

Student Name.....*SOLUTIONS*.....

Teacher (circle one) JOR CWE

Homegroup



MATHEMATICAL METHODS (CAS) UNIT 1

EXAMINATION 2

Wednesday, 3th June, 2015

Reading Time: 9.00 – 9.15 (15 minutes)

Writing time: 9.15 – 10.45 (90 minutes)

Instructions to students

This exam consists of Section 1 and Section 2.

Section 1 consists of **12** multiple-choice questions, to be answered on the separate answer sheet. It is worth **12** marks.

Section 2 consists of **12** extended-answer questions that should be answered in the spaces provided. It is worth **77** marks

There is a total of **89** marks available.

All questions in Section 1 and Section 2 should be answered.

Unless otherwise stated, diagrams in this exam are not drawn to scale.

Where more than one mark is allocated to a question, appropriate working must be shown.

Where an exact answer is required to a question, a decimal approximation will not be accepted.

Students may bring one bound reference into the exam.

Students may bring an approved CAS calculator.

Section 1: Multiple Choice: Choose the best answer and write in the box shown. (12 marks)

1	B
2	B
3	D
4	A
5	D
6	E
7	B
8	C
9	E
10	M C
11	D or A!
12	E

Student Name: SOLUTIONS Home Group: _____

Teacher (circle):

Ms O'Rielly

Ms Webb

SECTION 1: MULTIPLE-CHOICE QUESTIONS

- 1 Which of the following relations are functions?

I $(x-2)^2 + (y+1)^2 = 16$ ✗

II $y^2 = \frac{2}{3}x - 1$ ✗

III $y = -2x + 4$

IV $y = 4x^2$

- A I and III
☒ B III and IV
 C I and II
 D II and III
 E I and IV

- 2 If $f(x) = 2 + \frac{3}{x}$ then the value of $f(3) - f(6)$ is

$f(3) = 2 + \frac{3}{3} = 3$

- A. -3
☒ B. $\frac{1}{2}$
 C. $1\frac{1}{2}$
 D. 3
 E. $2\frac{1}{2}$

$f(6) = 2 + \frac{3}{6} = 2\frac{1}{2}$

$f(3) - f(6) = 3 - 2\frac{1}{2} = 0.5$

- 3 The expansion of $(x-3)^3(x+2)$ is given by

CAS

- A $x^2 + x - 12$
 B $x^3 - 2x^2 - 15x + 36$
 C $x^4 + 4x^3 - 27x - 54$
☒ D $x^4 - 7x^3 + 9x^2 + 27x - 54$
 E $x^4 - 5x^3 - 9x^2 + 81x - 108$

- 4 The graph of the parabola with equation $y = -(x+3)^2 - 2$ has a turning point with coordinates

- ☒ A (-3, -2)
 B (-3, 2)
 C (3, -2)
 D (9, -2)
 E (3, 2)

- 5 The equation $3x + 2y - 2 = 0$ has gradient and y-intercept respectively equal to:

$2y = -3x + 2$

$y = -\frac{3}{2}x + 1$

A 3, -2

B -2, 4

C $\frac{2}{3}, \frac{4}{3}$

☒ D $-\frac{3}{2}, 1$

E $-\frac{2}{3}, 4$

- 6 The variables a and b are related by the formula $a = \frac{4b}{b-1}$.

Rearrangement of the formula shows that b is equal to:

A $\frac{4a}{a+4}$

B $\frac{a}{a+4}$

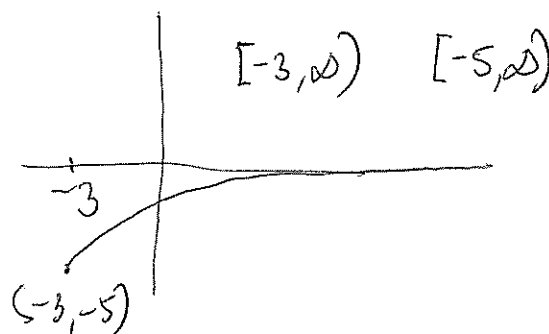
C $\frac{a+4}{a}$

D $\frac{4a}{4+a}$

☒ E $\frac{a}{a-4}$

- 7 A function has rule $f(x) = \sqrt{x+3} - 5$. The (implied) domain and range are:

- A domain: $[3, \infty)$; range: $[-5, \infty)$
☒ B domain: $[-3, \infty)$; range: $[-5, \infty)$
 C domain: $(3, \infty)$; range: $(-5, \infty)$
 D domain: $(-3, \infty)$; range: $(-5, \infty)$
 E domain: $[-3, \infty)$; range: R



- 8 The maximal domain and range for

$$g(x) = \frac{2}{1+3x}$$
 are respectively:

- A $R \setminus \left\{-\frac{3}{2}\right\}$ and $R \setminus \{0\}$
 B $R \setminus \left\{-\frac{2}{3}\right\}$ and $R \setminus \{4\}$
 C $R \setminus \left\{-\frac{1}{3}\right\}$ and $R \setminus \{0\}$
 D $R \setminus \left\{-\frac{2}{3}\right\}$ and $R \setminus \{0\}$
 E $R \setminus \left\{\frac{2}{3}\right\}$ and $R \setminus \{4\}$

- 9 The points (1, 4), (2, 0) and (4, p) lie on a straight line. The value of p is:

- A +2
 B -2
 C -4
 D -6
 E -8

$$m = \frac{-4}{1} = -4$$

$$C \Rightarrow A = -4(1) + C$$

$$A = -4 + C$$

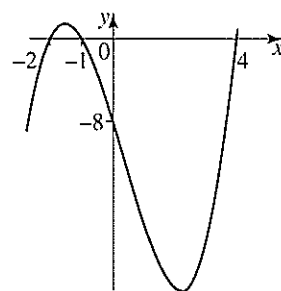
$$\Rightarrow C = 8$$

$$y = -4x + 8$$

$$= -1(4) + 8$$

$$= -8$$

10



$$(x-2)(x-1)(x+4)$$

The graph above represents the equation:

- A $f(x) = (x-2)(x-1)(x+4)$
 B $f(x) = -8(x+2)(x+1)(x-4)$
 C $f(x) = (x+2)(x+1)(x-4)$
 D $f(x) = (x-2)(x-1)(x+4)(x-8)$
 E $f(x) = (x+2)(x+1)(x-4)(x+8)$

11

The expression $\frac{(m^2n)^4}{(2m^5n^2)^3} \div \frac{(m^5n^2)^2}{2mn^5}$

can be simplified to:

- A $\frac{1}{4m^{16}n}$
 B $\frac{2^2}{m^{16}n}$
 C $\frac{1}{4m^8}$
 D $\frac{1}{4m^{16}n}$
 E $2^2m^{16}n$

$$\frac{m^8n^4 \times 2mn^5}{8m^{15}n^6 \times m^{10}n^4} = \frac{m^9n^9}{4m^{25}n^{10}} = \frac{1}{4m^{16}n}$$

12

The expression $\log_n \left(\frac{1}{n^4} \right)$ equals:

- A $\frac{n}{4}$
 B $4n$
 C 4
 D n^{-4}
 E -4

$$= \log_n n^{-4} = -4$$

SECTION 2 EXTENDED-ANSWER QUESTIONS

13. A line joins the points with coordinates $(-2, 5)$ and $(6, 9)$.

Find:

a) The equation of the line that joins the 2 points.

$$m = \frac{9-5}{6-(-2)} = \frac{4}{8} = \frac{1}{2}$$

$$y = mx + c$$

$$5 = \frac{1}{2}(-2) + c$$

$$5 = -1 + c$$

$$c = 6$$

$$y = \frac{1}{2}x + 6$$

or

$$2y - x - 12 = 0$$

b) The exact value (in simplest form) of the direct distance between the 2 points.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(6 - (-2))^2 + (9 - 5)^2}$$

$$= \sqrt{8^2 + 4^2}$$

$$= \sqrt{64 + 16}$$

$$= \sqrt{80} = \sqrt{16 \times 5}$$

$$= 4\sqrt{5}$$

c) The midpoint of the line.

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left(\frac{-2 + 6}{2}, \frac{5 + 9}{2} \right)$$

$$= (2, 7)$$

d) The equation of the perpendicular bisector of the line.

$$m_1 = \frac{1}{2} \Rightarrow m_2 = -2$$

$$y = mx + c$$

$$7 = -2(2) + c$$

$$7 = -4 + c$$

$$11 = c$$

$$y = -2x + 11$$

or $2x + y - 11 = 0$

e) State the domain and range of the line segment joining the points $(-2, 5)$ and $(6, 9)$

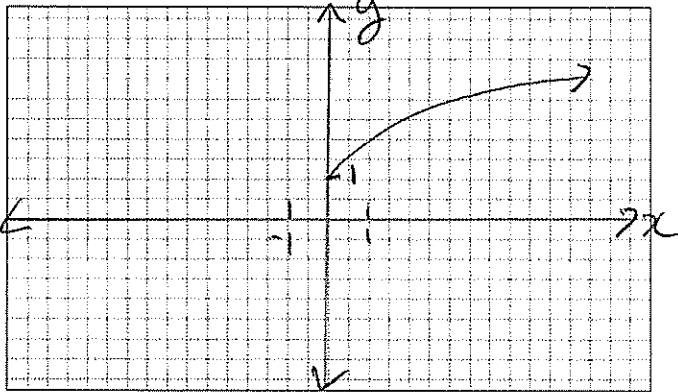
$$\text{domain} = [-2, 6]$$

$$\text{range} = [5, 9]$$

2 + 2 + 2 + 3 + 2 = 11 marks

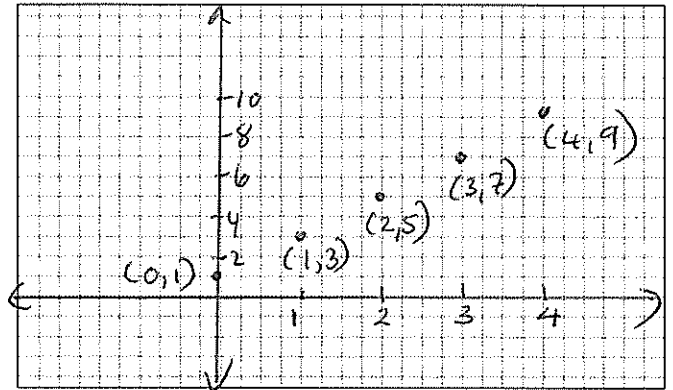
14. Sketch the graphs of each of the following equations. State the domain and range of each.

a) $y = \sqrt{x} + 1$, where $x \in \mathbb{R}$



Domain: $[0, \infty)$
Range: $[1, \infty)$

b) $y = 2x + 1$, where $x \in \{0, 1, 2, 3, 4\}$



Domain: $\{0, 1, 2, 3, 4\}$
Range: $\{1, 3, 5, 7, 9\}$

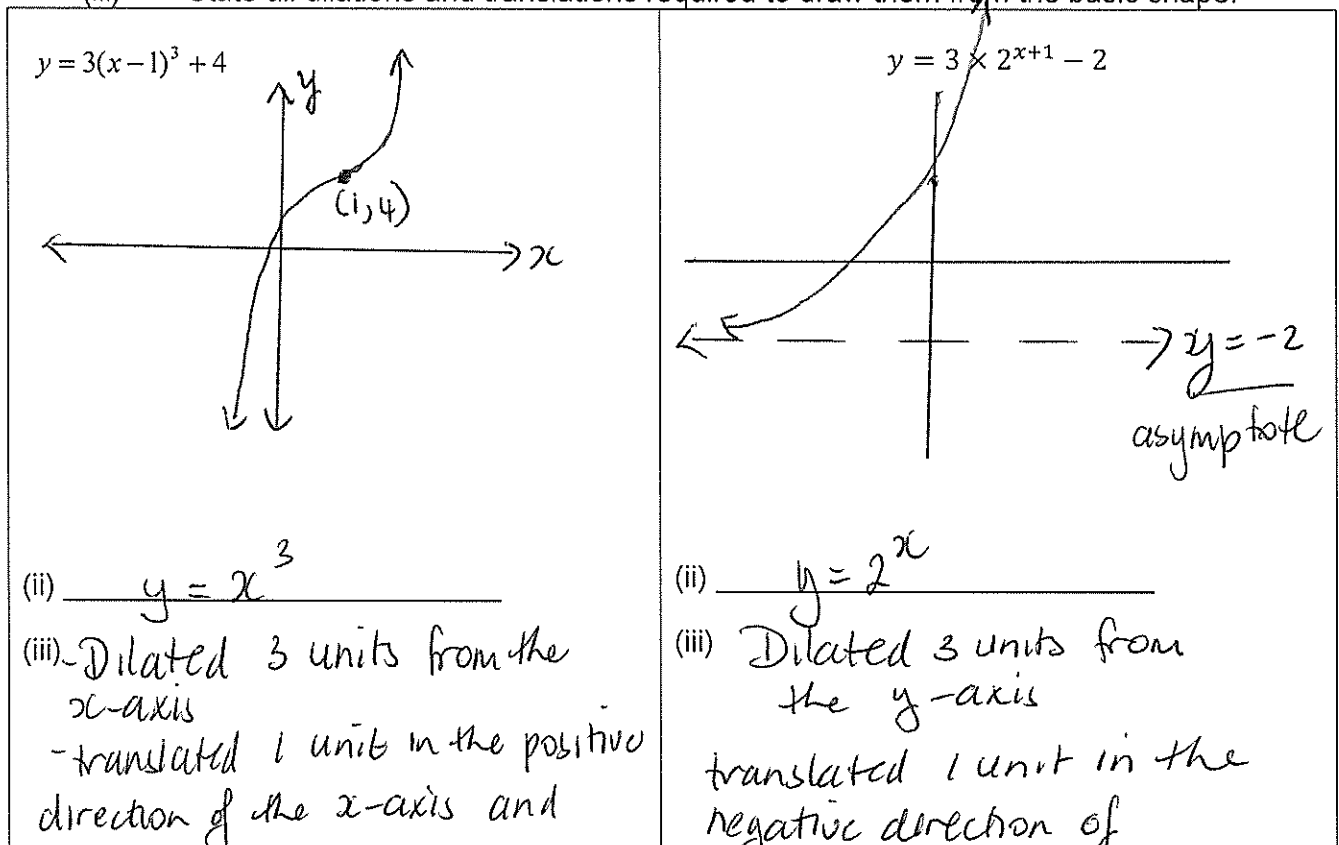
3 + 3 = 6 marks

15. (i) Sketch each of the following graphs, labelling the point of inflection or equation of any asymptotes where appropriate.

NOTE: You do not need to calculate any x or y intercept.

(ii) State the equation of the relevant basic shape graph.

(iii) State all dilations and translations required to draw them from the basic shape.



3 + 3 = 6 marks

16. a) Convert the following quadratic into turning point form: $y = x^2 + 4x - 7$

$$\begin{aligned} y &= x^2 + 4x - 7 \\ &= (x^2 + 4x + (2)^2) - (2)^2 - 7 \\ &= (x+2)^2 - 11 \end{aligned}$$

a) Hence, state the co-ordinates of the turning point.

$$(-2, -11)$$

b) State the domain and range

$$\text{domain : } \mathbb{R} \quad \text{range } [-11, \infty)$$

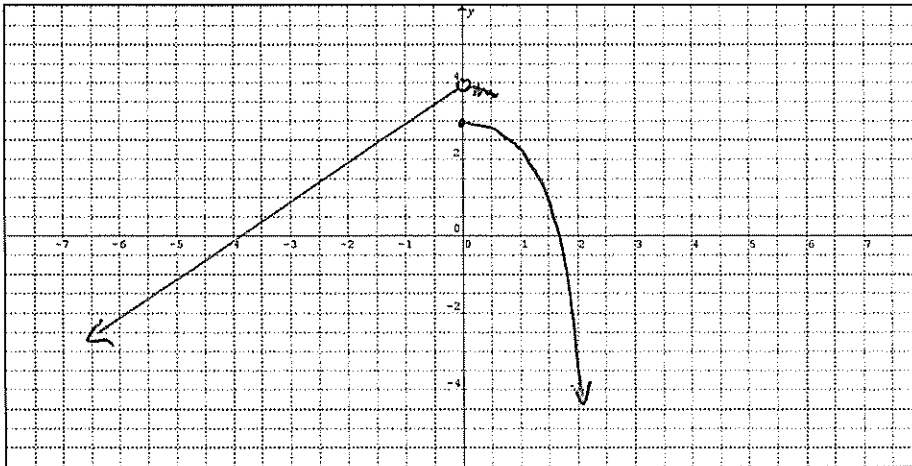
c) What translation would map the parabola $y = x^2$ onto $y = x^2 + 4x - 7$?

translation of 2 units in the negative direction of the x -axis and 11 units in the negative direction of the y -axis.

2 + 1 + 2 + 2 = 7 marks

17. If $f(x) = \begin{cases} 3 - x^2 & , x \geq 0 \\ x + 4 & , x < 0 \end{cases}$

Sketch this graph.



Find:

a) the range of $f(x)$ and

$$(-\infty, 4)$$

b) the value for $f(-2)$.

$$\begin{aligned} f(-2) &= -2 + 4 \\ &= 2 \end{aligned}$$

3+2 = 5 marks

18. (a) Factorise $x^3 - 5x^2 - 4x + 20$

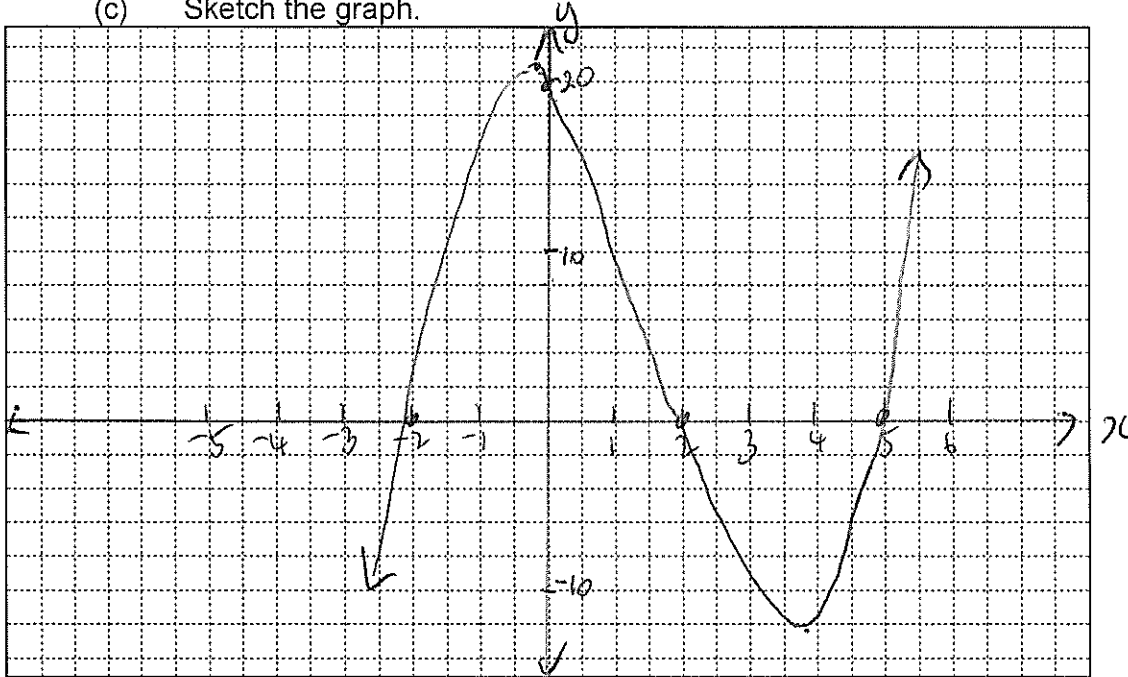
$$(x+2)(x-2)(x-5)$$

(b) What are the co-ordinates of the axes intercepts for the graph of $y = x^3 - 5x^2 - 4x + 20$?

x -intercepts $(-2, 0)$ $(2, 0)$ $(5, 0)$

y -intercept $(0, 20)$

(c) Sketch the graph.



(d) What are the co-ordinates of the turning points, correct to correct to 2 decimal places.

$(-0.36, 20.75)$ $(3.69, -12.60)$

1 + 2 + 2 + 2 = 7 marks

19. Rewrite these in interval notation:

(a) $\mathbb{R}^+ \setminus \{5\}$ $(0, 5) \cup (5, \infty)$

(b) $\mathbb{R}^+ \setminus \{1 \leq x < 4\}$ $(0, 1) \cup [4, \infty)$

(c) $\mathbb{R}^+ \cup \{-5 < x < -3\}$ $(-5, -3) \cup (0, \infty)$

3 marks

20. If $f(x) = 3 - x^2$, find:

a) $f(-2)$

$$\begin{aligned} f(-2) &= 3 - (-2)^2 \\ &= 3 - 4 \\ &= -1 \end{aligned}$$

$$\begin{aligned} \text{b) } f(m-3) &= 3 - (m-3)^2 \\ &= 3 - (m^2 - 6m + 9) \\ &= 3 - m^2 + 6m - 9 \\ &= -m^2 + 6m - 6 \end{aligned}$$

1 + 2 = 3 marks

21. Write in simplest index notation:

a) $3^{n+1} \times 9^{2n+3} \div 27^{1-3n}$

$$\begin{aligned} &= 3^{n+1} \times (3^2)^{2n+3} \div (3^3)^{1-3n} \\ &= 3^{n+1} \times 3^{4n+6} \div 3^{3-9n} \\ &= 3^{n+1+4n+6-(3-9n)} \\ &= 3^{5n+7-3+9n} \\ &= 3^{14n+4} \end{aligned}$$

b) $\frac{(a^{-3}\sqrt{b^3})^4 \times (\sqrt{2a^4b^{-3}})^3}{\sqrt{2}(ab^{-2})^{-2}}$

$$\begin{aligned} &= \frac{a^{-12} b^{\frac{3}{2} \times 4} \times 2^{\frac{3}{2}} a^{12} b^{-9}}{2^{\frac{1}{2}} a^{-2} b^4} \\ &= \frac{2^{\frac{3}{2}} b^{-3}}{2^{\frac{1}{2}} a^{-2} b^4} \\ &= \frac{2a^2}{b^7} \end{aligned}$$

2 + 3 = 5 marks

22. Solve for x in the following equations:

a) $3^{4x+1} = 243$

$$\begin{aligned} 3^{4x+1} &= 3^5 \\ 4x+1 &= 5 \\ 4x &= 4 \\ x &= 1 \end{aligned}$$

b) $5^{2x} - 6(5^x) + 5 = 0$

$$\begin{aligned} (5^x)^2 - 6(5^x) + 5 &= 0 \\ \text{Let } a &= 5^x \\ a^2 - 6a + 5 &= 0 \\ (a-5)(a-1) &= 0 \\ a &= 5 \text{ or } a = 1 \\ \therefore 5^x &= 5 \text{ or } 5^x = 1 \\ 5^x &= 5^1 \text{ or } 5^x = 5^0 \\ x &= 1 \text{ or } x = 0 \end{aligned}$$

2 + 3 = 5 marks

23.

a) Evaluate $\log_2(256)$, showing all working

$$\begin{aligned} &= \log_2 2^8 \\ &= 8 \log_2 2 \\ &= 8 \times 1 = 8 \end{aligned}$$

b) Simplify $4 \log_{10} 2 - 2 \log_{10} 8$

$$\begin{aligned} &4 \log_{10} 2^4 - \log_{10} 8^2 \\ &= \log_{10} \frac{16}{64} \\ &= \log_{10} \left(\frac{1}{4} \right) \end{aligned}$$

c) Solve for x where $\log_5(2x - 3) = 2$

$$\begin{aligned} 5^2 &= 2x - 3 \\ 2x - 3 &= 25 \\ 2x &= 28 \\ x &= 14 \end{aligned}$$

2 + 2 + 2 = 6 marks

24. The number of rabbits that are left on a farm t weeks after a virus is released is given by the function

$$N(t) = 15 + \frac{96}{t+3} \text{ rabbits per hectare.}$$

- (a) How many rabbits per hectare were on the farm when the virus was released?

$$N(0) = 15 + \frac{96}{3} = \underline{47} \text{ rabbits/hectare}$$

- (b) How many rabbits per hectare are there 13 weeks after the virus was released?

$$\begin{aligned} N(13) &= 15 + \frac{96}{13+3} \\ &= 21 \text{ rabbits/hectare.} \end{aligned}$$

- (c) How long after the virus is released are there 23 rabbits per hectare?

$$23 = 15 + \frac{96}{t+3}$$

$$8 = \frac{96}{t+3}$$

$$t+3 = \frac{96}{8} \text{ or}$$

$$t+3 = 12 \Rightarrow t = 9 \text{ weeks}$$

or solve on CAS

- (d) Will the virus kill all the rabbits? Explain your answer.

No, as t increases, the function

approaches the asymptote of $N(t) = 15$. $1 + 1 + 2 + 2 = 6$ marks
The no. rabbits will not go below 15 rabbits/hectare.

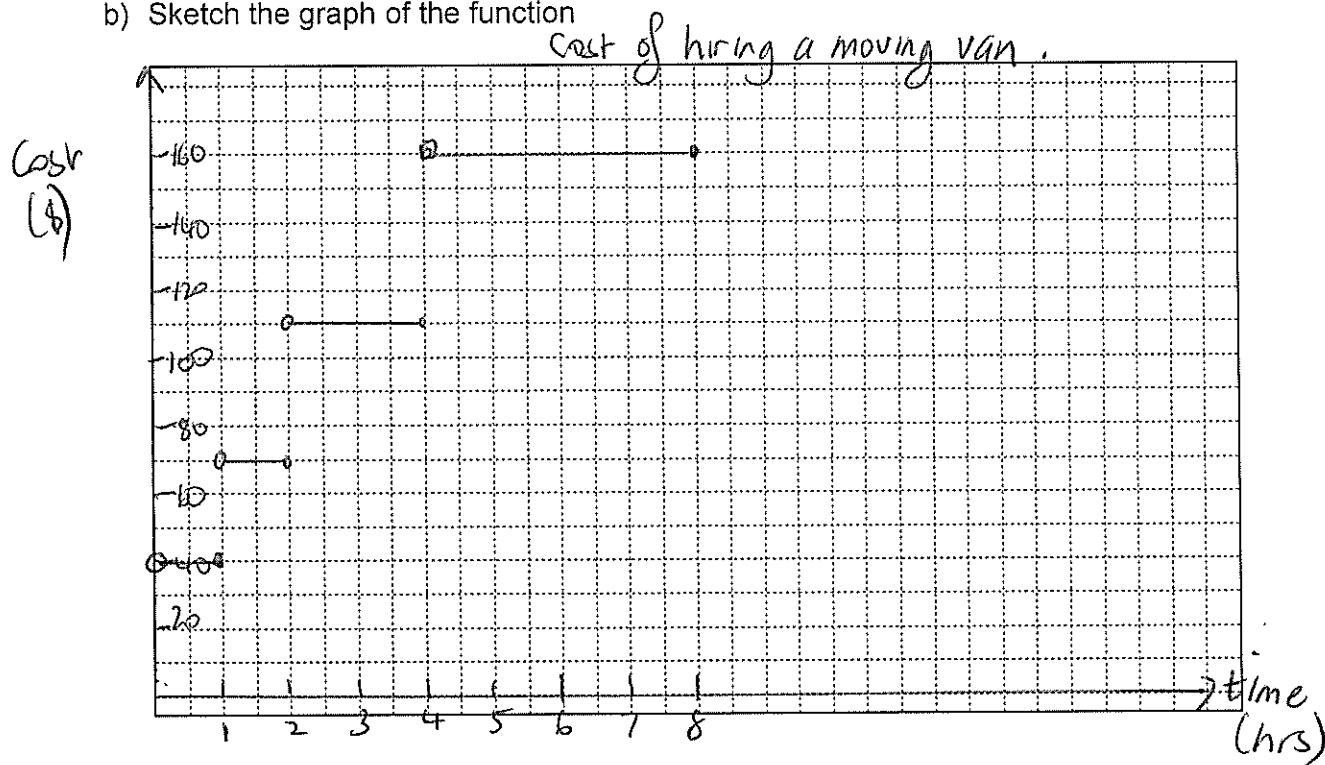
25. The cost of hiring a moving van is described in the table below:

Hours of Hire	Cost
Up to 1	\$40
Over 1 up to 2	\$70
Over 2 up to 4	\$110
Over 4 up to 8	\$160

a) State the cost function, $C(t)$, for hiring up to 8 hours

$$C(t) = \begin{cases} 40 & 0 < t \leq 1 \\ 70 & 1 < t \leq 2 \\ 110 & 2 < t \leq 4 \\ 160 & 4 < t \leq 8 \end{cases}$$

b) Sketch the graph of the function



c) State the domain and range of the function.

domain : $(0, 8]$

range $\{ 40, 70, 110, 160 \}$

2 + 3 + 2 = 7 marks

END OF PAPER