

Student Name *Answers McAnswerface*

Teacher (circle one) DKI JOR VNA

Homegroup *Trenzalore*



MATHEMATICAL METHODS (CAS) UNIT 1

EXAMINATION 2

Monday June 6th 2016

Reading Time: 11.30-11.45 (15 minutes)

Writing time: 11.45-1.15 (90 minutes)

Instructions to students

This exam consists of Section 1 and Section 2.

Section 1 consists of **12** multiple-choice questions, to be answered on the separate answer sheet. It is worth **12** marks.

Section 2 consists of **13** extended-answer questions that should be answered in the spaces provided. It is worth **73** marks

There is a total of **85** marks available.

All questions in Section 1 and Section 2 should be answered.

Unless otherwise stated, diagrams in this exam are not drawn to scale.

Where more than one mark is allocated to a question, appropriate working must be shown.

Where an exact answer is required to a question, a decimal approximation will not be accepted.

Students may bring one bound reference into the exam.

Students may bring an approved CAS calculator.

MATHEMATICAL METHODS (CAS) UNIT 1

EXAM 2

SECTION 1

ANSWER SHEET

Student Name.....*Topof DaClass*.....

Teacher (circle one) DKI JOR. VNA

Homegroup.....*Siluria*.....

Question	Answer (A – E)
1.	<i>E</i>
2.	<i>B</i>
3.	<i>C</i>
4.	<i>B</i>
5.	<i>A</i>
6.	<i>B</i>
7.	<i>D</i>
8.	<i>B</i>
9.	<i>C</i>
10.	<i>B</i>
11	<i>A</i>
12	<i>E</i>

MULTIPLE-CHOICE QUESTIONS

Question 1

The equation of the line passing through the points $(-1, -3)$ and $(3, 4)$ is:

- A $4x + 7y + 25 = 0$
- B $4x - 7y + 5 = 0$
- C $4x - 7y - 17 = 0$
- D $7x - 4y + 17 = 0$
- ☒ E $7x - 4y - 5 = 0$

Question 2

The exact distance between the points $(-1, 5)$ and $(5, -1)$ is:

- A $2\sqrt{6}$
- ☒ B $6\sqrt{2}$
- C $4\sqrt{2}$
- D 12
- E $2\sqrt{3}$

Question 3

The expanded form of $(3x + 2)^3$ is:

- A $3x^3 + 8$
- B $3x^3 + 12x + 8$
- ☒ C $27x^3 + 54x^2 + 36x + 8$
- D $27x^3 + 27x^2 + 3x + 8$
- E $3x^3 + 6x^2 + 12x + 8$

Question 4

Solutions to the quadratic equation $2x^2 + 4x - 3 = 0$ are:

- A $x = \{-1 \pm \sqrt{40}\}$
- ☒ B $x = \left\{ \frac{-2 \pm \sqrt{10}}{2} \right\}$
- C $x = \{-1 \pm \sqrt{10}\}$
- D $x = \left\{ \frac{4 \pm \sqrt{40}}{4} \right\}$
- E $x = \{1 \pm \sqrt{40}\}$

Question 5

The graph with equation $y = 6x^2 + x - 2$ has x -intercepts:

- ☒ A $x = \left\{ -\frac{2}{3}, \frac{1}{2} \right\}$
- B $x = \left\{ -\frac{2}{3}, -\frac{1}{2} \right\}$
- C $x = \left\{ -\frac{1}{2}, \frac{2}{3} \right\}$
- D $x = \left\{ \frac{1}{2}, \frac{2}{3} \right\}$
- E $x = \{1, -2\}$

Question 6

The radius measure of the circle $9(x - 5)^2 + 9(y + 2)^2 = 81$ is:

- A 9
- ☒ B 3
- C $3\sqrt{2}$
- D $2\sqrt{3}$
- E 81

Question 7

The maximal domain and range for $f(x) = \sqrt{5 - 2x}$ respectively are:

- A $(-\infty, 2.5)$ and $(0, \infty)$
- B $(-\infty, 2.5)$ and $[0, \infty)$
- C $(-\infty, 2.5]$ and $(0, \infty)$
- ☒ D $(-\infty, 2.5]$ and $[0, \infty)$
- E $(-\infty, 2.5]$ and $[2.5, \infty)$

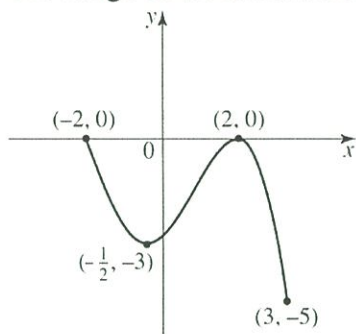
Question 8

When $6x^3 - 5x^2 + x + 3$ is divided by $2x + 3$, the quotient and remainder respectively are:

- A $3x^2 - 7x - 11, -30$
- ☒ B $3x^2 - 7x + 11, -30$
- C $3x^2 + 7x + 11, -30$
- D $3x^2 + 7x - 11, -30$
- E $3x^2 - 7x - 11, -32$

Question 9

The range of the function shown below is:



- A $[-2, 3]$
- B $[-2, 2]$
- ☒ C $[-5, 0]$
- D $[-3, 0]$
- E $[-5, -3]$

Question 10

The expression $\frac{3}{x^2 - 5x - 6}$ is undefined when:

- A $x = 0$
- ☒ B $x = -1, x = 6$
- C $x = -3, x = -2$
- D $x = -6, x = 1$
- E $x = 2, x = 3$

Question 11

The logarithmic form of the statement $729 = 9^3$ is:

- ☒ A $\log_9(729) = 3$
- B $\log_3(729) = 9$
- C $\log_3(9) = 729$
- D $\log_{729}(3) = 9$
- E $\log_{729}(9) = 3$

Question 12

$\frac{(3m^{-2}n^{\frac{1}{2}})^2}{4(m^6n^3)^{\frac{1}{2}}}$ is equal to:

- A $\frac{9m}{2n^{\frac{1}{2}}}$
- B $\frac{9n^{\frac{1}{2}}}{4m}$
- C $\frac{3m^7}{2n^{\frac{1}{2}}}$
- D $\frac{9n^{\frac{1}{2}}}{4m^7}$
- ☒ E $\frac{9}{4m^7n^{\frac{1}{2}}}$

SECTION 2 EXTENDED-ANSWER QUESTIONS

Question 13

Find the equation of the line which passes through the point $(-1, 2)$ and is perpendicular to the line with equation $2x - 4y + 3 = 0$.

$$4y = 2x + 3$$

$$y = \frac{1}{2}x + \frac{3}{4} \Rightarrow m = \frac{1}{2}, m_{\perp} = -2$$

$$\text{so } y - 2 = -2(x + 1)$$

$$y - 2 = -2x - 2$$

$$y = -2x$$

4 marks

Question 14

Write down an expression to represent the midpoint of the line joining the point $(3k, 3h + 1)$ with $(4 - 5k, 2h - 5)$. Then simplify if possible.

$$\text{midpoint } \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$\Rightarrow \left(\frac{(3k) + (4 - 5k)}{2}, \frac{(3h + 1) + (2h - 5)}{2} \right)$$

$$\Rightarrow \left(\frac{4 - 2k}{2}, \frac{5h - 4}{2} \right)$$

$$\Rightarrow \left(2 - k, \frac{5h - 4}{2} \right)$$

3 marks

Question 15

- (a) For the parabola with equation $y = x^2 + 5x - 1$, find the vertex by completing the square.

$$\begin{aligned} y &= \left(x^2 + 5x + \left(\frac{5}{2}\right)^2\right) - \left(\frac{5}{2}\right)^2 - 1 \\ &= \left(x + \frac{5}{2}\right)^2 - \frac{25}{4} - \frac{4}{4} \\ &= \left(x + \frac{5}{2}\right)^2 - \frac{29}{4} \end{aligned}$$

so vertex at $\left(-\frac{5}{2}, -\frac{29}{4}\right)$

- (b) Hence find the domain and range.

Domain: $\mathbb{R}$

Range: $\left[-\frac{29}{4}, \infty\right)$

- (c) What translation would map the parabola $y = x^2$ onto $y = x^2 + 5x - 1$?

Translation of $\frac{5}{2}$ units negatively along the x -axis

Translation of $\frac{29}{4}$ units vertically down along the y -axis

2+2+2=6 marks

Question 16

- a) Write down the numbers in row 4 of Pascal's triangle.

1 4 6 4 1

- a) Hence, expand $(2x - 3)^4$, using the binomial coefficients from Pascal's triangle. You must show at least 1 line of algebraic working.

$$\begin{aligned} (2x-3)^4 &= 1(2x)^4 + (4(2x)^3(-3)^1) + 6(2x)^2(-3)^2 + \\ &\quad 4(2x)^1(-3)^3 + (-3)^4 \\ &= 16x^4 - 96x^3 + 216x^2 - 216x + 81 \end{aligned}$$

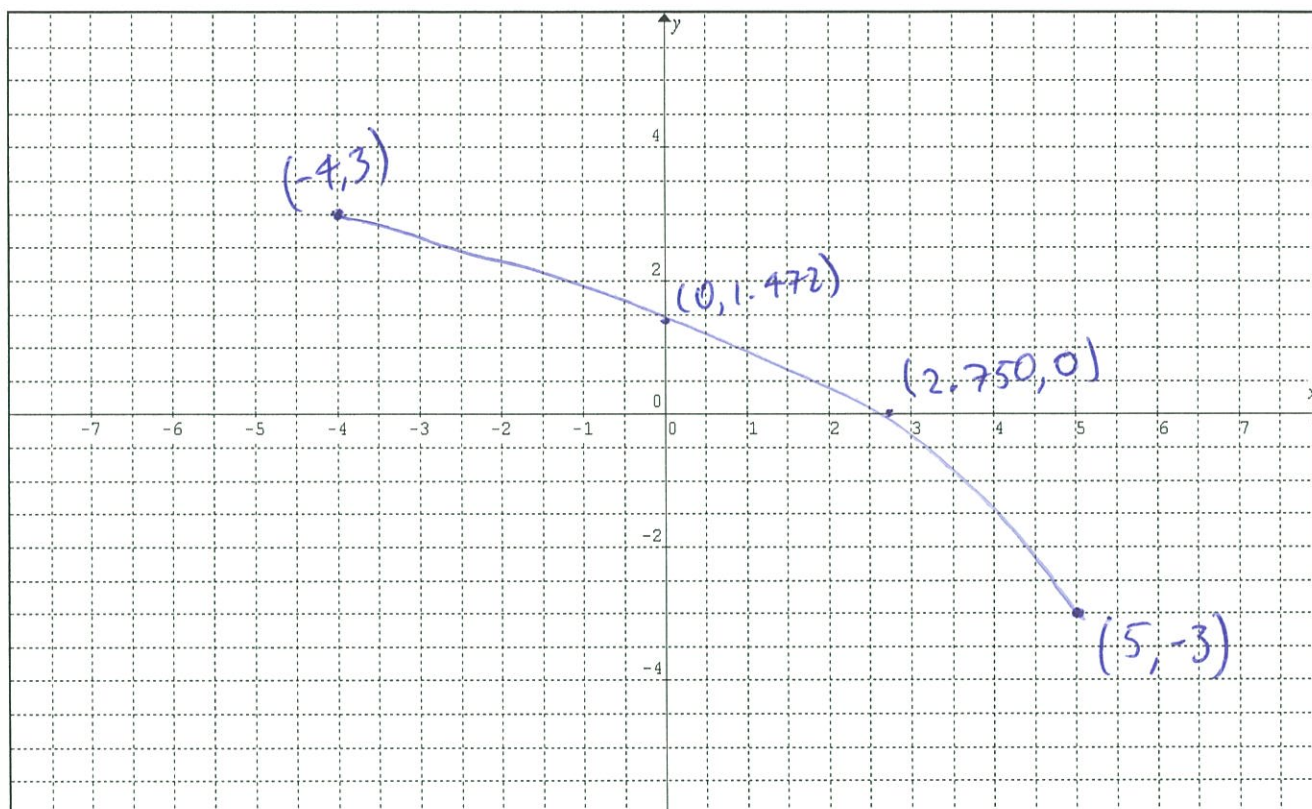
(1 + 2 = 3 marks)

Question 17

a. i. On the set of axes below sketch the graph of the relation

$$f : [-4, 5] \rightarrow \mathbb{R} \text{ where } f(x) = 2(\sqrt{5-x}) - 3.$$

Indicate clearly on the graph any intercepts or endpoints to **3 decimal places**.



ii. Write down the domain and range of this relation.

$$\text{Domain: } [-4, 5]$$

$$\text{Range: } [-3, 3]$$

iii. The basic shape of this graph was $y = \sqrt{x}$. Clearly state the transformations (dilations, reflections and translations that would be needed to give $y = 2(\sqrt{5-x}) - 3$

$$y = 2(\sqrt{-(x-5)}) - 3$$

1. Dilation of a factor of 2 away from x-axis
2. Reflection in y axis
3. Translation of 5 units in positive direction along x-axis
4. Translation of 3 units down along y axis

Question 18

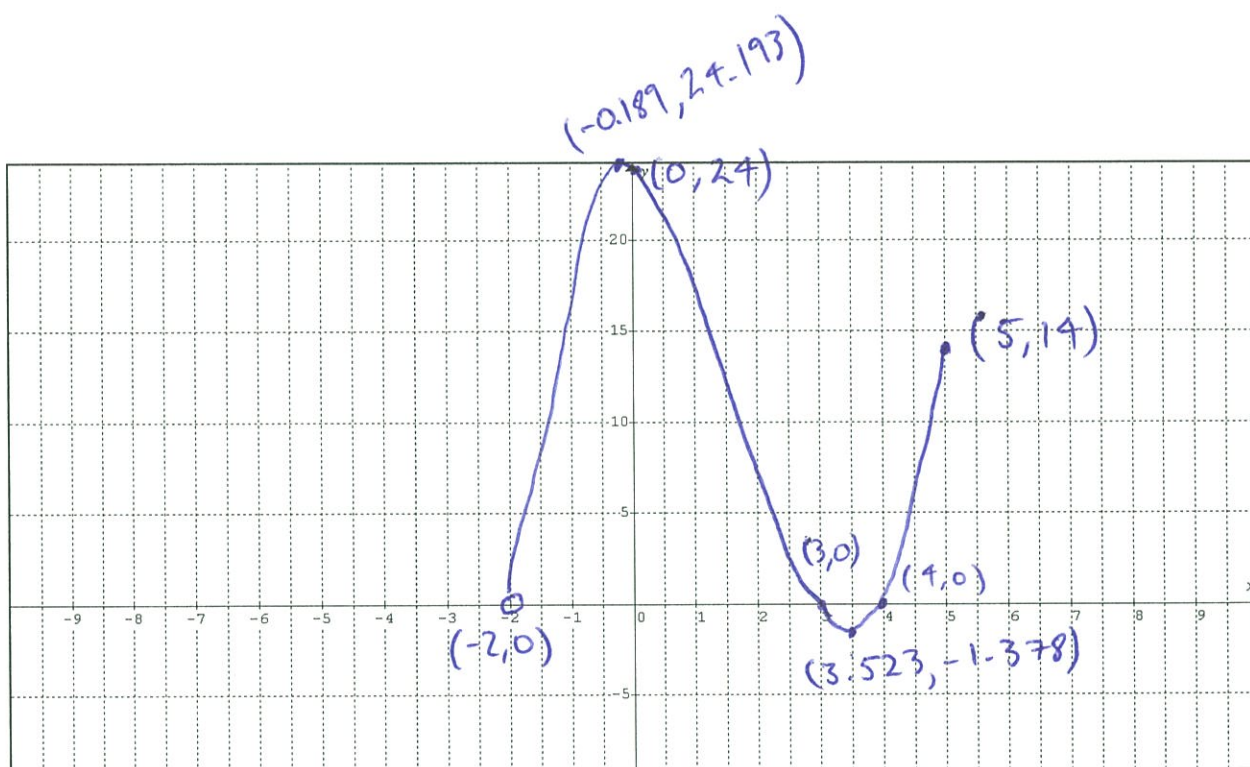
- a) Factorise $x^3 - 5x^2 - 2x + 24$, then sketch the graph of

$g: (-2, 5] \rightarrow \mathbb{R}$ where $g(x) = x^3 - 5x^2 - 2x + 24$.

Clearly label the **coordinates** of the intercepts and endpoints.

$$g(x) = (x+2)(x-3)(x-4)$$

Endpoints: $(-2, 0)$ (open), $(5, 14)$ (closed)



- b) Using CAS find the **coordinates** of the turning points to **3 decimal places**. Label these on your graph

T.P. $(-0.189, 24.193)$

T.P. $(3.523, -1.378)$

- c) Hence find the domain and range. (**3 decimal places**.)

Domain $[-2, 5]$

Range $[-1.378, 24.193]$

Question 18

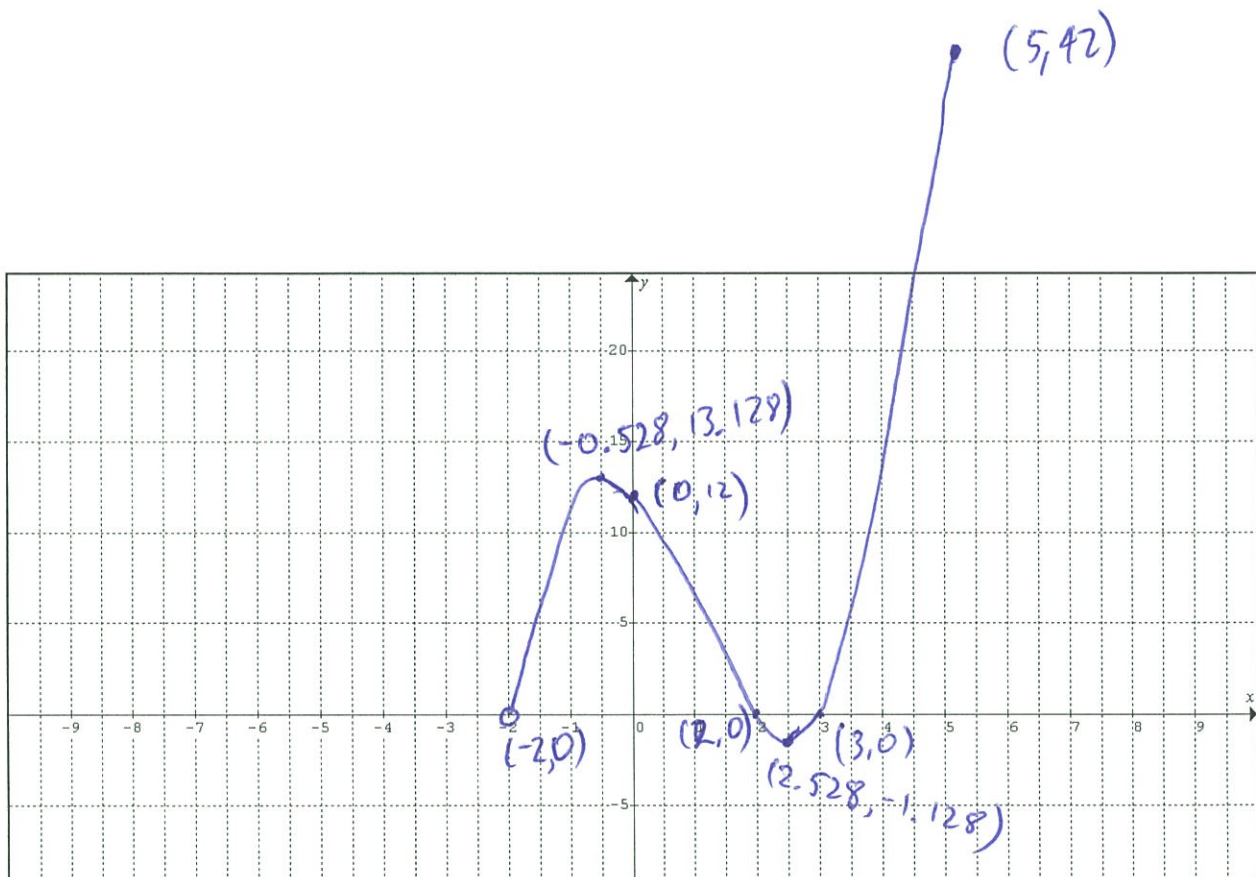
- a) Factorise $x^3 - 5x^2 - 2x + 24$, then sketch the graph of

$$g: (-2, 5] \rightarrow \mathbb{R} \text{ where } g(x) = x^3 - 3x^2 - 4x + 12.$$

Clearly label the **coordinates** of the intercepts and endpoints.

$$g(x) = (x+2)(x-2)(x-3)$$

$$\text{Endpoints: } (-2, 0), (5, 42)$$



- b) Using CAS find the **coordinates** of the turning points to **3 decimal places**. Label these on your graph

$$\text{T.P. } (-0.528, 13.128)$$

$$\text{T.P. } (2.528, -1.128)$$

- c) Hence find the domain and range. (**3 decimal places**.)

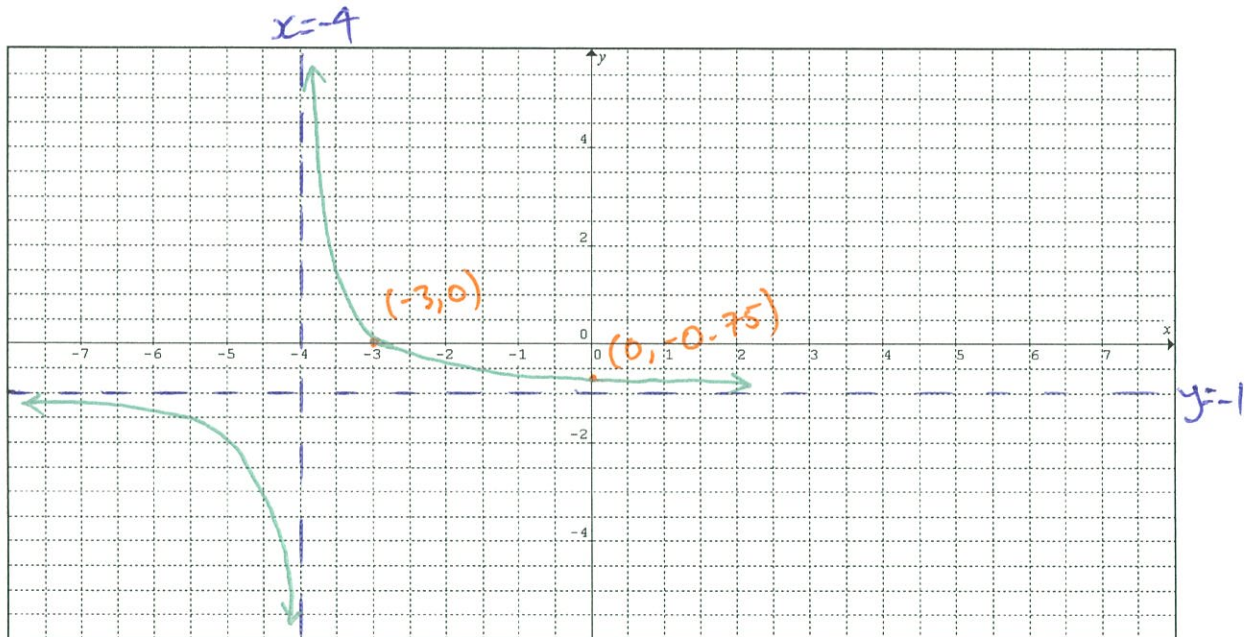
$$\text{Domain } [-2, 5]$$

$$\text{Range } [-1.128, 42]$$

Question 19

- a) i. On the set of axes below sketch the graph of the relation $y = \frac{1}{(x+4)} - 1$.

Clearly label the **coordinates** of the intercepts and the equations of any asymptotes.



- ii. State whether or not this relation is a function. Give the reason for your answer.

Function

Passes vertical line test

Each x-value has one unique y-value

- iii. State the domain and Range of y

Domain: $\mathbb{R} \setminus \{-4\}$

Range: $\mathbb{R} \setminus \{-1\}$

3+2+2=7 marks

Question 20

- a) Give the equation for the upper half of a circle with centre (0,0) and radius 6.

circle: $x^2 + y^2 = 36$ upper half: $y = \sqrt{36 - x^2}$

- b) Give the equation for the lower half of a circle with centre (4,-3) and radius 2.

circle: $(x-4)^2 + (y+3)^2 = 4$

$(y+3)^2 = 4 - (x-4)^2$

lower half: $y = -\sqrt{4 - (x-4)^2} - 3$

2 marks

Question 21

$$\text{If } f(x) = \begin{cases} x+1, & x \in [-5, 0) \\ 5-x^2, & x \in [0, 3) \\ x, & x \in [3, 6) \end{cases}$$

Find

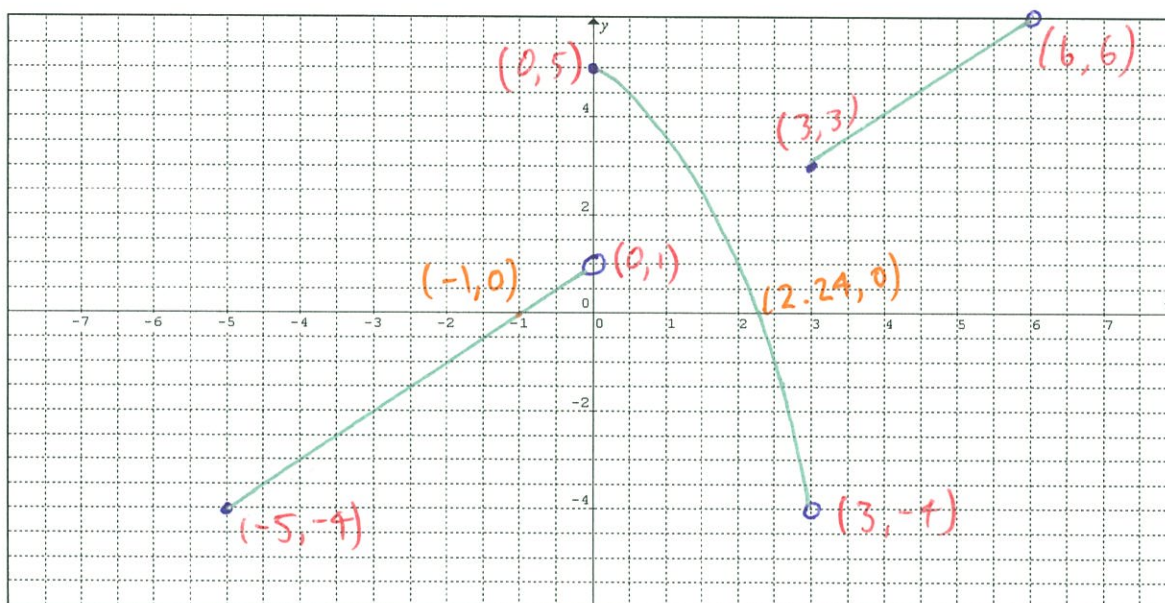
a) the value for $f(3)$.

$$f(3) = 3$$

b) The range of, $f(x)$

$$\text{Range: } [-4, 6)$$

c) Sketch this graph. Place the **coordinates** on each end point and intercept.



1+1+3 = 5 marks

Question 22

Write in simplest index notation:

a) $\frac{(a^2b^3)^2 \times (4ab^{-2})^3}{16(a^{-3}b^5)^2}$

$$= \frac{a^4b^6 \times 4^3a^3b^{-6}}{16a^{-6}b^{10}}$$

$$= \frac{4^3a^7}{16a^{-6}b^{10}}$$

$$= \frac{4a^{13}}{b^{10}}$$

b) $\frac{(-3a^nb^{2n-2})^3 \times 4(ab^{1-n})^2}{(2a^{n+2}b^3)^6}$

$$= \frac{(-3)^3a^{3n}b^{6n-6} \times 4a^2b^{2-2n}}{2^6a^{6n+12}b^{18}}$$

$$= \frac{-27a^{3n+2}b^{4n-4} \times 4}{16 \cancel{4}a^{6n+12}b^{18}}$$

$$= \frac{-27a^{-3n-10}b^{4n-22}}{16}$$

$$= \frac{-27b^{4n-22}}{a^{3n+10}}$$

3 + 3 = 6 marks

Question 23

22. Solve for x in the following equations:

a) $2^{2x-3} - 64^x = 0$

$$2^{2x-3} = 64^x$$

$$2^{2x-3} = (2^6)^x$$

$$2^{2x-3} = 2^{6x}$$

$$2x-3 = 6x$$

$$-3 = 4x$$

$$x = -\frac{3}{4}$$

b) $\log_x(2) - \log_x(128) = -3$

$$\log_x\left(\frac{2}{128}\right) = -3$$

$$x^{-3} = \frac{1}{64}$$

$$\frac{1}{x^3} = \frac{1}{64}$$

$$x^3 = 64$$

$$x^3 = 4^3$$

$$x = 4$$

2+2=4 marks

Question 24

- (a) What are the values k can take for which the equation $(k+2)x^2 + (3k+3)x + 2k+2 = 0$ has only one rational solution?

$$\Delta = 0$$
$$\Delta = b^2 - 4ac$$

$$(3k+3)^2 - 4(k+2)(2k+2) = 0$$

$$k = -1 \text{ or } 7$$

- (b) For the values of k calculated in part (a), solve the equation.

$$k = 7 \Rightarrow 9x^2 + 24x + 16 = 0 \Rightarrow x = -\frac{4}{3}$$

$$k = -1 \Rightarrow x^2 = 0 \Rightarrow x = 0$$

- (c) Solve the equation when $k = -3$. Give your answer in EXACT Values.

$$(-3+2)x^2 + (3(-3)+3)x + 2(-3)+2 = 0$$

$$-x^2 - 6x - 4 = 0 \Rightarrow x^2 + 6x + 4 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow x = \frac{-6 \pm \sqrt{36 - 16}}{2}$$

$$= \frac{-6 \pm \sqrt{20}}{2}$$

$$= \frac{-6 \pm 2\sqrt{5}}{2}$$

$$= -3 \pm \sqrt{5}$$

3+2+2=7 marks

Question 25

A cup of coffee is left to cool on a kitchen table. The temperature of the coffee T ($^{\circ}\text{C}$) after t minutes is measured and recorded in a table:

T (mins)	1	2	10	15
T ($^{\circ}\text{C}$)	80.5	76.4	50.9	39.3

- a. Use your calculator to generate the equation of the line of best fit, $y = a \times b^x$, giving the coefficients to 2 d.p.

$$T = \underline{84.70} \times \underline{0.95^t}$$

- b. According to your equation, what is the initial temperature of the coffee, to the nearest degree?

$$85^{\circ}$$

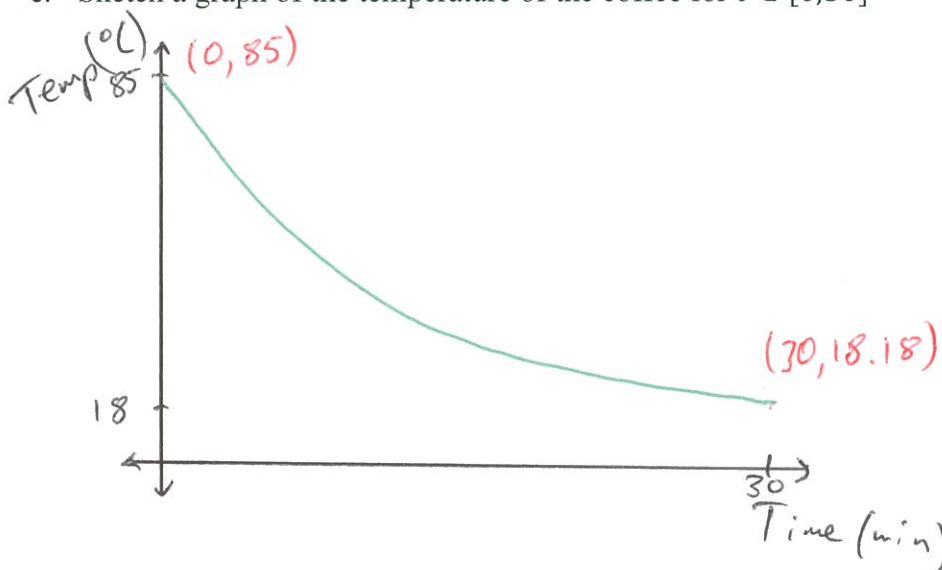
- c. By how many degrees does the coffee cool in 5 minutes? Give your answer to the nearest degree

when $t=5 \Rightarrow (5, 65.54)$
Cools down by 20°C

- d. How long (in minutes, accurate to 2 decimal places) does it take the coffee to cool down to 48°C ?

$x = 11.07$
 $\therefore$ it takes 11 min to cool to get to 48°C

- e. Sketch a graph of the temperature of the coffee for $t \in [0, 30]$



2 + 1 + 2 + 1 + 2 = 8 marks

END OF EXAMINATION