

Student Name.....

Teacher (circle one) JOR CWE

Homegroup



MATHEMATICAL METHODS (CAS) UNIT 1

EXAMINATION 2

Wednesday, 3th June, 2015

Reading Time: 9.00 – 9.15 (15 minutes)

Writing time: 9.15 – 10.45 (90 minutes)

Instructions to students

This exam consists of Section 1 and Section 2.

Section 1 consists of **12** multiple-choice questions, to be answered on the separate answer sheet. It is worth **12** marks.

Section 2 consists of **12** extended-answer questions that should be answered in the spaces provided. It is worth **77** marks

There is a total of **89** marks available.

All questions in Section 1 and Section 2 should be answered.

Unless otherwise stated, diagrams in this exam are not drawn to scale.

Where more than one mark is allocated to a question, appropriate working must be shown.

Where an exact answer is required to a question, a decimal approximation will not be accepted.

Students may bring one bound reference into the exam.

Students may bring an approved CAS calculator.

SECTION 1: MULTIPLE-CHOICE QUESTIONS

- 1 Which of the following relations are functions?

I $(x-2)^2 + (y+1)^2 = 16$

II $y^2 = \frac{2}{3}x - 1$

III $y = -2x + 4$

IV $y = 4x^2$

- A I and III
B III and IV
C I and II
D II and III
E I and IV

- 2 If $f(x) = 2 + \frac{3}{x}$ then the value of $f(3) - f(6)$ is

- A. -3
B. $\frac{1}{2}$
C. $1\frac{1}{2}$
D. 3
E. $2\frac{1}{2}$

- 3 The expansion of $(x-3)^3(x+2)$ is given by

- A $x^2 + x - 12$
B $x^3 - 2x^2 - 15x + 36$
C $x^4 + 4x^3 - 27x - 54$
D $x^4 - 7x^3 + 9x^2 + 27x - 54$
E $x^4 - 5x^3 - 9x^2 + 81x - 108$

- 4 The graph of the parabola with equation $y = -(x+3)^2 - 2$ has a turning point with coordinates

- A $(-3, -2)$
B $(-3, 2)$
C $(3, -2)$
D $(9, -2)$
E $(3, 2)$

- 5 The equation $3x + 2y - 2 = 0$ has gradient and y-intercept respectively equal to:

- A $3, -2$
B $-2, 4$
C $\frac{2}{3}, \frac{4}{3}$
D $-\frac{3}{2}, 1$
E $-\frac{2}{3}, 4$

- 6 The variables a and b are related by the formula $a = \frac{4b}{b-1}$.

Rearrangement of the formula shows that b is equal to:

- A $\frac{4a}{a+4}$
B $\frac{a}{a+4}$
C $\frac{a+4}{a}$
D $\frac{4a}{4+a}$
E $\frac{a}{a-4}$

- 7 A function has rule $f(x) = \sqrt{x+3} - 5$. The (implied) domain and range are:

- A domain: $[3, \infty)$; range: $[-5, \infty)$
B domain: $[-3, \infty)$; range: $[-5, \infty)$
C domain: $(3, \infty)$; range: $(-5, \infty)$
D domain: $(-3, \infty)$; range: $(-5, \infty)$
E domain: $[-3, \infty)$; range: R

- 8 The maximal domain and range for

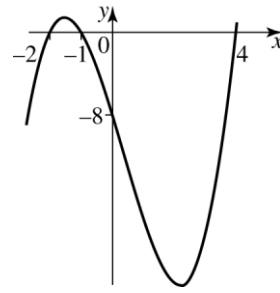
$$g(x) = \frac{2}{1+3x} \text{ are respectively:}$$

- A $R \setminus \left\{-\frac{3}{2}\right\}$ and $R \setminus \{0\}$
 B $R \setminus \left\{-\frac{2}{3}\right\}$ and $R \setminus \{4\}$
 C $R \setminus \left\{-\frac{1}{3}\right\}$ and $R \setminus \{0\}$
 D $R \setminus \left\{-\frac{2}{3}\right\}$ and $R \setminus \{0\}$
 E $R \setminus \left\{\frac{2}{3}\right\}$ and $R \setminus \{4\}$

- 9 The points $(1, 4)$, $(2, 0)$ and $(4, p)$ lie on a straight line. The value of p is:

- A +2
 B -2
 C -4
 D -6
 E -8

10



The graph above represents the equation:

- A $f(x) = (x-2)(x-1)(x+4)$
 B $f(x) = -8(x+2)(x+1)(x-4)$
 C $f(x) = (x+2)(x+1)(x-4)$
 D $f(x) = (x-2)(x-1)(x+4)(x-8)$
 E $f(x) = (x+2)(x+1)(x-4)(x+8)$

11

The expression $\frac{(m^2n)^4}{(2m^5n^2)^3} \div \frac{(m^5n^2)^2}{2mn^5}$

can be simplified to:

- A $\frac{1}{4m^{16}n}$
 B $\frac{2^2}{m^{16}n}$
 C $\frac{1}{4m^8}$
 D $\frac{1}{4m^{16}n}$
 E $2^2 m^{16}n$

12

The expression $\log_n \left(\frac{1}{n^4} \right)$ equals:

- A $\frac{n}{4}$
 B $4n$
 C 4
 D n^{-4}
 E -4

SECTION 2 EXTENDED-ANSWER QUESTIONS

13. A line joins the points with coordinates $(-2, 5)$ and $(6, 9)$.

Find:

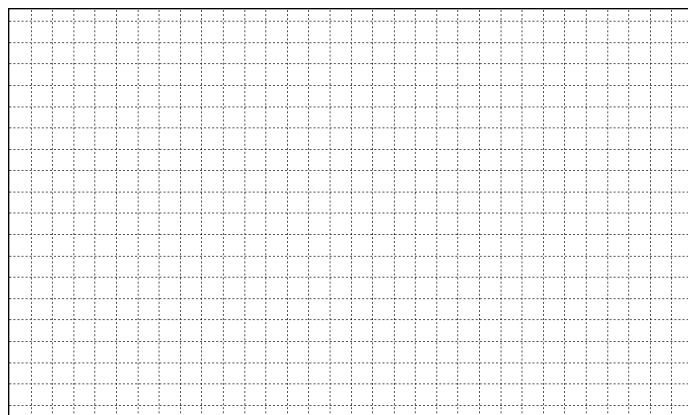
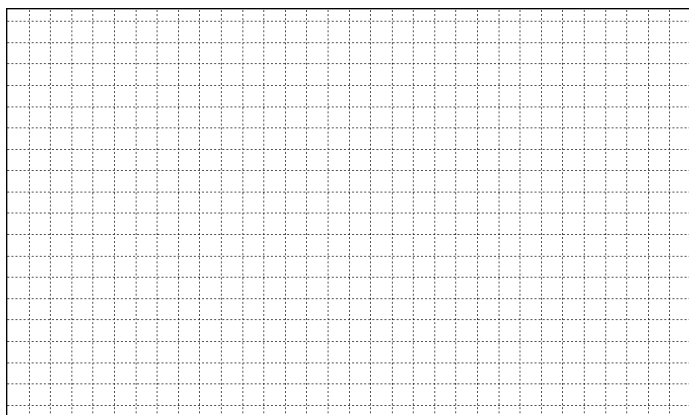
- The equation of the line that joins the 2 points.
- The exact value (in simplest form) of the direct distance between the 2 points.
- The midpoint of the line.
- The equation of the perpendicular bisector of the line.
- State the domain and range of the line segment joining the points $(-2, 5)$ and $(6, 9)$

2 + 2 + 2 + 3 + 2 = 11 marks

14. Sketch the graphs of each of the following equations. State the domain and range of each.

a) $y = \sqrt{x} + 1$, where $x \in R$

b) $y = 2x + 1$, where $x \in \{0, 1, 2, 3, 4\}$



Domain:
Range:

Domain:
Range:

3 + 3 = 6 marks

15. (i) Sketch each of the following graphs, labelling the point of inflection or equation of any asymptotes where appropriate.

NOTE: You do **not** need to calculate any x or y intercept.

(ii) State the equation of the relevant basic shape graph.

(iii) State all dilations and translations required to draw them from the basic shape.

$$y = 3(x - 1)^3 + 4$$

$$y = 3 \times 2^{x+1} - 2$$

(ii) _____

(iii)

(ii) _____

(iii)

3 + 3 = 6 marks

16. a) Convert the following quadratic into turning point form: $y = x^2 + 4x - 7$

a) Hence, state the co-ordinates of the turning point.

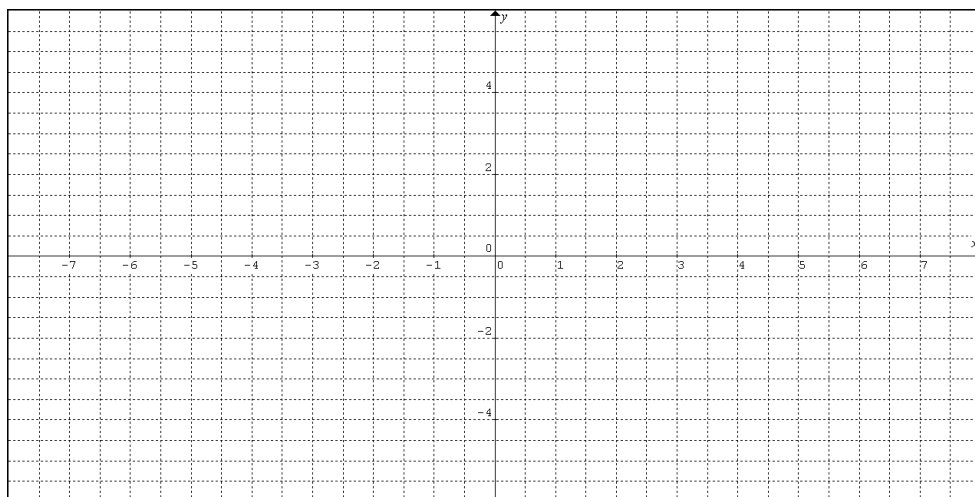
b) State the domain and range

c) What translation would map the parabola $y = x^2$ onto $y = x^2 + 4x - 7$?

2 + 1 + 2 + 2 = 7 marks

17. If $f(x) = \begin{cases} 3 - x^2 & , \quad x \geq 0 \\ x + 4 & , \quad x < 0 \end{cases}$

Sketch this graph.



Find:

a) the range of $f(x)$ and

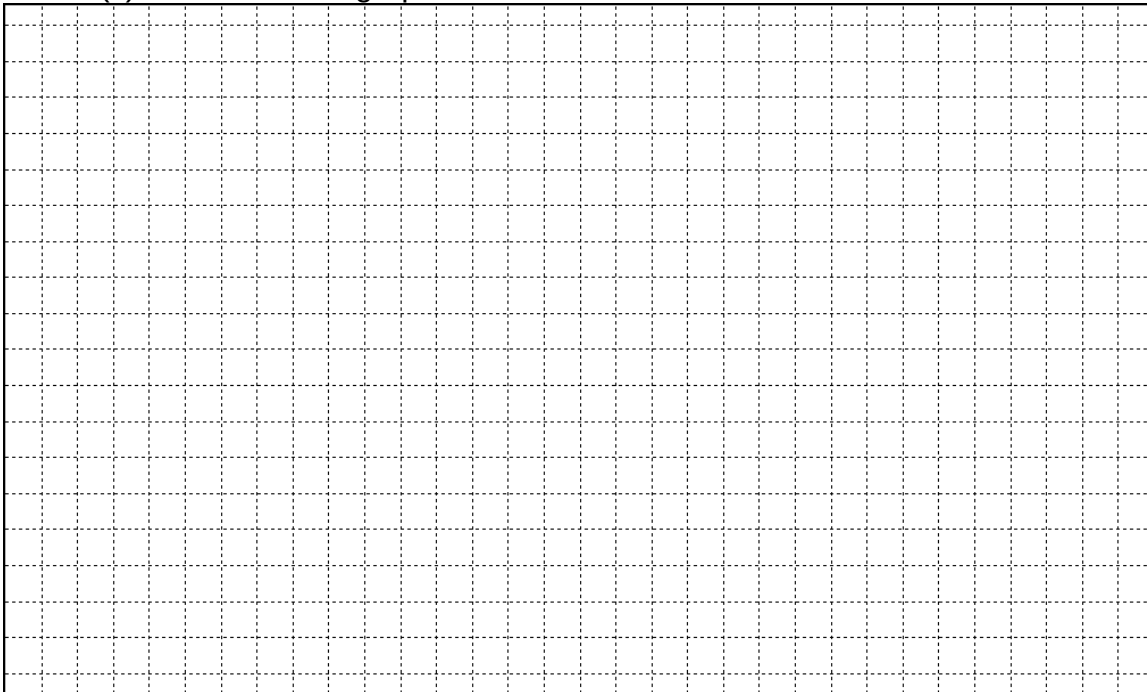
b) the value for $f(-2)$.

3+2 = 5 marks

18. (a) Factorise $x^3 - 5x^2 - 4x + 20$

(b) What are the co-ordinates of the axes intercepts for the graph of $= x^3 - 5x^2 - 4x + 20$?

(c) Sketch the graph.



(d) What are the co-ordinates of the turning points, correct to correct to 2 decimal places.

1 + 2 + 2 + 2 = 7 marks

19. Rewrite these in interval notation:

(a) $R^+ \setminus \{5\}$ _____

(b) $R^+ \setminus \{1 \leq x < 4\}$ _____

(c) $R^+ \cup \{-5 < x < -3\}$ _____

3 marks

20. If $f(x) = 3 - x^2$, find:

a) $f(-2)$

b) $f(m - 3)$

1 + 2 = 3 marks

21. Write in simplest index notation:

a) $3^{n+1} \times 9^{2n+3} \div 27^{1-3n}$

b) $\frac{(a^{-3}\sqrt{b^3})^4 \times (\sqrt{2}a^4b^{-3})^3}{\sqrt{2}(ab^{-2})^{-2}}$

2 + 3 = 5 marks

22. Solve for x in the following equations:

a) $3^{4x+1} = 243$

b) $5^{2x} - 6(5^x) + 5 = 0.$

2 + 3 = 5 marks

23.

a) Evaluate $\log_2(256)$, showing all working

b) Simplify $4\log_{10} 2 - 2\log_{10} 8$

c) Solve for x where $\log_5(2x - 3) = 2$

$2 + 2 + 2 = 6$ marks

24. The number of rabbits that are left on a farm t weeks after a virus is released is given by the function

$$N(t) = 15 + \frac{96}{t+3} \text{ rabbits per hectare.}$$

- (a) How many rabbits per hectare were on the farm when the virus was released?

- (b) How many rabbits per hectare are there 13 weeks after the virus was released?

- (c) How long after the virus is released are there 23 rabbits per hectare?

- (d) Will the virus kill all the rabbits? Explain your answer.

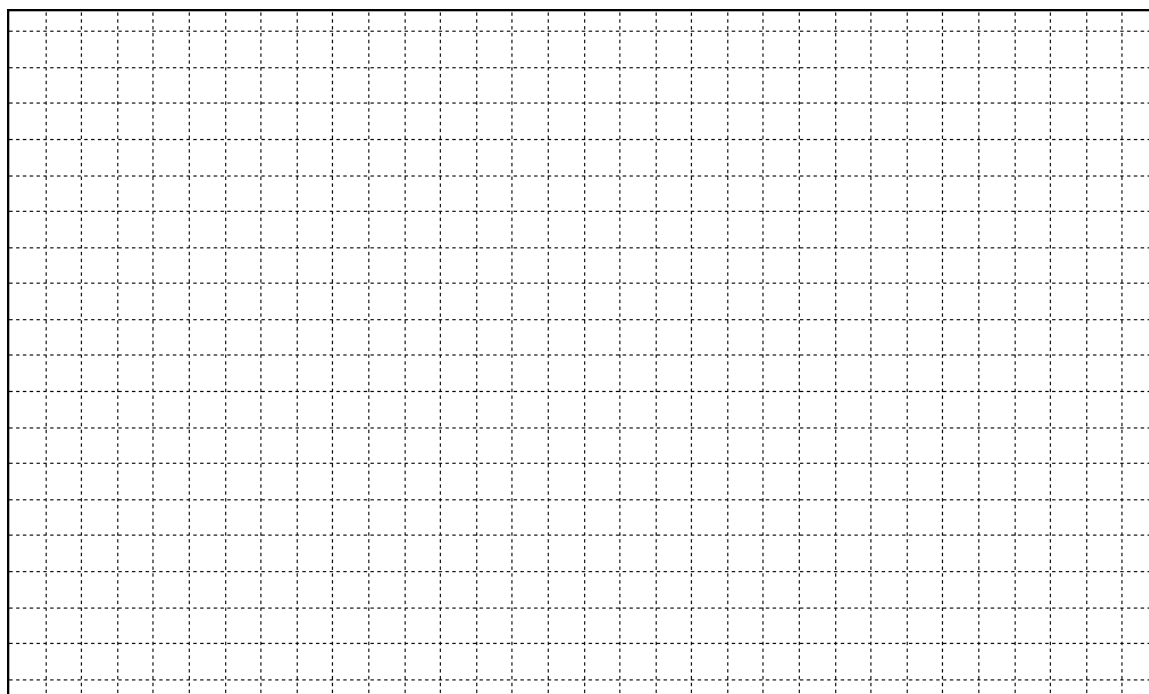
1 + 1 + 2 + 2 = 6 marks

25. The cost of hiring a moving van is described in the table below:

Hours of Hire	Cost
Up to 1	\$40
Over 1 up to 2	\$70
Over 2 up to 4	\$110
Over 4 up to 8	\$160

a) State the cost function, $C(t)$, for hiring up to 8 hours

b) Sketch the graph of the function



c) State the domain and range of the function.

2 + 3 + 2 = 7 marks

END OF PAPER