

Student Name.....

Teacher (circle one)

JOR

CWE

Homegroup



MATHEMATICAL METHODS (CAS) UNIT 1

EXAMINATION 1

Thursday, 4th June, 2015

Reading Time: 5 minutes

Writing time: 1 hour

Instructions to students

This exam consists of **13** questions.

All questions should be answered in the spaces provided.

There is a total of **61** marks available.

A decimal approximation will not be accepted if an exact answer is required.

Where more than one mark is allocated to a question working must be shown.

Students **may not** bring any notes or any calculators into this exam.

Diagrams in this exam are not to scale except where otherwise stated.

FORMULAS

Function and Graphs

Distance formula $d_{AB} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Midpoint formula $x_M, y_M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$

Straight line graphs

General equation $y = mx + c$

Gradient $m = \frac{y_2 - y_1}{x_2 - x_1}$

Equation through point (x_1, y_1) given by $y - y_1 = m(x - x_1)$

Difference/sum of squares and cubes

$$a^2 - b^2 = (a + b)(a - b)$$

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

Expansions

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$

1 Solve these equations for x :

(a) $3x^3 + 81 = 0$

$$x^3 + 27 = 0$$

$$x^3 = -27$$

$$x^3 = (-3)^3$$

$$x = -3$$

(b) $\frac{1}{x+2} = \frac{2}{6x-5}$

$$6x-5 = 2(x+2)$$

$$6x-5 = 2x+4$$

$$4x = 9$$

$$x = \frac{9}{4}$$

2 + 3 = 5 marks

2 Use the factor theorem and division to factorise $W(x) = x^3 - 5x^2 - 2x + 24$

$$P(2) = 8 - 20 - 4 + 24 \neq 0$$

$$P(-2) = -8 - 20 + 4 + 24 = 0 \Rightarrow (x+2) \text{ is a factor}$$

$$\begin{array}{r} x^2 - 7x + 12 \\ x+2 \overline{) x^3 - 5x^2 - 2x + 24} \\ \underline{-x^3 + 2x^2} \\ -7x^2 - 2x \\ \underline{-7x^2 - 14x} \\ 12x + 24 \\ \underline{12x + 24} \\ 0 \end{array}$$

$$\begin{aligned} P(x) &= (x+2)(x^2 - 7x + 12) \\ &= (x+2)(x-4)(x-3) \end{aligned}$$

5 marks

3 Let $f(x) = -3x^4 + 2x^2 - 3$

Evaluate:

i) $f(-1) = -3(-1)^4 + 2(-1)^2 - 3$

$$= -3 \times 1 + 2 \times 1 - 3$$

$$= -3 + 2 - 3 = \underline{-4}$$

ii) $f(\sqrt{2})$

$$= -3(2^{\frac{1}{2}})^4 + 2(2^{\frac{1}{2}})^2 - 3$$

$$= -3 \times 2^2 + 2 - 3$$

$$= -12 + 2 - 3 = \underline{-13}$$

2 marks

4 Expand

(a) $(2x-1)(x+1)(1-x)$ $= (2x-1)(1-x^2)$ $= 2x - 2x^3 - 1 + x^2$ $= -2x^3 + x^2 + 2x - 1$	(b) $(3x+2)^3$ $= (3x)^3 + 3(3x)^2(2) + 3(3x)(2)^2 + (2)^3$ $= 27x^3 + 54x^2 + 36x + 8$
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2 + 2 = 4 marks

5 Factorise the following completely

$4x^2 + 2x - 2$ $= 2(2x^2 + x - 1)$ $= 2(2x^2 + 2x - x - 1)$ $= 2(2x(x+1) - (x+1))$ $= 2(x+1)(2x-1)$	$3x^3y - 12xy^3$ $= 3xy(x^2 - 4y^2)$ $= 3xy(x+2y)(x-2y)$	$x^3 - 8$ $= (x-2)(x^2 + 2x + 4)$
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3 x 2 = 6 marks

6 Simplify this expression using appropriate logarithm or index laws

a) $\frac{25^{x+3} \times 5^{6x}}{125^{2x-1}}$

$$= \frac{(5^2)^{x+3} \times 5^{6x}}{(5^3)^{2x-1}} = \frac{5^{2x+6} \times 5^{6x}}{5^{6x-3}}$$

$$= \frac{5^{8x+6}}{5^{6x-3}}$$

$$= 5^{8x+6-(6x-3)}$$

$$= 5^{14x+9}$$

b) $3\log_3 18 + \log_3 2 - 2\log_3 12$

$$= \log_3 18^3 + \log_3 2 - 2\log_3 12^2$$

$$= \log_3 \frac{18^3 \times 2}{12^4}$$

$$= \log_3 (3 \times 3 \times 9)$$

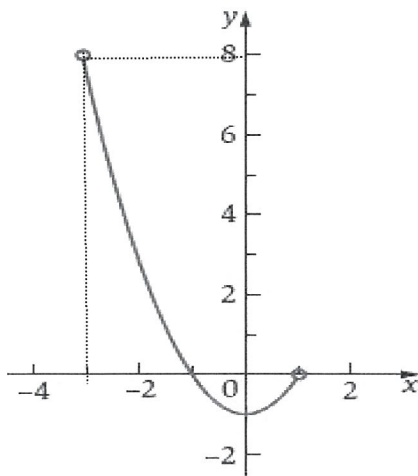
$$= \log_3 81$$

$$= \log_3 3^4$$

$$= 4$$

3+3=6 marks

7 State the domain and range of this graph

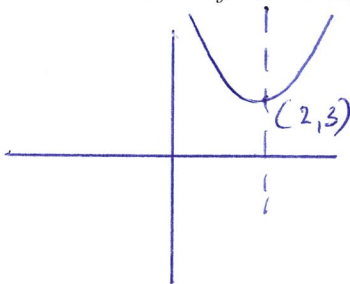


Domain $(-3, 1)$

Range $[-1, 8)$

2 marks

8 For the function $f: D \rightarrow R, f(x) = 2(x-2)^2 + 3$ find D , the largest domain for which the function is 1:
1



2 answers possible:

either $[-\infty, 2]$ or $[2, \infty)$

2 marks

9 List all the transformations that have been applied to the graph of $y = x^2$ to transform it into the graph of $y = \frac{1}{2}(x+4)^2 + 1$

1. Dilation $\frac{1}{2}$ unit from x -axis

2. Translation 4 units in the negative direction of the x -axis

3. Translation 1 unit in the positive direction of the y -axis

3 marks

10 Consider the points: A (5,-1) and B (1,3)

- (a) Find the distance from A to B.
Express your answer in simplest surd form.

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(1 - 5)^2 + (3 - (-1))^2} \\ &= \sqrt{4^2 + 4^2} = \sqrt{32} = \sqrt{16 \times 2} = 4\sqrt{2} \end{aligned}$$

- (b) Find the midpoint of the line segment AB

$$\begin{aligned} M &= \left(\frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2} \right) \\ &= \left(\frac{1 + 5}{2}, \frac{3 + (-1)}{2} \right) \\ &= (3, 1) \end{aligned}$$

- (c) Show that the point A(5,-1) lies on the line with equation $y = 2x - 11$.

$$\begin{aligned} \text{If } x &= 5, \quad y = 2(5) - 11 \\ &= -1 \\ &= \text{RHS} \end{aligned}$$

$\therefore (5, -1)$ does lie on the equation

- (d) Find the equation of the line that passes through the point A(5,-1) and is perpendicular to the line $y = 2x - 11$. Leave your answer in the form $ay + bx + c = 0$

For 2 lines perpendicular: $m_1 m_2 = -1$

Line 1: $m_1 = 2$

$$\therefore m_2 = -\frac{1}{2}$$

$y = mx + c$ Sub in (5, -1)

$$-1 = -\frac{1}{2}(5) + c$$

$$-1 + \frac{5}{2} = c$$

$$\frac{3}{2} = c$$

$$y = -\frac{1}{2}x + \frac{3}{2}$$

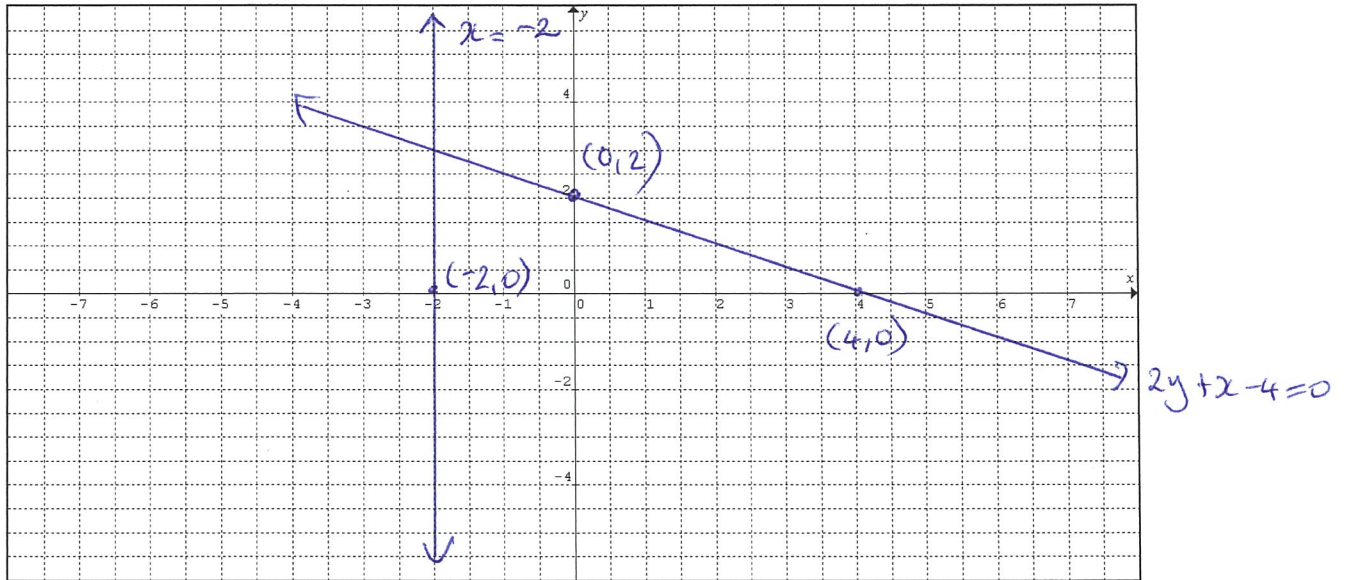
$$2y = -x + 3$$

$$\underline{x + 2y - 3 = 0}$$

2 + 1 + 1 + 3 = 7 marks

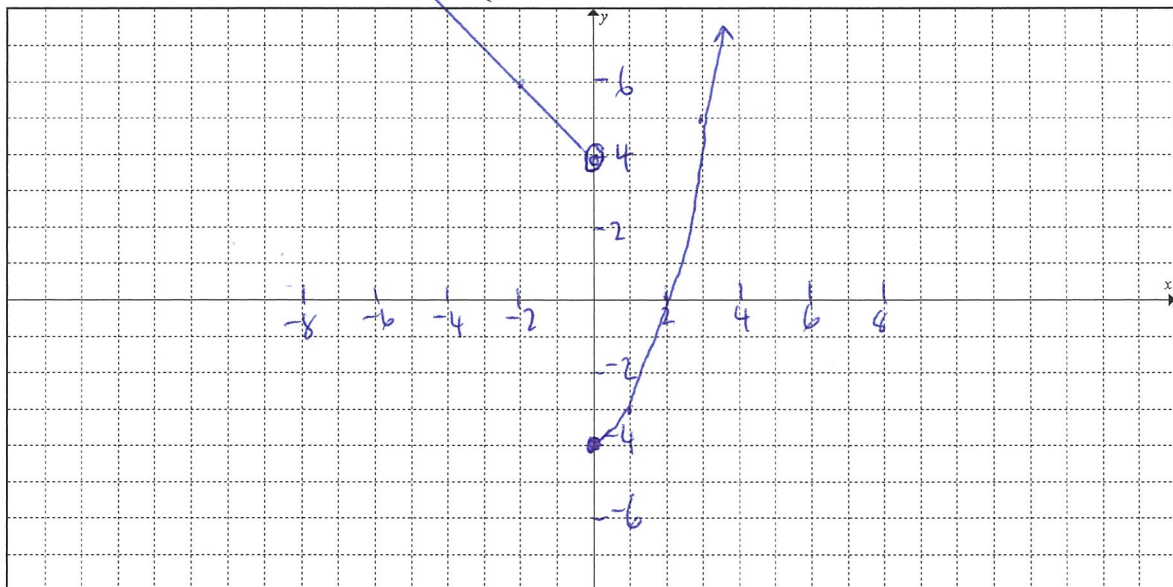
$$\begin{array}{l|l}
 2y+x-4=0 & y \text{ int } x=0 \\
 x-4=0 & 2y-4=0 \\
 x=4 & 2y=4 \Rightarrow y=2
 \end{array}
 \begin{array}{l}
 (4,0) \\
 (0,2)
 \end{array}$$

- 11 Sketch the graphs of the following, labelling axes intercepts with their coordinates.
 (a) $2y+x-4=0$ (b) $x=-2$



2 marks

- 12 (a) Sketch the graph of $f(x) = \begin{cases} 4-x & , x < 0 \\ x^2-4 & , x \geq 0 \end{cases}$



- (b) What is the domain and range of $f(x)$?

domain: $\mathbb{R}$ range $[-4, \infty)$

- (c) Find the value of $f(3)$. $f(3) = (3)^2 - 4 = 5$

- (d) Is $f(x)$ a function or a relation? Give reasons

function - each x value has only 1 unique y value.

3 + 2 + 1 + 2 = 8 marks

13 Consider the curve with equation. $y = \frac{1}{x-3} - 4$

(a) State the equations of the asymptotes.

$$x = 3, y = -4$$

(b) What are the coordinates of any axes intercepts?

x int $y=0$

$$0 = \frac{1}{x-3} - 4$$

$$4 = \frac{1}{x-3}$$

$$x-3 = \frac{1}{4} \quad (3.75, 0)$$

$$\Rightarrow x = 3\frac{3}{4}$$

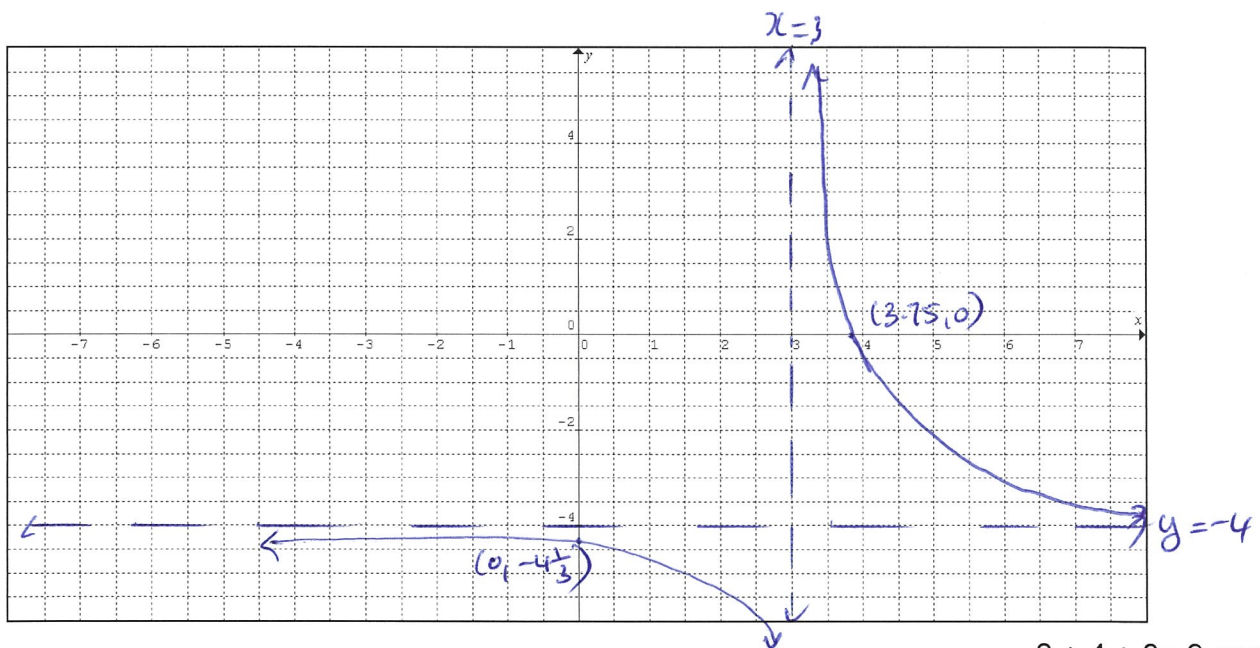
y-int $x=0$

$$y = \frac{1}{-3} - 4$$

$$= -4\frac{1}{3}$$

$$(0, -4\frac{1}{3})$$

(c) Sketch the curve. Label all axes intercepts with their co-ordinates and asymptotes with their equations.



2 + 4 + 3 = 9 marks

END OF PAPER