3 \quad \therefore \text{Breaking regulations.}$$

$$= 3\frac{1}{3}$$

$\therefore$ Spectator is $\frac{1}{3}$ metre south of track

2 marks

d) What is the exact distance from the spectator to the Ticket and Information Booth?



2 marks

$$\begin{aligned} \text{distance} &= \sqrt{(0-3)^2 + (0+5)^2} \\ &= \sqrt{9+25} \\ &= \sqrt{34} \text{ metres.} \end{aligned}$$

End of question booklet.