

Student Name.....Answers!

Teacher (circle one) DKI JOR VNA

Homegroup



MATHEMATICAL METHODS (CAS) UNIT 1

EXAMINATION 1

Wednesday November 2nd 2016

Reading Time: 1:00 – 1:15pm (15 minutes)

Writing time: 1:15 – 2:15pm (1 hour)

Instructions to students

This exam consists of **17** questions.

All questions should be answered in the spaces provided.

There are **65** marks available in this examination.

A decimal approximation will not be accepted if an exact answer is required.

Where more than one mark is allocated to a question working must be shown.

Students **may not** bring any notes or any calculators into this exam.

Diagrams in this exam are not to scale except where otherwise stated.

FORMULAS

$$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$$

$$\Pr(A \cap B) = \Pr(A) \cdot \Pr(B)$$

$$\Pr(A | B) = \frac{\Pr(A \cap B)}{\Pr(B)}$$

$${}^nC_r = \frac{n!}{(n-r)!r!}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Newton's Iterative formula for approximating roots of a polynomial:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

1) Given $A = \begin{bmatrix} -2 & 4 \\ -6 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 0 \\ -2 & 7 \end{bmatrix}$, calculate the following:

a) $B - 2A$ $= \begin{bmatrix} 5 & 0 \\ -2 & 7 \end{bmatrix} - \begin{bmatrix} -4 & 8 \\ -12 & 2 \end{bmatrix}$ $= \begin{bmatrix} 9 & -8 \\ 10 & 5 \end{bmatrix}$	b) AB $= \begin{bmatrix} -2 & 4 \\ -6 & 1 \end{bmatrix} \begin{bmatrix} 5 & 0 \\ -2 & 7 \end{bmatrix}$ $= \begin{bmatrix} -19 & 28 \\ -32 & 7 \end{bmatrix}$
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(2 + 2 = 4 marks)

2) Consider the set of simultaneous equations:

$$5x - 6y = 21$$

$$x - 2y = 5$$

a Write the set of equations as a matrix equation.

$$\begin{bmatrix} 5 & -6 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 21 \\ 5 \end{bmatrix}$$

b Use a matrix method to solve the equations and hence determine the values of x and y .

$$\begin{aligned} \begin{bmatrix} x \\ y \end{bmatrix} &= \begin{bmatrix} 5 & -6 \\ 1 & -2 \end{bmatrix}^{-1} \begin{bmatrix} 21 \\ 5 \end{bmatrix} \\ &= \frac{1}{-10+6} \begin{bmatrix} -2 & 6 \\ -1 & 5 \end{bmatrix} \begin{bmatrix} 21 \\ 5 \end{bmatrix} \\ &= -\frac{1}{4} \begin{bmatrix} -12 \\ 4 \end{bmatrix} \\ &= \begin{bmatrix} 3 \\ 1 \end{bmatrix} \end{aligned}$$

$$\text{So } x = 3, y = 1$$

(1 + 3 = 4 marks)

3)

- a) Calculate the coordinates of the image of the point $(17, -5)$ under the translation defined by $T = \begin{bmatrix} -8 \\ 9 \end{bmatrix}$.

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 17 \\ -5 \end{bmatrix} + \begin{bmatrix} -8 \\ 9 \end{bmatrix}$$

$$= \begin{bmatrix} 9 \\ 4 \end{bmatrix}$$

$$\Rightarrow (9, 4)$$

- b) Calculate the coordinates of the image of the point $(6, -13)$ under the linear transformation defined by the matrix

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$\Rightarrow$ reflection in x -axis.

$$\text{so } (x', y') = (6, 13)$$

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 6 \\ -13 \end{bmatrix} = \begin{bmatrix} 6 \\ 13 \end{bmatrix}$$

$$\Rightarrow (x', y') = (6, 13)$$

- c) Describe the transformation defined by the matrix $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$. (See part b) above)

reflection in x -axis

(2 + 2 + 1 = 5 marks)

- 4) Find the exact values of

a) $\sin 60^\circ$

$$\frac{\sqrt{3}}{2}$$

b) $\tan \frac{2\pi}{3}$

$$-\sqrt{3}$$

c) $\cos\left(-\frac{\pi}{6}\right)$

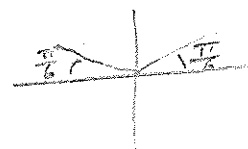
$$\frac{\sqrt{3}}{2}$$

(3 marks)

- 5) Solve the following equation $2 \sin x = \sqrt{3}, -2\pi \leq x \leq 2\pi$

$$\sin x = \frac{\sqrt{3}}{2}$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}, -\frac{5\pi}{6}, -\frac{\pi}{6}$$

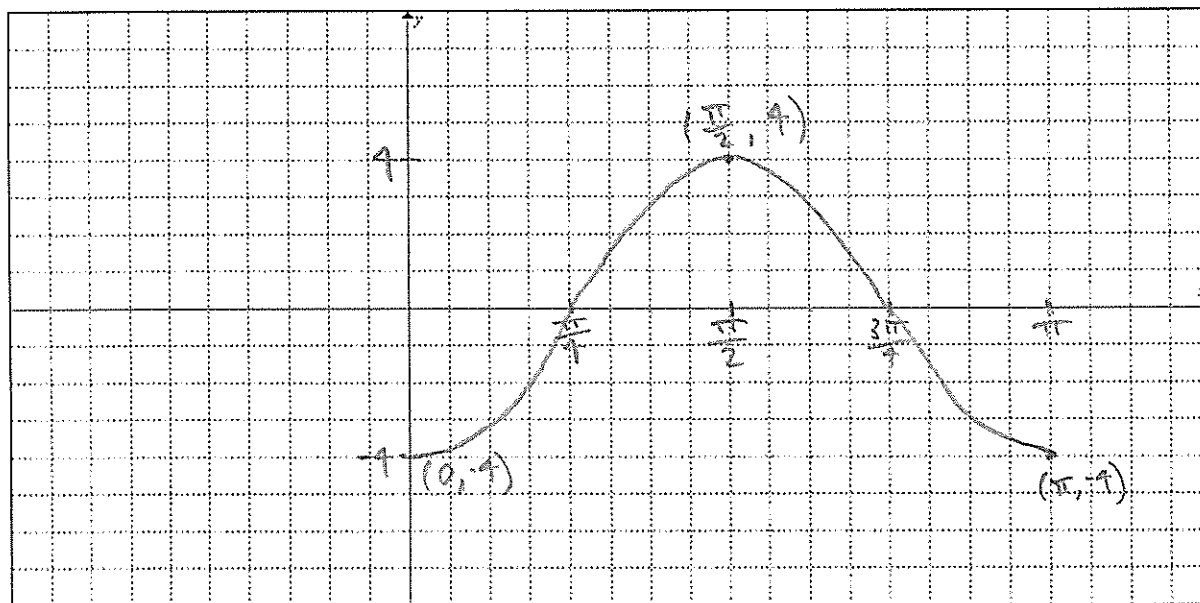


(4 marks)

- 6) a) What is the period and the amplitude of the graph of $y = -4 \cos 2x$?

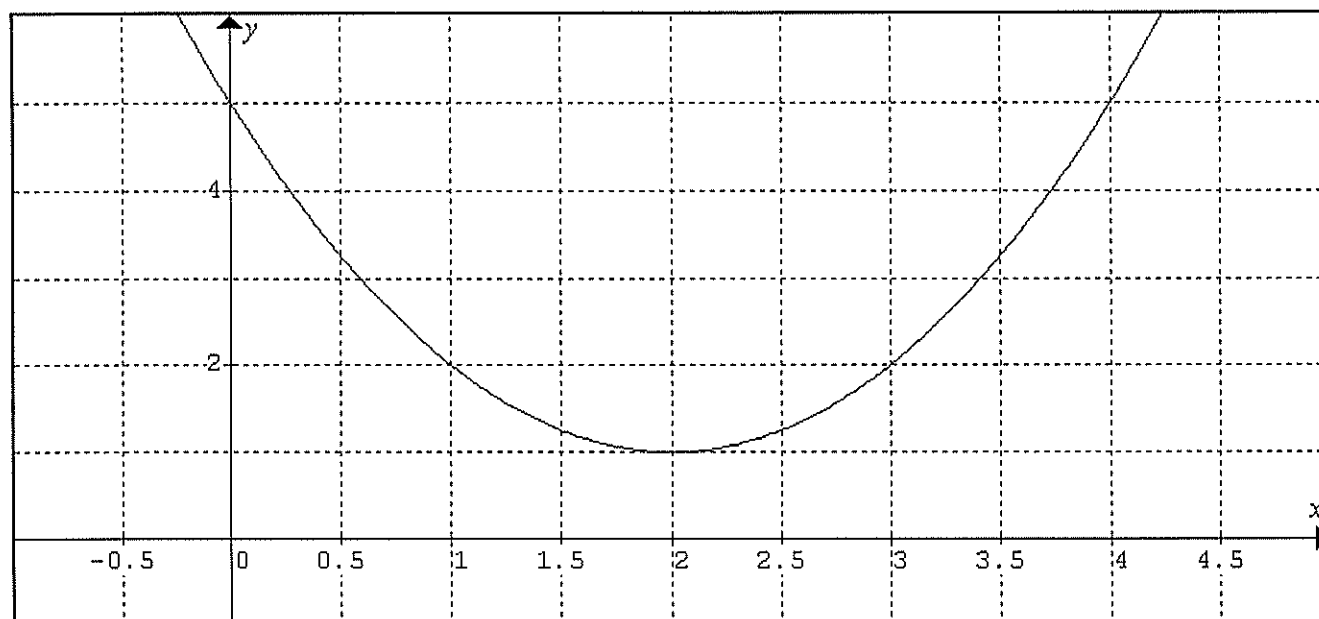
Period = π Amp!itude = 4

- b) Sketch the graph, showing one complete cycle. Clearly label key points.



(2 + 3 = 5 marks)

- 7) Part of the graph of the function $f: \mathbb{R} \rightarrow \mathbb{R}, f(x) = (x-2)^2 + 1$ is shown below.



- a) Find the average rate of change of $y = f(x)$ with respect to x , between $x = 0$ and $x = 3$.

$$\begin{aligned} \text{Avg change} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{2 - 5}{3 - 0} \\ &= -1 \end{aligned}$$

b) Find the instantaneous rate of change of $y = f(x)$ with respect to x at the point where $x = 5$.

$$f(x) = x^2 - 4x + 4 + 1$$

$$f'(x) = 2x - 4$$

$$\text{so } f'(5) = 10 - 4 = 6$$

(2 + 2 = 4 marks)

8) Find, using first principles, the derivative of $y = x^2 + 5x + 1$

$$\begin{aligned} \frac{dy}{dx} &= \lim_{h \rightarrow 0} \frac{(x+h)^2 + 5(x+h) + 1 - x^2 - 5x - 1}{h} \\ &= \lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 + 5x + 5h - x^2 - 5x}{h} \\ &= \lim_{h \rightarrow 0} \frac{2xh + h^2 + 5h}{h} \\ &= \lim_{h \rightarrow 0} 2x + h + 5 \end{aligned}$$

$$\text{so } \frac{dy}{dx} = 2x + 5$$

(3 marks)

9) If $f(x) = x^2(3x^2 - x) + 7$, find $f'(-1)$.

$$f(x) = 3x^4 - x^3 + 7$$

$$f'(x) = 12x^3 - 3x^2$$

$$\begin{aligned} f'(-1) &= 12(-1)^3 - 3(-1)^2 \\ &= -12 - 3 \\ &= -15 \end{aligned}$$

(3 marks)

10) Find the derivatives of

with positive indices?

a) $y = \frac{7x^2 - 2x}{x}$
 $= 7x - 2$
 $\Rightarrow \frac{dy}{dx} = 7$

b) $f(x) = \frac{4}{3x^4}$
 $= \frac{4}{3} x^{-4}$
 $\Rightarrow f'(x) = \frac{-16}{3} x^{-5}$
 $= \frac{-16}{3x^5}$

(2 + 2 = 4 marks)

11) Simplify

a) $\int (5x^3 + 3x^2 + 2) dx$
 $= \frac{5}{4} x^4 + x^3 + 2x + C$

b) $\int \sqrt[3]{x^2} dx$
 $= \int x^{\frac{2}{3}} dx$
 $= \frac{3}{5} x^{\frac{5}{3}} + C$
 $= \frac{3}{5} \sqrt[3]{x^5} + C$

★ more room?

(2 + 2 = 4 marks)

12) Find $\lim_{x \rightarrow 2} \frac{x^2 + 3x - 10}{x^2 - 4}$
 $= \lim_{x \rightarrow 2} \frac{(x+5)(x-2)}{(x+2)(x-2)}$
 $= \lim_{x \rightarrow 2} \frac{x+5}{x+2}$
 $= \frac{7}{4}$

(2 marks)

13) A particle moves in a straight line such that its displacement, x metres, from a fixed origin at time t seconds is modelled by $x = t^2 - 6t + 8, t \geq 0$.

a Identify its initial position.

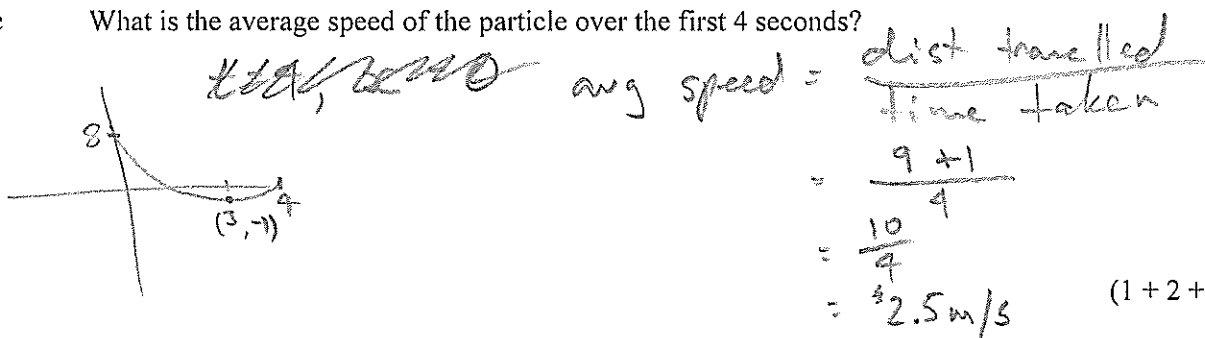
$t = 0, x = 8$

b Show, using calculus, that the particle is momentarily at rest at $t = 3$ seconds.

$v = 2t - 6$ at rest $\Rightarrow v = 0$

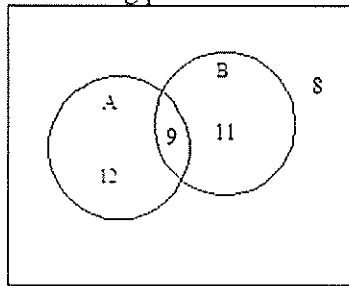
$v = 0 \quad 0 = 2t - 6$
 $6 = 2t$
 $t = 3$

c What is the average speed of the particle over the first 4 seconds?



(1 + 2 + 3 = 6 marks)

- 14) Use this Venn Diagram to find the following probabilities:



a) $\Pr(B' \cap A)$	b) $\Pr(B A)$	c) $\Pr(A' \cup B)$
$\frac{12}{40} = \frac{3}{10}$	$\frac{9}{21}$	$\frac{11+8+9}{40} = \frac{28}{40} = \frac{7}{10}$

(3 marks)

- 15) If $\Pr(B) = 0.42$, $\Pr(A' \cap B) = 0.16$ and $\Pr(A') = 0.51$,

a) complete this probability table

	B	B'	
A	0.26	0.23	0.49
A'	0.16	0.35	0.51
	0.42	0.58	1

b) Find $\Pr(A \cap B')$

$$0.23$$

c) Find $\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$
 $= 0.49 + 0.42 - 0.26$
 $= 0.65$

d) Find $\Pr(A' | B)$

$$= \frac{\Pr(A' \cap B)}{\Pr(B)}$$

$$= \frac{0.16}{0.42} = \frac{8}{21}$$

(2 + 1 + 1 + 2 = 6 marks)

- 16) Mr Oates needs two students to take some parents on a school tour. He chooses them randomly from a group of ten that were standing near his office. How many different groups of two could he choose?

$${}^{10}C_2 = \frac{10!}{2!8!}$$

$$= \frac{10 \times 9}{2 \times 1}$$

$$= \frac{90}{2}$$

$$= 45$$

2 marks

17)

- a) Define an iterative formula using Newton's Method for the function $f(x) = 6x^3 + 4x - 3$.

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \quad f'(x) = 18x^2 + 4$$

So
$$x_{n+1} = x_n - \frac{6x_n^3 + 4x_n - 3}{18x_n^2 + 4}$$

- b) Use this to calculate the value of x_1 when $x_0 = 1$. Give your answer as an exact value.

$$\begin{aligned} x_0 &= 1, \text{ then } x_1 = x_0 - \frac{6x_0^3 + 4x_0 - 3}{18x_0^2 + 4} \\ &= 1 - \frac{6 + 4 - 3}{18 + 4} \\ &= 1 - \frac{7}{22} \\ &= \frac{15}{22} \end{aligned}$$

(2 + 1 = 3 marks)

END OF EXAMINATION