

Student Name A. Plus

Teacher (circle one) DKI JOR VNA

Homegroup Asgard



MATHEMATICAL METHODS (CAS) UNIT 1

EXAMINATION 1

Thursday June 9th 2016

Reading Time: 2:00 – 2:15pm (15 minutes)

Writing time: 2:15 – 3:15pm (1 hour)

Instructions to students

This exam consists of **13** questions.

All questions should be answered in the spaces provided.

There is a total of **65** marks available.

A decimal approximation will not be accepted if an exact answer is required.

Where more than one mark is allocated to a question working must be shown.

Students **may not** bring any notes or any calculators into this exam.

Diagrams in this exam are not to scale except where otherwise stated.

FORMULAS

Function and Graphs

Distance formula $d_{AB} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Midpoint formula $x_M, y_M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$

Straight line graphs

General equation $y = mx + c$

Gradient $m = \frac{y_2 - y_1}{x_2 - x_1}$

Equation through point (x_1, y_1) given by $y - y_1 = m(x - x_1)$

Difference/sum of squares and cubes

$$a^2 - b^2 = (a + b)(a - b)$$

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

Expansions

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$

1. Solve these equations for x

(a) $6(x+2) = 4x-1$ $6x+12 = 4x-1$ $2x = -13$ $x = -\frac{13}{2}$	(b) $\frac{3}{x+2} = \frac{5}{x}$ $3x = 5(x+2)$ $3x = 5x+10$ $-2x = 10$ $x = -5$
---	--

2X2 = 4 marks

2. Use the factor theorem and division to factorise $P(x) = 2x^3 - x^2 - 7x + 6$

$P(1) = 2 - 1 - 7 + 6 = 0 \Rightarrow (x-1)$ is a factor

$$\begin{array}{r}
 2x^2 + x - 6 \\
 x-1 \overline{) 2x^3 - x^2 - 7x + 6} \\
 \underline{- 2x^3 - 2x^2} \\
 x^2 - 7x \\
 \underline{- x^2 - x} \\
 -6x + 6 \\
 \underline{- -6x + 6} \\
 0
 \end{array}$$

So $P(x) = (x-1)(2x^2 + x - 6)$
 $= (x-1)(2x^2 + 4x - 3x - 6)$
 $= (x-1)(2x(x+2) - 3(x+2))$
 $= (x-1)(x+2)(2x-3)$

5 marks

3. Expand

(a) $(2x+3)^3 = (2x)^3 + 3(2x)^2(3) + 3(2x)(3)^2 + 3^3$ $= 8x^3 + 36x^2 + 54x + 27$	(b) $(2x-1)(x+2)(x-3)$ $= (2x-1)(x^2 - 3x + 2x - 6)$ $= (2x-1)(x^2 - x - 6)$ $= 2x^3 - 2x^2 - 12x - x^2 + x + 6$
---	--

$= 2x^3 - 3x^2 - 11x + 6$

2+2=4 marks

4. Factorise the following completely

(a) $2x^3 + 54$ $= 2(x^3 + 27)$ $= 2(x^3 + 3^3)$ $= 2(x+3)(x^2 - 3x + 4)$	(b) $12a^2 + 8a - 15$ $= 12a^2 + 18a - 10a - 15$ $= 6a(2a+3) - 5(2a+3)$ $= (6a-5)(2a+3)$
(c) $18a^2b - 2b$ $= 2b(9a^2 - 1)$ $= 2b((3a)^2 - 1)$ $= 2b(3a+1)(3a-1)$	(d) Use completion of the square method Leave your answer in exact form $w^2 + 2w - 3$ $= (w^2 + 2w + 1) - 1 - 3$ $= (w+1)^2 - 4$ $= (w+1+2)(w+1-2)$ $= (w+3)(w-1)$

4x2=8 marks

5. Use the discriminant to predict the number of solutions and the type for the following equations DO NOT SOLVE

a) $x^2 - 6x + 8 = 0$ $a=1 \quad b=-6 \quad c=8$ $\Delta = b^2 - 4ac$ $= 36 - 32$ $= 4$ 2 solutions	b) $5v^2 - 2v + 1 = 0$ $a=5 \quad b=-2 \quad c=1$ $\Delta = 4 - 20$ $= -16$ 0 solutions	c) $3g^2 - 10 = 0$ $a=3 \quad b=0 \quad c=-10$ $\Delta = b^2 - 4ac$ $= 0 + 120$ $= 120$ 2 solutions
--	--	--

3X2 = 6 marks

6. State the domain and range of $(x+1)^2 + (y-4)^2 = 169$

circle centre $(-1, 4)$, $r=13$

domain: $[-1-13, -1+13] = [-14, 12]$

range: $[4-13, 4+13] = [-9, 17]$

2 marks

7. (a) Sketch the function $y = 1 - \frac{4}{(x-2)^2}$, where $-2 < x < 6$. Include any intercepts, endpoints and any other main feature(s)

asymptotes $y=1, x=2$

Intercepts

$$\begin{aligned} y=0 & \quad x=0 \\ \Rightarrow 1 - \frac{4}{(x-2)^2} = 0 & \quad \Rightarrow y = 1 - \frac{4}{(-2)^2} \\ & \quad = 0 \end{aligned}$$

$$1 = \frac{4}{(x-2)^2}$$

$$(x-2)^2 = 4$$

$$x-2 = \pm 2$$

$$\Rightarrow x = 0, 4$$

Endpoints

at $x = -2, x = 6$

$$y = 1 - \frac{4}{16}$$

$$= 1 - \frac{1}{4}$$

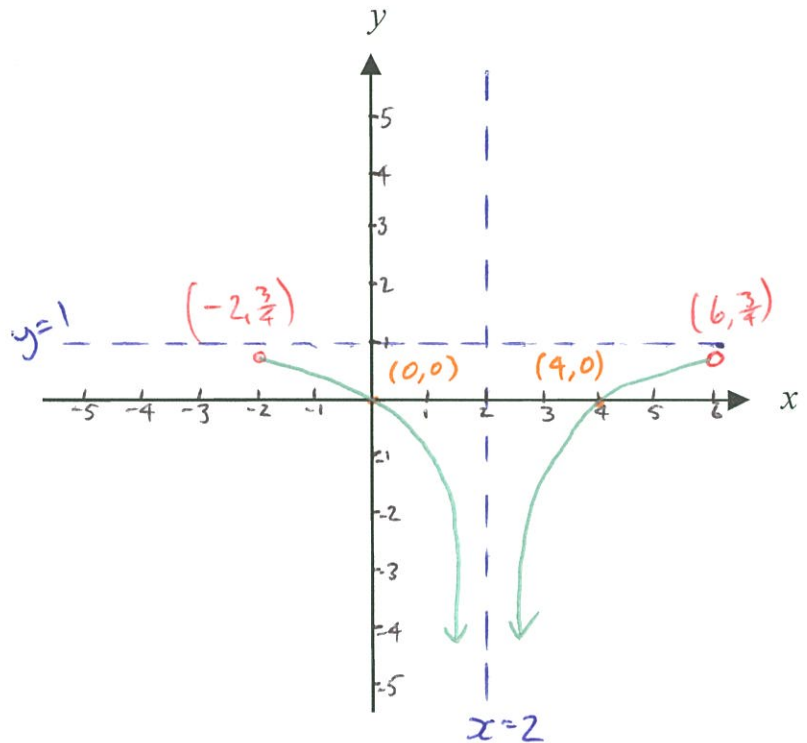
$$= \frac{3}{4}$$

$$(-2, \frac{3}{4})$$

$$y = 1 - \frac{4}{16}$$

$$= \frac{3}{4}$$

$$(6, \frac{3}{4})$$



- b) What is this shape called?

Truncus

- c) What type of relation is this?

Function, many to one

- d) What is the domain and range of this relation.

Domain: $(-2, 2) \cup (2, 6)$

Range: $(-\infty, \frac{3}{4})$

4+1+1+1=7 marks

8. State all the transformations required to transform the cubic $y = \sqrt{x}$ to $y = \sqrt{5x-2} + 3$

$$y = \sqrt{5(x - \frac{2}{5})} + 3$$

① Dilation of $\frac{1}{5}$ units from y-axis

② translation of $\frac{2}{5}$ units positive direction of x-axis

③ translation of 3 units positive direction of y-axis

3 marks

9. The perpendicular bisector of a line segment is the line passing through its mid point and perpendicular to the line segment. Find the equation of the perpendicular bisector of the line interval joining the points $(-2, 5)$ and $(4, -3)$.

$$m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3 - 5}{4 - (-2)} = \frac{-8}{6} = -\frac{4}{3} \Rightarrow m_2 = \frac{3}{4}$$

$$\text{Mid point} = \left(\frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2} \right) = \left(\frac{-2 + 4}{2}, \frac{5 + (-3)}{2} \right) = (1, 1)$$

$$\text{Perpendicular Bisector: } y = mx + c$$

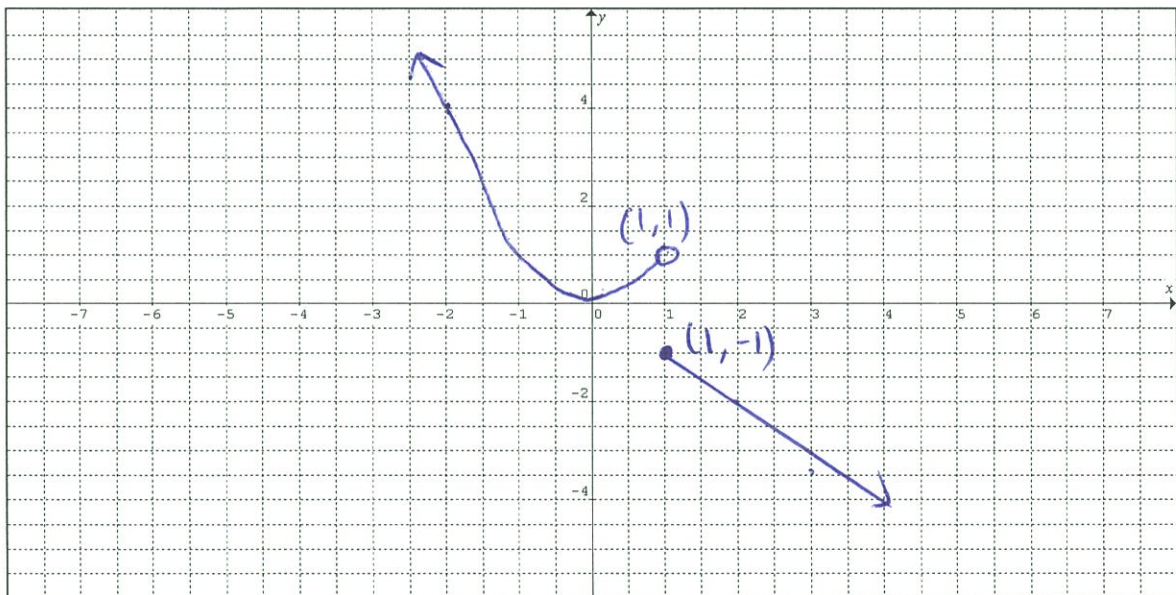
$$\text{sub in } m_2 = \frac{3}{4}, (1, 1) \Rightarrow 1 = \frac{3}{4}(1) + c$$

$$c = 1 - \frac{3}{4} = \frac{1}{4}$$

$$\text{so } y = \frac{3}{4}x + \frac{1}{4}$$

5 marks

10. a) Sketch the graph of $h(x) = \begin{cases} x^2 & , x < 1 \\ -x & , x \geq 1 \end{cases}$



- b) What is the domain and range of $h(x)$?

$$\text{Domain: } \mathbb{R}$$

$$\text{Range: } (-\infty, -1] \cup [0, \infty)$$

- c) Find the value of $h(1)$.

$$h(1) = -(1) = -1$$

- d) State any value of x for which the function is not continuous.

$$x = 1$$

3+2+1+2=8 marks

11. Consider the equation for a circle: $x^2 - 4x + y^2 + 4y - 1 = 0$

a) State the coordinates of the centre of the circle and the size of the radius.

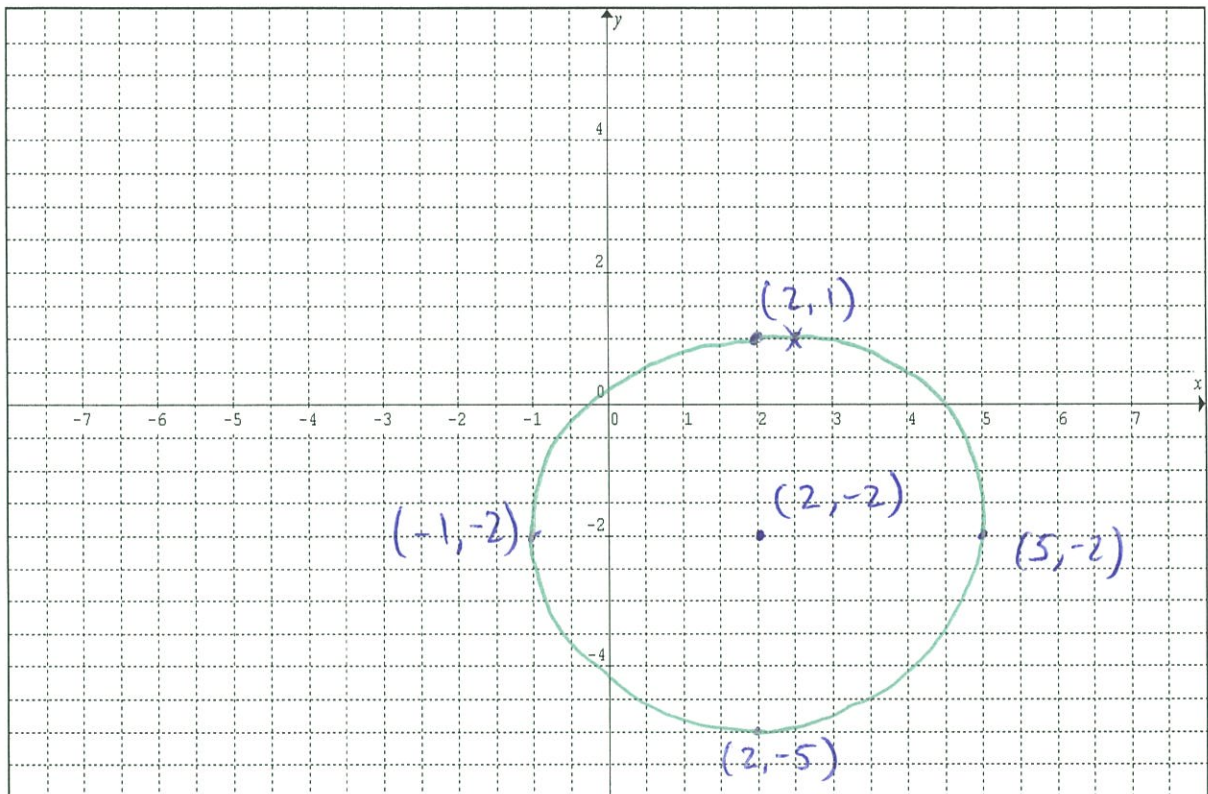
$$\underline{x^2 - 4x + 4 + y^2 + 4y + 4 - 4 - 4 - 1 = 0}$$

$$(x-2)^2 + (y+2)^2 - 9 = 0$$

$$(x-2)^2 + (y+2)^2 = 3^2$$

centre: $(2, -2)$, radius = 3

b) Sketch the circle (do not calculate intercepts for this question)



12. a) Simplify $\frac{9^{2n} \times 8^n \times 16^n}{6^n}$

$$= \frac{(3^2)^{2n} \times (2^3)^n \times (2^4)^n}{(2 \times 3)^n}$$

$$= \frac{3^{4n} \times 2^{3n} \times 2^{4n}}{2^n \times 3^n}$$

$$= 3^{3n} \times 2^{6n}$$

b) Simplify $2\log_{10} 3 + \log_{10} 4$

$$= \log_{10}(3)^2 + \log_{10}(4)$$

$$= \log_{10}(9 \times 4)$$

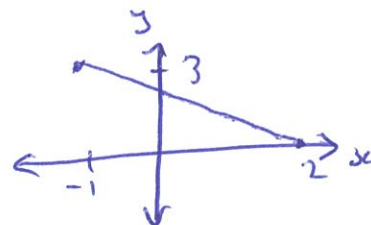
$$= \log_{10}(36)$$

4 marks

13. Consider the function $h: [-1, 2] \rightarrow \mathbb{R}, h(x) = 2 - x$

a) State the range of $h(x)$

$$[0, 3]$$



b) In full notation, write the inverse of $h(x)$

$$y = 2 - x$$

so inverse:

$$x = 2 - y$$

$$\Rightarrow y = 2 - x$$

$$\text{so } f^{-1}: [0, 3] \rightarrow \mathbb{R}, f^{-1}(x) = 2 - x$$

2+2=4 marks

END OF EXAMINATION 1