



Student Name

Teacher (circle one) DKI JOR VNA

Homegroup

MATHEMATICAL METHODS (CAS) UNIT 1

EXAMINATION 2

Section	# of questions	# of questions to be answered	Number of marks
A	16	16	16
B	9	9	62
		Total	78

Monday November 7th 2016

Reading Time: 11:45 – 12:00pm (15 minutes)

Writing time: 12:00 – 1:30pm (90 minutes)

Instructions to students

This exam consists of **16** multiple choice and **9** short answer questions.
Answers to the Multiple Choice questions should be recorded on the separate answer sheet.
Section B should be answered in the spaces provided.
There are **78** marks available in this examination.
A decimal approximation will not be accepted if an exact answer is required.
Where more than one mark is allocated to a question working must be shown.
Students **may not** bring any notes or any calculators into this exam.
Students may bring one bound reference book into the exam.
Students may bring a CAS and a Scientific calculator into the exam.
No paper or electronic dictionaries may be used.
Diagrams in this exam are not to scale except where otherwise stated.

FORMULAS

$$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$$

$$\Pr(A \cap B) = \Pr(A) \cdot \Pr(B)$$

$$\Pr(A | B) = \frac{\Pr(A \cap B)}{\Pr(B)}$$

$${}^nC_r = \frac{n!}{(n-r)!r!}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Newton's Iterative formula for approximating roots of a polynomial:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Section A: Multiple Choice. Choose the best answer and RECORD ON ANSWER SHEET.

1 If $\mathbf{A} = \begin{bmatrix} 2 & 7 \\ -5 & 4 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 11 & 3 \\ -2 & 0 \end{bmatrix}$ then $2\mathbf{A}$

+ $3\mathbf{B}$ is equal to

A $\begin{bmatrix} -29 & 5 \\ -4 & 8 \end{bmatrix}$

B $\begin{bmatrix} 13 & 10 \\ -7 & 4 \end{bmatrix}$

C $\begin{bmatrix} 28 & 27 \\ -19 & 12 \end{bmatrix}$

D $\begin{bmatrix} 37 & 23 \\ -4 & 8 \end{bmatrix}$

E $\begin{bmatrix} 37 & 23 \\ -16 & 8 \end{bmatrix}$

2 The determinant of the matrix $\begin{bmatrix} 5 & 3 \\ 1 & 2 \end{bmatrix}$ is

A -7

B -1

C 1

D 7

E 13

3 The matrix which determines the transformation, a dilation from the x -axis of factor 2 followed by reflection in the line $y = x$ is

A $\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$

B $\begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix}$

C $\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

D $\begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}$

E $\begin{bmatrix} 1 & 0 \\ 2 & 0 \end{bmatrix}$

4 The matrix which describes the composition of mappings

- dilation of factor 3 from the x -axis
- reflection in the line $y = x$
- reflection in the x -axis

is

A $\begin{bmatrix} 3 & 0 \\ -1 & 3 \end{bmatrix}$

B $\begin{bmatrix} 0 & 0 \\ -1 & 3 \end{bmatrix}$

C $\begin{bmatrix} 3 & 0 \\ -1 & 0 \end{bmatrix}$

D $\begin{bmatrix} 0 & 0 \\ -3 & 1 \end{bmatrix}$

E $\begin{bmatrix} 0 & 3 \\ -1 & 0 \end{bmatrix}$

5 The exact value of $\cos(-330^\circ)$ is:

A $\frac{1}{2}$

B $-\sqrt{3}$

C $-\frac{\sqrt{3}}{2}$

D $\frac{\sqrt{3}}{2}$

E $\sqrt{3}$

6 If $\sin a = 0.2$, then $\cos a$ must equal:

A 0.80

B 0.96

C 0.98

D $\sqrt{0.96}$

E 0.2

7 The following limits $\lim_{x \rightarrow -3} (3x^2 - 4)$ and

$\lim_{x \rightarrow -3} (4 - 3x^2)$ evaluate respectively to:

- A 23 and 9
- B -23 and 31
- C -31 and 31
- D -23 and 23
- E 23 and -23

8 If $y = -2x^3 + 6x^2 - 4x + 7$, then $\frac{dy}{dx}$ is equal to:

- A $-2x^2 + 6x - 4$
- B $-3x^2 + 2x - 4$
- C $-5x^2 + 8x - 4$
- D $4x^2 + 12x - 4$
- E $-6x^2 + 12x - 4$

9 If $y = x^3 - 2x^2 + 3x - 4$, then the gradient of the *perpendicular* to this curve at $x = -2$ is:

- A $3x^2 - 4x + 3$
- B $\frac{-1}{23}$
- C $\frac{-1}{3x^2 - 4x + 3}$
- D $\frac{1}{23}$
- E -1

10 If a curve passes through the point $(2, -3)$ and at any point has a gradient given by $3x^2 + 2x - 1$, then the y -intercept of the curve must be:

- A -17
- B -13
- C -9
- D -5
- E -1

11 A colony of organisms multiplies at a rate according to $N(t) = 4t^2 + 1$. The average rate of increase between $t = 3$ and $t = 4$ is:

- A 28
- B 37
- C 65
- D 18
- E 38

12 For the missile fired with displacement equation $s(t) = 800 - 16t - 3t^2$, the instantaneous rate of change (velocity) at $t = 3$ is:

- A -16
- B -6
- C 725
- D -34
- E 34

13 The height, h , of a netball is given by the equation $h(x) = -0.1x^2 + 0.9x + 1$ where x is the horizontal displacement of the ball. The horizontal displacement when the ball reaches its maximum height is:

- A 9
- B 4
- C 5
- D 4.7
- E 4.5

14 An arc subtends an angle of 48° at the centre of a circle of radius 8 cm. The length of the arc (in cm) is:

- A 360
- B 6.70
- C 0.68
- D 5.34
- E 53.61

15 The function with the rule $f(x) = 4 \tan\left(\frac{\pi x}{2}\right)$ has period

- A 4
- B $\frac{1}{2}$
- C 2
- D $\frac{\pi}{2}$
- E $\frac{2}{\pi}$

16 A box contains six red blocks and four blue blocks. Three blocks are drawn from the box without replacement.

The probability that all of the blocks are red is

- A $\frac{8}{125}$
- B $\frac{3}{500}$
- C $\frac{24}{29}$
- D $\frac{27}{125}$
- E $\frac{1}{6}$

Section B: Short and Extended response questions. Answer in the space provided.

1. (4 marks)

Evaluate the root of the equation $x^3 = x + 8$ by carrying out two iterations of Newton's Method, starting at $x_0 = 2$. Give your answer to 4 decimal places.

2. (5 marks)

The curve with equation $y = \sin(x)$ is subject to the following transformations;

- a dilation in the y -direction (from the x axis) of 3 units
- a reflection in the x axis

a What is the equation of the transformed graph?

b i) What is the maximum value of y on the transformed curve?

ii) When does this maximum occur (exact answer)?

c What is the exact value of y when $x = \frac{\pi}{3}$? (Show working)

1 + 1 + 1 + 2 = 5 marks

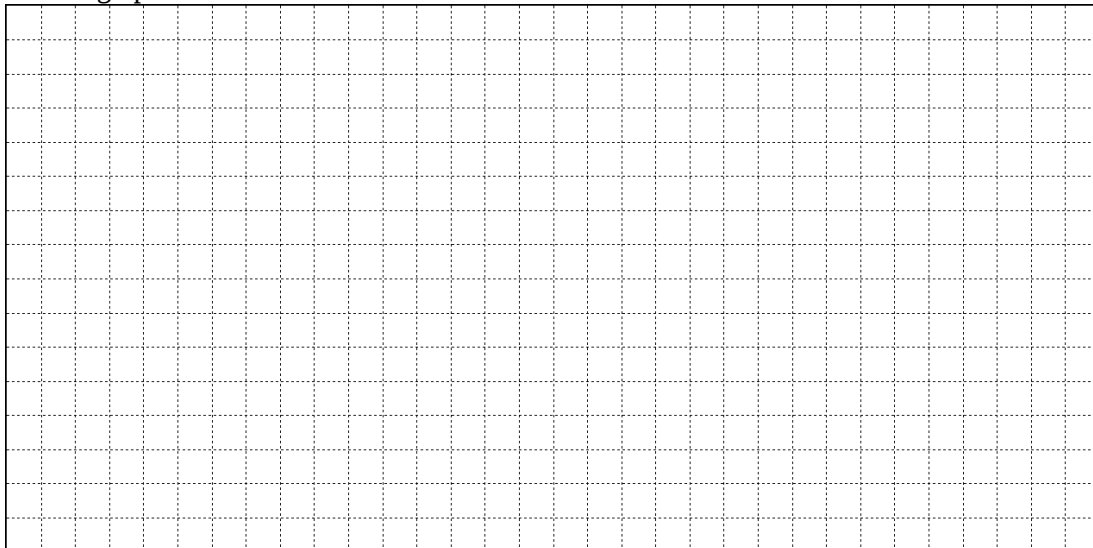
3. (8 marks)

Harry hits a ball; which was resting on flat ground, up into a nearby tree. The ball becomes stuck at a point 5 metres horizontally from where it was hit. The path of the ball is given by

$$y = -x^2 + 8x, \quad x \geq 0, y \geq 0$$

where x is the horizontal distance in metres from the point where the ball rested on the ground and y is the height in metres of the ball above the ground.

- a. Sketch a graph which shows this situation.



2 marks

- b. How high vertically above the ground is the ball when it becomes stuck? (label this on your graph)

1 mark

- c. What is the maximum height that the ball reaches above the ground before it becomes stuck? (label this on your graph)

2 marks

- d. Had the ball not become stuck in the tree, how far horizontally from its starting position would it have landed?

1 mark

- e. During the ball's flight, over what horizontal distance was the ball 15 metres or higher above the ground?

2 marks

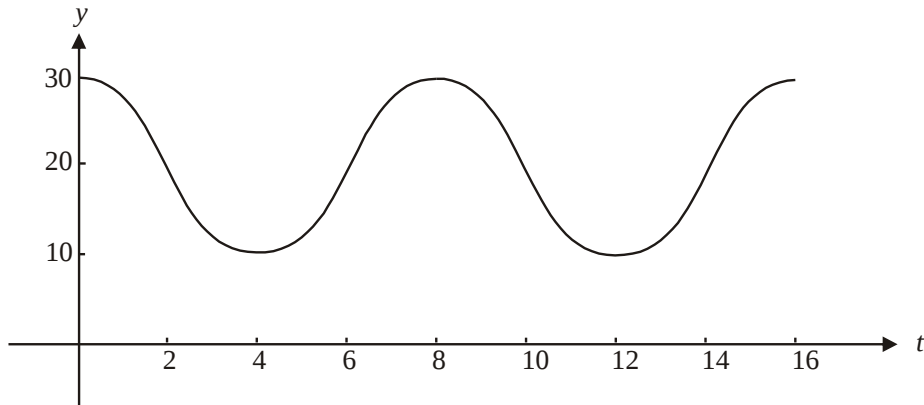
4. (13 marks)

In a laboratory, the temperature in a controlled environment is given by

$$y(t) = a \cos(bt) + c$$

where a , b and c are constants, y represents the temperature in degrees Celsius and t represents the time in hours where $t=0$ corresponds to noon on Wednesday.

The graph of the function is shown below.



a. Explain why

a. $a=10$

b. $b = \frac{\pi}{4}$

c. $c=20$

b. Write down:

1+1+1=3 marks

a. the minimum temperature in the controlled environment.

b. the mean temperature in the controlled environment.

1+1=2 marks

c. Write down the temperature in the controlled environment at 3pm on Wednesday.
Express your answer in degrees Celsius correct to 2 decimal places.

1 mark

- d. At what time did the temperature in the controlled environment first reach 25°C?

2 marks

- e. Hence find for what period of time the temperature is greater than or equal to 25°C between noon and midnight on Wednesday?

3 marks

In a second controlled environment the temperature is given by $p(t) = 8 \cos\left(\frac{\pi t}{4}\right) + 20$, $t \geq 0$ where p represents the temperature in degrees Celsius and t represents the time in hours where $t=0$ corresponds to noon on Wednesday.

- f. When is the temperature in the original controlled environment first one degree higher than in the second controlled environment?
Explain how you found your answer.

2 marks

5. (3 marks)

If $\sin \theta = 0.3$, write down the value of:

a $\sin (\pi - \theta)$	b $\sin (\pi + \theta)$	c $\sin (-\theta)$

6. (9 marks)

A particle moves in a straight line and starts from a fixed point O on the line.

Its acceleration $a \text{ m/s}^2$ is given by $a = 24t - 72$ for any time t seconds, $t \geq 0$.

a What is its acceleration at $t = 5$?

1 mark

b If its initial velocity is 96 m/s , what is its velocity for any time t seconds?

2 marks

c When is its velocity zero?

1 mark

d What is its displacement from O at any time t seconds?

1 mark

e Find the total distance covered by the particle in the first 6 seconds.

4 marks

7. (4 marks)

From a group of 12 female students, 2 female staff, 10 male students and 3 male staff, a committee of 6 is to be formed.

Find the number of different committees if:

- a. there are no restrictions.
- b. All committee members must be students
- c. There is an equal number of males and females on the committee
- d. The committee must comprise of 1 male staff member, 1 female staff member, 2 female students and 2 male students.

8. (14 marks)

In an animated computer game an adventurer can travel from his base to a treasure chest by three different routes. The route is chosen randomly.

The probability of the adventurer travelling on the desert route is $\frac{1}{2}$.

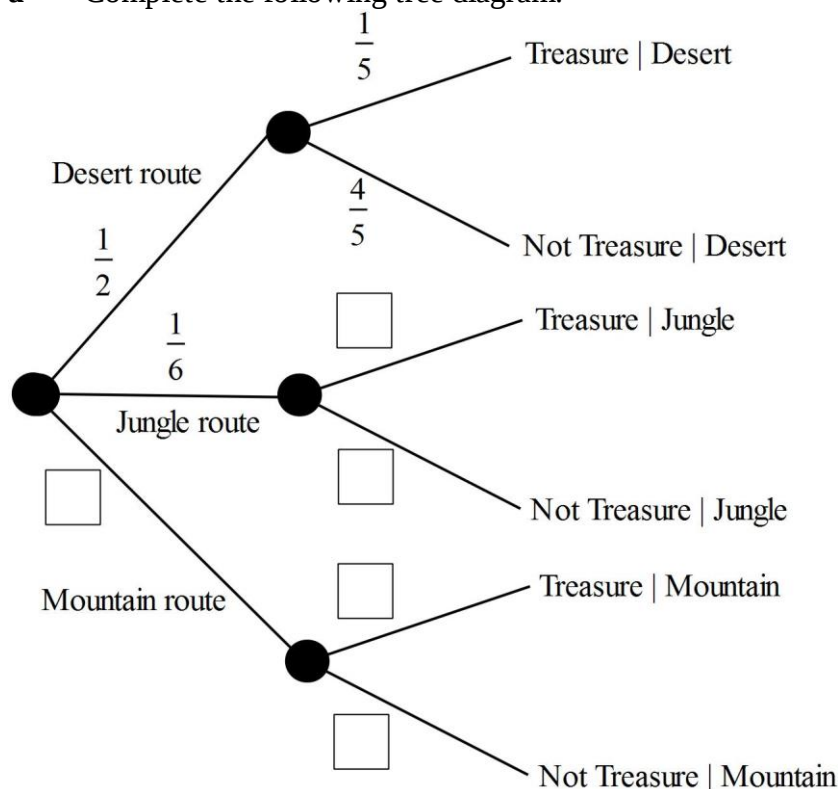
The probability of the adventurer travelling on the jungle route is $\frac{1}{6}$.

The probability of the adventurer travelling on the mountain route is $\frac{1}{3}$.

The probability that the adventurer obtains the treasure by a given route is given in the table below.

Treasure given desert route	Treasure given jungle route	Treasure given mountain route
$\frac{1}{5}$	$\frac{2}{5}$	$\frac{1}{10}$

a Complete the following tree diagram.



3 marks

b Find the probability that the adventurer follows the desert route and obtains the treasure.

1 mark

- c** Find the following probabilities:
- i** the adventurer obtains the treasure

2 marks

- ii** the adventurer does not obtain the treasure

2 marks

- d** Find the probability that, given the adventurer obtains the treasure, he has travelled:
- i** the desert route

2 marks

- ii** the jungle route

2 marks

- iii** the mountain route.

2 marks

9. (2 marks)

Find all possible rational x intercepts of $3x^3 + 4x^2 - 7x + 6$

END OF EXAMINATION

EXAM 2

MULTIPLE CHOICE

ANSWER SHEET

Student Name.....

Teacher (circle one) DKI JOR. VNA

Homegroup.....

Question	Answer (A – E)
1.	
2.	
3.	
4.	
5.	
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