Name:
Teacher:

Warrnambool COLLEGE

Unit 1 Maths Methods (CAS) Exam 2014

Thursday June 5 - 1.50 pm

Reading time: 10 Minutes
Writing time: 80 Minutes

Instruction to candidates:
Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a single bound exercise book containing notes and class-work, CAS calculator.

Materials Supplied:
Question and answer booklet, detachable multiple choice answer sheet at end of booklet.

Instructions:

- Write your name and that of your teacher in the spaces provided.
- Answer all questions in this book where indicated.
- Answer the questions from your choice of detailed study on the separate answer sheet.
- Always show your full working where spaces are provided.
- All written answers must be in English.
- Students are NOT permitted to bring mobile phones and/or any other electronic communication devices into the examination room.

Section A	Section B	Total exam
/20	/30	/50

Section A – Multiple choice questions (20 marks)

Question 1

Which of the following correctly shows the transformation of the equation $E = \frac{1}{2}mv^2$ to make v the subject?

a) $v = \frac{1}{2}mE^2$

b) $v = 2mE^2$

c) $v = \sqrt{\frac{2E}{m}}$

d) $v = \sqrt{\frac{E}{2m}}$

e) $v = \frac{2E}{m}$

Question 2

A straight line has the equation $y = \frac{-x}{3} + 5$. Another form of this equation is:

a) $x - 3y = 5$

b) $3x + y = 15$

c) $x + 3y = 5$

d) $x - 3y = 15$

e) $x + 3y = 15$

Question 3

A straight line segment joins the points (2,4) and (6,14). The midpoint of the segment and distance between the points are respectively:

a) $(8,18)$ and $\sqrt{116}$

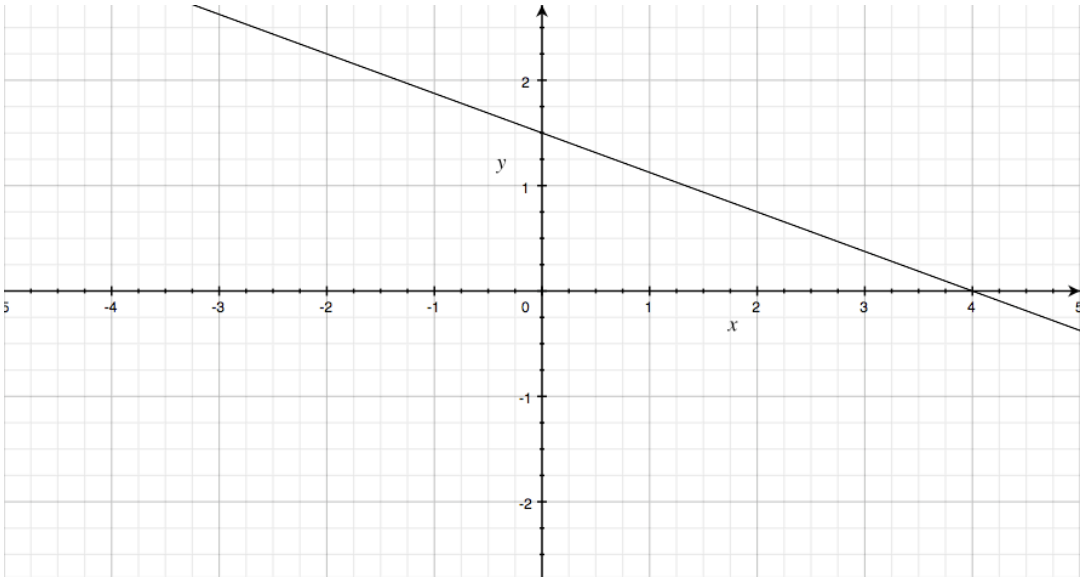
b) $(4,9)$ and $\sqrt{116}$

c) $(4,9)$ and 116

d) $(2,12)$ and $\sqrt{116}$

e) $(4,9)$ and 116^2

Question 4
The equation of the graph shown here is:



a) $y = \frac{3}{8}x + \frac{3}{2}$

b) $y = -\frac{3}{2}x + \frac{3}{8}$

c) $y = \frac{3}{2}x - \frac{3}{8}$

d) $y = \frac{3}{8}x - \frac{3}{2}$

e) $y = -\frac{3}{8}x + \frac{3}{2}$

Question 5

The straight lines $y = x - 3$ and $y = 2 - \frac{x}{4}$ intersect at the point:

a) $(2,3)$

b) $(3,2)$

c) $(1,4)$

d) $(4,1)$

e) $(5,12)$

Question 6

Which one of the following is **perpendicular** to the line $y = 2x + 3$?

a) $y = \frac{x}{2} - 5$

b) $y = \frac{x}{2} - \frac{1}{3}$

c) $y = \frac{x}{2} - 3$

d) $y = 5 - \frac{x}{2}$

e) $y = \frac{x}{3} + \frac{1}{2}$

Question 7

Which of the following is the **turning point form** of the quadratic $y = 2(x^2 + 4x - 12)$?

a) $y = 2(x - 2)(x + 6)$

b) $y = 2(x + 2)^2 - 32$

c) $y = (x + 2)^2 - 16$

d) $y = 2(x + 2)^2 - 8$

e) $y = 2(x - 2)^2 - 32$

Question 8

The equation $x^2 - 8x + 16 = 0$ has:

a) no real solutions
b) one rational solution
c) one irrational solution
d) two irrational solutions
e) two irrational solutions

Question 9

What is the turning point of the quadratic function $y = -x^2 + 6x - 5$?

a) $(3,4)$

b) $(-3,4)$

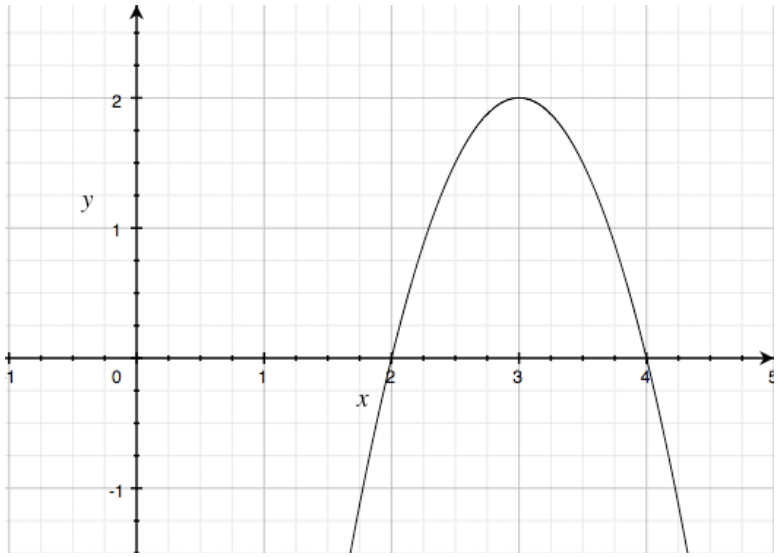
c) $(4,3)$

d) $(1,5)$

e) $(5,1)$

Question 10

A parabola is shown below. Which of the following equations correctly defines the parabola?



a) $y = -2(x+2)(x+4)$

b) $y = 2(x+2)(x+4)$

c) $y = (x-2)(x-4)$

d) $y = -(x-3)^2 + 2$

e) $y = -2(x-3)^2 + 2$

Question 11

Which of the following correctly lists all of the factors of the cubic function $y = x^3 - x^2 - 8x + 12$?

a) $(x-2)^2(x+3)$

b) $(x+2)^2(x+3)$

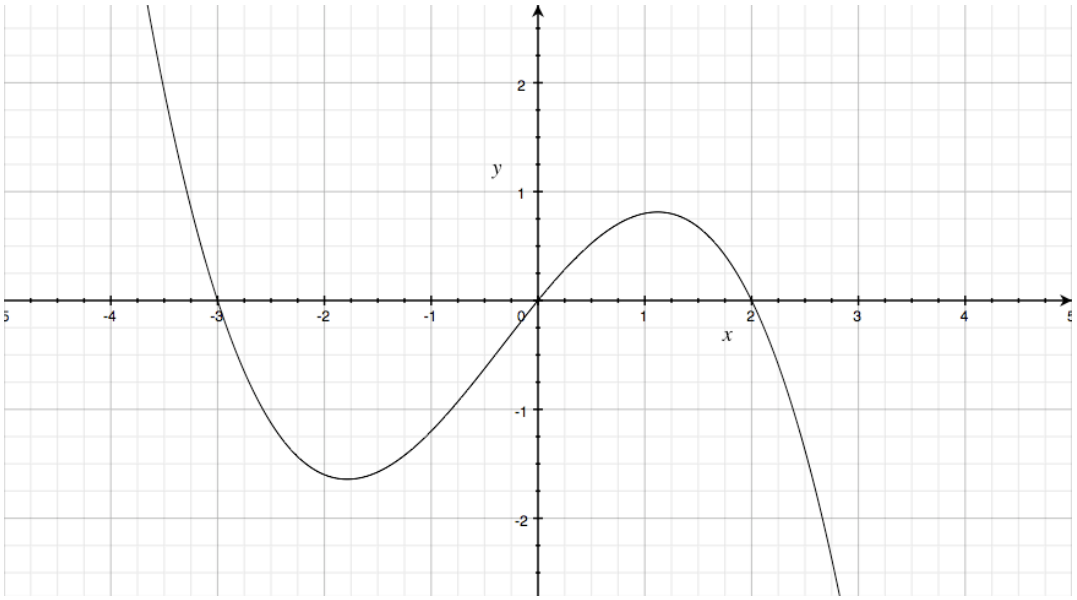
c) $(x-3)^2(x+2)$

d) $(x-2)(x-3)(x+2)$

e) $(x-2)(x+2)(x+3)$

Question 12

The graph of a cubic function is shown below. Which of the following equations describes the graph?



a) $y = -\frac{x}{5}(x-2)(x+3)$

b) $y = -\frac{x}{5}(x+2)(x-3)$

c) $y = x(x+2)(x-3)$

d) $y = -\frac{x^2}{5}(x+2)(x+3)$

e) $y = \frac{x}{5}(x-2)(x+3)$

Question 13

The complete set of solution(s) to the equation $27 = 64x^3$ is:

a) $x = \frac{4}{3}$

b) $x = -\frac{4}{3}$

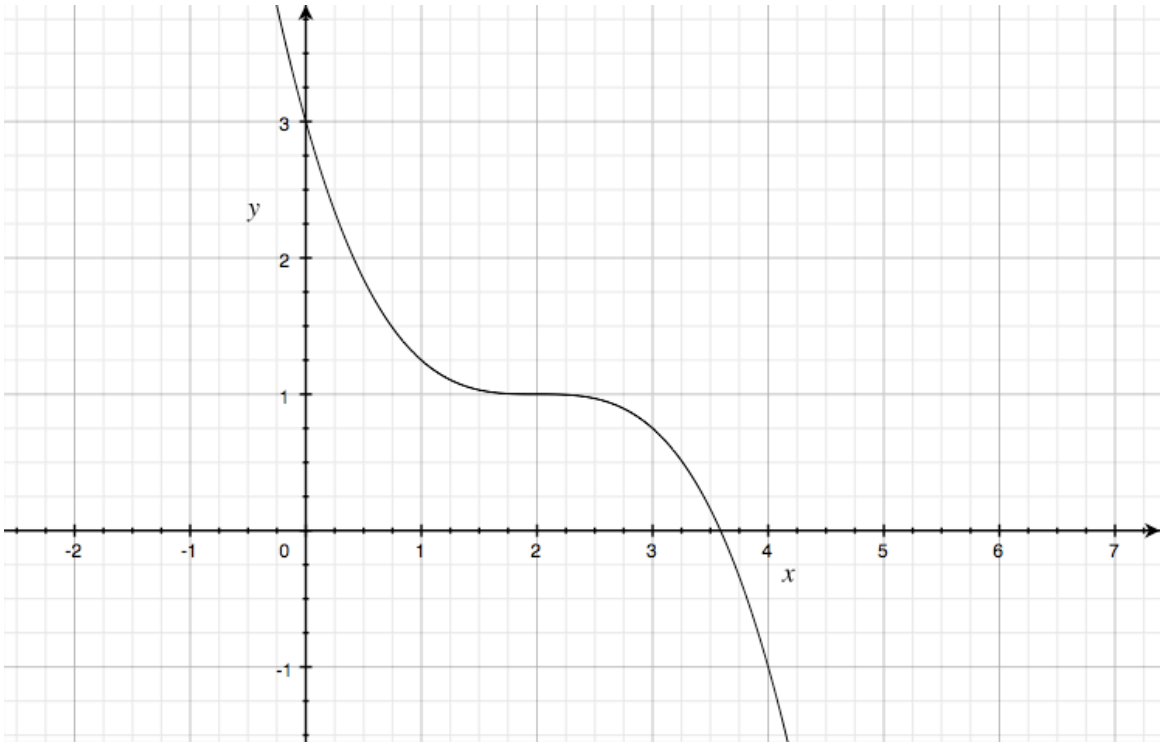
c) $x = \frac{3}{4}$

d) $x = \frac{3}{4}$ and $x = -\frac{3}{4}$

e) $x = \frac{3}{4}$ and $x = \frac{4}{3}$

Question 14

The graph of a cubic function is shown below. Which of the following equations describes the graph?



a) $y = -\frac{1}{4}(x-2)^3 + 1$

b) $y = \frac{1}{4}(x-2)^3 + 1$

c) $y = -(x-2)^3 + 1$

d) $y = -\frac{1}{4}(x+2)^3 - 1$

e) $y = \frac{1}{4}(x-2)^2 + 1$

Question 15

Which of the following correctly shows the method to solve the equation $7^x = 16807$?

a) $\log_x 16807 = 5$

b) $\log_7 16807 = x$

c) $\log_x \frac{7}{16807} = 5$

d) $\log_{16807} 7 = x$

e) $\log_7 \frac{7}{16807} = x$

Question 16

The expression $\frac{\log_5 b^2}{\log_5 b}$ is equal to:

a) -2
b) 2
c) $b^{\frac{3}{2}}$
d) b
e) 5^b

Question 17

A particular strain of bacteria is able to double in number every half hour given the correct conditions. This could be modelled by the equation $N = 2^{2t}$, where N is the number of bacteria and t is the time in hours.

The time taken for a such a bacterial colony to grow in number by a thousand times is closest to:

a) 5 hours
b) 10 hours
c) 50 hours
d) 100 hours
e) 1000 hours

Question 18

$x^{\frac{5}{2}}$ can be also be expressed as:

a) $\frac{x^5}{x^2}$
b) $x^5 - x^2$
c) $x^{-\frac{2}{5}}$
d) $\sqrt{x^5}$
e) $\sqrt[5]{x^2}$

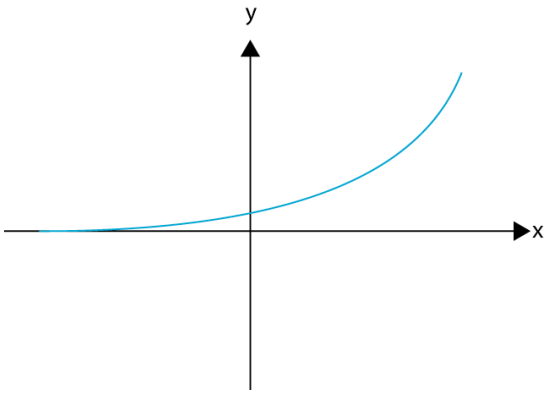
Question 19

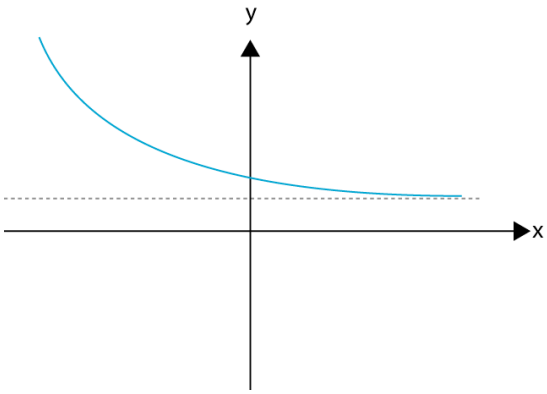
The solution to the equation $125 = 5^{4x-5}$ is:

a) $x = 1$
b) $x = 2$
c) $x = 3$
d) $x = 4$
e) $x = 5$

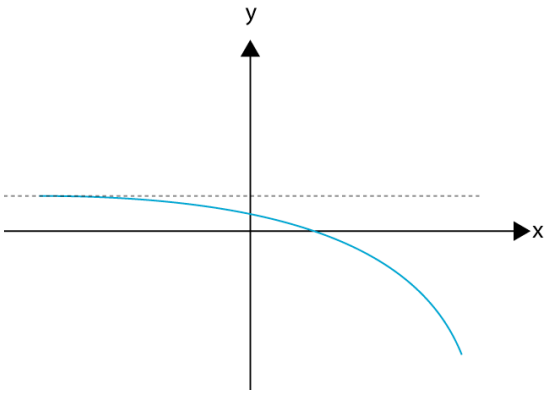
Question 20

The graph of the function $y = a^x$ is shown here. Which of the following graphs below correctly shows the function $y = a^{-x} + 2$?

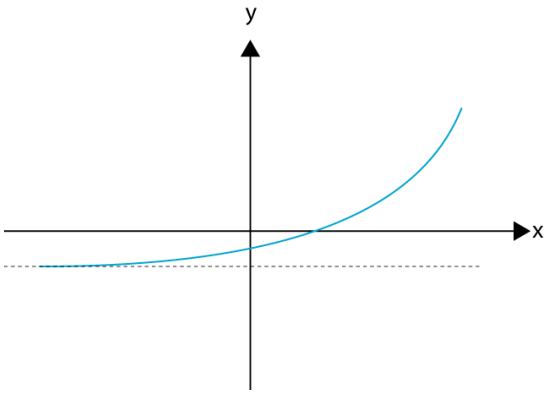




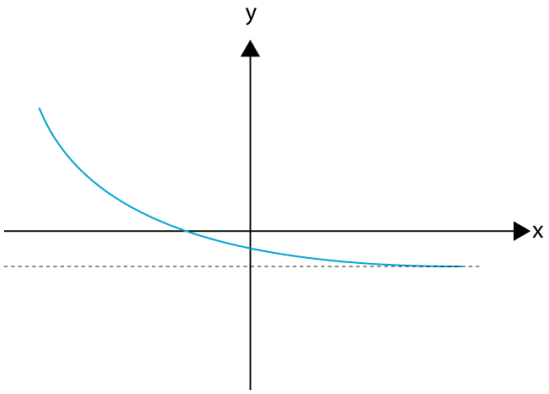
a)



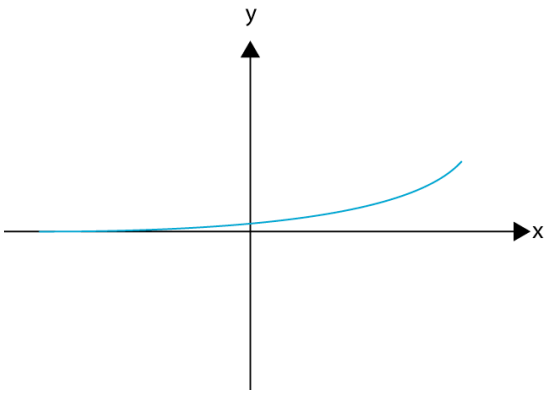
b)



c)



d)



e)

Section B – Short answer questions (30 marks)

Question 1

A potential buyer is investigating the overall costs of purchasing and running a new car. There are two models of the car:
Model P (Petrol engine): $24,000 and costs $12 / 100 km for fuel.
Model D (Diesel engine): $26,000 and costs $8 / 100 km for fuel.
All other costs for the two models are the same.

a) What is the **cost of fuel** for driving each of the cars for 20,000 km and 100,000 km?

	Model P	Model D
20,000 km		
100,000 km		

(2 marks)

b) Write an equation c(x) that gives the **total cost of purchase and fuel** as a function of each kilometre travelled for Model P.

(1 mark)

c) How many more kilometres can be driven in the diesel car (rather than petrol) using $1000 worth of fuel?

(2 marks)

d) Find how many kilometres of driving it will take for the overall cost of the diesel car (car + fuel) to be less than the petrol model.

(2 marks)

Question 2

The path taken by a human cannonball is given by the equation $h(x) = -\frac{1}{40}\left(x^2 - 50x\right)$, where h is the height above the ground and x is the horizontal distance travelled from the cannon.

a) Find the horizontal distance that the human cannonball would travel before hitting the ground.

(2 marks)

b) Find the the height of the human cannonball when he has travelled 10 m horizontally.

(2 marks)

c) Show that the maximum height reached by the human cannonball is 15.63 m above the ground.

(2 marks)

d) The landing net is 3 m above the ground. Using a CAS calculator or other means, find where the centre of the net should be placed from the cannon for a safe landing.

(2 marks)

Question 3

The shape of a slide is defined by the cubic function $y = -\frac{1}{250}(x^3 - 40x^2 + 500x - 2000)$, where y is the height above ground level and x is the horizontal distance. The slide starts at x = 0 and ends where the curve goes below ground level.

a) Find the initial starting height of the curve.

_____

_____

_____

(2 marks)

b) Find the height of the slide where x = 10.

_____

_____

_____

(2 marks)

c) Using a CAS calculator or other means, find the value of x (to two decimal places) where the height of the slide is 4 m.

_____

_____

(1 mark)

d) Using a CAS calculator or other means, find the value of x where the slide ends.

_____

_____

(1 mark)

Question 4

The polynomial function $p(x) = 2x^3 - 5x^2 - 39x - 18$ has an x intercept where x = 6.

Use polynomial long division to factorise p(x).

(3 marks)

Question 5

A bike tyre is slowly leaking and losing air pressure. The equation that describes the pressure (psi) over time (hours) after the leak was found is given by: $P(t) = 80 \times 10^{-0.02t}$.

a) Find the initial pressure in the tyre at the time that the leak was found.

(1 mark)

b) Find the pressure (to two decimal places) 24 hours after the leak was noticed.

(1 mark)

c) Find the time (to the nearest hour) at which the pressure had dropped to 40 psi.

(2 marks)

d) The cyclist knows that the tyre started at a pressure of 110 psi. How many hours before the leak was found did the tyre start to lose pressure?

(2 marks)

Answer sheet for section A

1.	a	b	c	d	e
2.	a	b	c	d	e
3.	a	b	c	d	e
4.	a	b	c	d	e
5.	a	b	c	d	e
6.	a	b	c	d	e
7.	a	b	c	d	e
8.	a	b	c	d	e
9.	a	b	c	d	e
10.	a	b	c	d	e
11.	a	b	c	d	e
12.	a	b	c	d	e
13.	a	b	c	d	e
14.	a	b	c	d	e
15.	a	b	c	d	e
16.	a	b	c	d	e
17.	a	b	c	d	e
18.	a	b	c	d	e
19.	a	b	c	d	e
20.	a	b	c	d	e

Summary

- **Students didn't do as well in the short answer questions (average ~50%) vs multiple choice questions (average ~75%). This is probably due to an emphasis on short, single skill questions in the text book rather than completing the longer application questions at the end of each chapter.**
- **Workings should be shown for the short answer questions to show an understanding rather than a calculator solution.**
- **As a general rule, any question worth more than one mark must have workings. Just an answer only gets one mark. (I have let a few of the more difficult questions through without full workings.)**
- **Workings also allow consequential marks to be given, where a small mistake in the process can be seen in the correct process for solving the problem.**
- **Draw a graph on your calculator! For four of the five short answer questions, your starting point should be graphing the function eg f1(x) to an appropriate scale. The graph tells you about the relationship and possible values.**
  Then: substituting in x values eg f1(25) to find y values, using solve to find x values eg (solve f1(x)=3,x). Graph trace is OK, but not as accurate.
- **Question 1: Think about your answer. Does it really cost $12 per km to drive a car?**
- **Question 2: Draw the graph, factorise to find where h = 0, maximum height occurs half-way between the x intercepts.**
- **Question 3: Draw a graph. The slide touches ground level at 10m, but then goes up & back down to 20m.**
- **Question 4: You were given an intercept of x = 6, this means that there is a factor (x-6).**
  If the question asks you to factorise, then there should always be a 0 remainder. (Factorise function on the CAS calculator!)
- **Question 5: This is an exponential decay, values decrease as x increases. Part d asks you to solve back in time, so the solution for x will actually be negative.**

Answer sheet for section A

1.	a	b	**c**	d	e
2.	a	b	c	d	**e**
3.	a	**b**	c	d	e
4.	a	b	c	d	**e**
5.	a	b	c	**d**	e
6.	a	b	c	**d**	e
7.	a	**b**	c	d	e
8.	a	**b**	c	d	e
9.	**a**	b	c	d	e
10.	a	b	c	d	**e**
11.	**a**	b	c	d	e
12.	**a**	b	c	d	**e**
13.	**a**	b	**c**	d	e
14.	**a**	b	c	d	e
15.	a	**b**	**c**	d	e
16.	a	**b**	c	d	e
17.	**a**	b	c	d	**e**
18.	a	b	**c**	**d**	e
19.	a	**b**	**c**	**d**	e
20.	**a**	b	c	**d**	e

Section B – Short answer questions (30 marks)

Question 1

A potential buyer is investigating the cost of purchasing and running a new car. There are two models of the car:
Model P (Petrol engine): $24,000 and costs $12 / 100 km for fuel.
Model D (Diesel engine): $26,000 and costs $8 / 100 km for fuel.
All other costs for the two models are the same.

a) What is the **cost of fuel** for driving the cars for 20,000 km and 100,000 km?

	Model P	Model D
20,000 km	$2,400	$1600
100,000 km	$12,000	$8,000

(2 marks)

b) Write an equation c(x) that gives the **total cost of purchase and fuel** as a function of each kilometre travelled for Model P.

$C(x) = 24,000 + 0.12x$

(1 mark)

c) How many more kilometres can be driven in the diesel car (rather than petrol) using $1000 worth of fuel?

Diesel:	Petrol:	The diesel car can travel 4167 more km.
$1000 = 0.08x$	$1000 = 0.12x$	
$x = 12,500km$	$x = 8,333km$	

(2 marks)

d) Find how many kilometres of driving it will take for the overall cost of the diesel car (car + fuel) to be less than the petrol model.

$24,000 + 0.12x = 26,000 + 0.08x$

$2,000 = 0.04x$

$x = 50,000$

(2 marks)

Question 2

The path taken by a human cannonball is given by the equation $h(x) = -\frac{1}{40}\left(x^2 - 50x\right)$, where h is the height above the ground and x is the horizontal distance travelled from the cannon.

a) Find the horizontal distance that the human cannonball would travel before hitting the ground.

$$h(x) = -\frac{1}{40}\left(x^2 - 50x\right) = 0$$

$$0 = -\frac{1}{40}x\left(x - 50\right)$$

$x = 50$ m

(2 marks)

b) Find the the height of the human cannonball when he has travelled 10 m horizontally.

$$h(10) = -\frac{\left(10^2 - 50 \times 10\right)}{40} = 10$$

(2 marks)

c) Show that the maximum height reached by the human cannonball is 15.63 m above the ground.

The maximum value of h(x) is when x = 25m

$$h(25) = -\frac{\left(25^2 - 50 \times 25\right)}{40} = 15.63$$

(2 marks)

d) The landing net is 3 m above the ground. Using a CAS calculator or other means, find where the centre of the net should be placed from the cannon for a safe landing.

$$3 = -\frac{\left(x^2 - 50x\right)}{40}$$

$$0 = -x^2 - 50x - 120$$

$x = 47.5$ m

(2 marks)

Question 3

The shape of a slide is defined by the cubic function $y = -\frac{1}{250}(x^3 - 40x^2 + 500x - 2000)$, where y is the height above ground level and x is the horizontal distance. The slide starts at x = 0 and ends where the curve goes below ground level.

a) Find the initial starting height of the curve.

$$y = -\frac{1}{250}(-2000) = 8m$$

(2 marks)

b) Find the height of the slide where x = 10.

$$y = -\frac{1}{250}(10^3 - 40 \times 10^2 + 500 \times 10 - 2000) = 0m$$

(2 marks)

c) Using CAS calculator or other means, find the value of x (to two decimal places) where the height of the slide is 4 m.

$$4 = -\frac{1}{250}(x^3 - 40x^2 + 500x - 2000)$$

$$x = 2.45m$$

(1 mark)

d) Using CAS calculator or other means, find the value of x where the slide ends.

$$0 = -\frac{1}{250}(x^3 - 40x^2 + 500x - 2000)$$

$x = 10$m or 20m (The end is 20m)

(1 mark)

Question 4

The polynomial function $p(x) = 2x^3 - 5x^2 - 39x - 18$ has an x intercept where x = 6.

Use polynomial long division to factorise p(x).

$$
\begin{array}{r}
2x^2 + 7x + 3 \\
x - 6 \overline{)\,2x^3 - 5x^2 - 39x - 18} \\
\underline{2x^3 - 12x^2} \\
7x^2 - 39x - 18 \\
\underline{7x^2 - 42x} \\
3x - 18 \\
\underline{3x - 18} \\
0
\end{array}
$$

The other two factors are (2x + 1) and (x + 3).

(3 marks)

Question 5

A bike tyre is slowly leaking and losing air pressure. The equation that describes the pressure (psi) over time (hours) after the leak was found is given by: $P(t) = 80 \times 10^{-0.02t}$.

a) Find the initial pressure in the tyre at the time that the leak was found.

$P(0) = 80 \times 10^{-0.02 \times 0} = 100 \times 1 = 80 psi$

(1 mark)

b) Find the pressure (to two decimal places) 24 hours after the leak was noticed.

$P(24) = 80 \times 10^{-0.02 \times 24} = 80 \times 0.33 = 26.5 psi$

(1 mark)

c) Find the time (to the nearest hour) at which the pressure had dropped to 40 psi.

$40 = 80 \times 10^{-0.02t}$

$0.5 = 10^{-0.02t}$

$t = \dfrac{\log_{10} 0.5}{-0.02} = 15h$

(2 marks)

d) The cyclist knows that the tyre started at a pressure of 110 psi. How many hours before the leak was found did the tyre start to lose pressure?

$110 = 80 \times 10^{-0.02t}$

$1.375 = 10^{-0.02t}$

$t = \dfrac{\log_{10} 1.375}{-0.02} = -7h$

(2 marks)