

		STUDENT NUMBER								Letter
Figures										
Words										

MATHEMATICAL METHODS (CAS)

Written examination 1

Created by James Hancock, 2012

QUESTION AND ANSWER BOOK

Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
10	10	38

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: notes of any kind, blank sheets of paper, white out liquid/tape or a calculator of any type.

Materials supplied

- Question and answer book of 13 pages, with a detachable sheet of miscellaneous formulas in the centrefold.
- Working space is provided throughout the book.

Instructions

- Detach the formula sheet from the centre of this book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Instructions

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

a. Differentiate $x^3 e^{2x}$ with respect to x .

1 mark

b. Let $f(x) = \log_e(\tan(2x))$. Find $f'\left(\frac{\pi}{8}\right)$.

2 marks

Question 2

a. Find an antiderivative for $12xe^{3x^2}$.

1 mark

b. Solve $2\log_e \sqrt{x-2} + \log_e(x-4) = 1$ giving exact answers.

3 marks

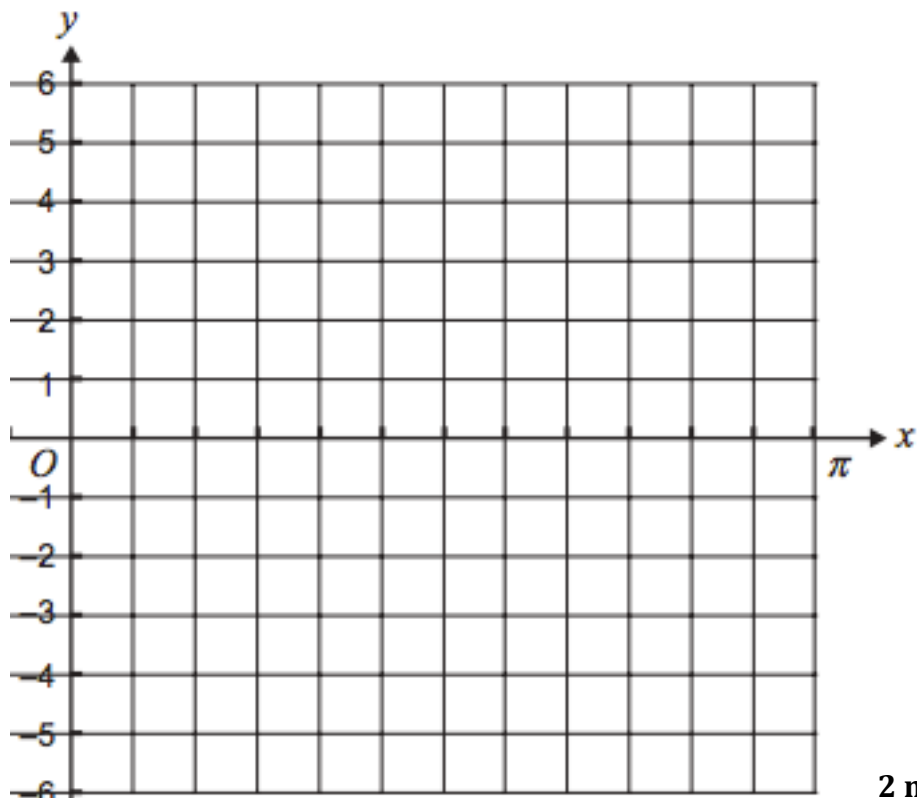
Question 3

a. Solve $f(x) = \tan\left(3x + \frac{\pi}{4}\right) = 0$ for $x \in \left[0, \frac{2\pi}{3}\right]$

This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and extend across the width of the page. There are no margins, text, or other markings on the paper.

2 marks

b. Plot $g(x) = 2f(x)$ for $x \in \left[0, \frac{2\pi}{3}\right]$ marking in all features.



2 marks

Question 4

a.

Given that $f(x) = \frac{1}{2x^3} - 2$, find $g(x) = f\left(\frac{1}{x}\right)$.

2 marks

b.

Find where $g(x) = 0$.

1 mark

Question 5

a.

Thomas drives to school each day and has to suffer through red lights. The probability that he will be stopped at a certain number of red lights is given in the discrete table below.

Pr(X=x)	m	0.2	0.4	0.15	k
x	0	1	2	3	4

Find k and m given that the expected number of lights he stops at is 2.05.

2 marks

b.

Find the probability that he stops at the number of lights he has to stop at in 2 consecutive days adds to 3.

2 marks

Question 6

a.

Brad has accidentally dropped his ice cream on the ground in a spherical blob. Unfortunately, it's a hot day and the ice cream begins to melt at a rate of $\frac{2T}{5} \text{ mm}^3/\text{s}$, where T is the outside temperature.

Assuming that the blob of ice cream remains spherical while melting, find the rate at which the radius changes in terms of T and r.

2 marks

b. Given that it is 25°C on this day, find the rate at which the radius is decreasing when the volume of the ice cream is $32\pi/3 \text{ mm}^3$.

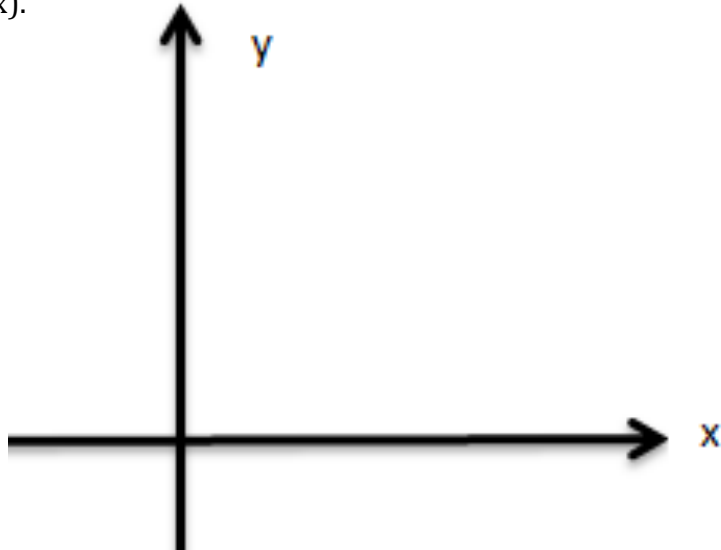
2 marks

Question 7

$$f(x) = \begin{cases} k \cos(x), & 0 \leq x \leq \frac{\pi}{2} \cup \frac{3\pi}{2} \leq x \leq 2\pi \\ 0, & \text{otherwise} \end{cases}$$

$f(x)$ describes a probability distribution function.

a. Sketch $f(x)$.



1 mark

b. Prove that $k = \frac{1}{2}$.

2 marks

c. Using information from previous steps or otherwise, find $\Pr(X < \frac{\pi}{2} \mid X < \frac{7\pi}{4})$

3 marks

Question 8

a.

Given that $y = -8x + a$ is normal to the equation $f(x) = \sqrt{x} + b$ at the point $(c, 9)$, find a , b and c .

4 marks

Question 9 –

An unfair coin has a probability p of getting landing on heads.

a.

Find the probability that out of 3 attempts, 3 tails show up.

1 mark

b.

Find the probability that out of 3 attempts, exactly 2 are heads.

1 mark

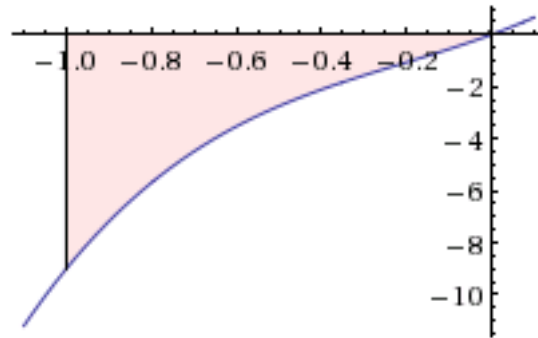
c.

Find the value of p such that the probability that makes the probability of 2 heads out of 3 coin flips a maximum.

2 marks

Question 10

If the area shaded below is equal to 19, and the graph of $f(x)$ passes through the point $(0,0)$ find a , b , c and d for $f(x) = a(x^3 + 5x^2 + 6x + d)$ where b and c are x -intercepts of the function $f(x)$.



4 marks