

James Hancock Methods Exam 1 –

Brackets denote mark distribution.

1.

$$2x^3 e^{2x} + 3x^2 e^{2x} \quad (1)$$

$$f'(x) = 2 \sec^2(2x) / \tan(2x) \quad (1)$$

$$f'\left(\frac{\pi}{8}\right) = \frac{2 \sec^2\left(\frac{\pi}{4}\right)}{\tan\left(\frac{\pi}{4}\right)} = 2 * \frac{2}{1} = 4 \quad (1)$$

2.

$$2e^{3x^2} + C \quad (+C \text{ is not necessary as it asked for AN antiderivative}) \quad (1)$$

$$2\log_e \sqrt{(x-2)} + \log_e(x-4) = 1$$

$$\log_e(x-2)(x-4) = \log_e(e) \quad (1) \text{ for correct log law usage}$$

$$(x-2)(x-4) = e$$

$$x^2 - 6x + 8 - e = 0$$

Using the quadratic formula:

$$x = \frac{6 \pm \sqrt{36 - 4(8 - e)}}{2} = \frac{6 \pm 2\sqrt{9 - (8 - e)}}{2} = 3 \pm \sqrt{1 + e} \quad (1) \text{ mark}$$

Providing an argument as to why $3 - \sqrt{1 + e}$ is an unacceptable solution (since $\ln(x-4)$ will have $x-4 < 0$) yields the third mark. (1mark)

3.

$$\tan\left(3x + \frac{\pi}{4}\right) = 0$$

$$3x + \frac{\pi}{4} = \tan^{-1}(0)$$

$$3x + \frac{\pi}{4} = 0, \pi, 2\pi \quad (1 \text{ mark})$$

$$3x = -\frac{\pi}{4}, \frac{3\pi}{4}, \frac{7\pi}{4}$$

$$x = \frac{\pi}{4}, \frac{7\pi}{12} \quad (1 \text{ mark})$$

Asymptote at $x = \pi/12$ and $5\pi/12$ (1 mark)

General tan shape (1 mark)

4.

$f(1/x) = 1/(2(1/x)^3) - 2 = x^3/2 - 2$ (2 marks, 1 for substitution, 1 for correct answer)

$$x^3/2 - 2 = 0$$

$$x^3 = 4$$

$$x = (4^{1/3}) = 2^{2/3} \quad (1 \text{ mark either is fine})$$

5.

a .

Given $E(X) = 2.05$

$$m * 0 + 0.2 + 0.8 + 0.45 + 4k = 2.05$$

$$4k = 0.6$$

$$k = 0.15 \quad (1 \text{ mark})$$

Since the sum of all probabilities = 1, $m = 0.1$ (1 mark)

b.

4 combinations (writing them down gives one mark, or having them in the form Pr(0 and 3) etc gives one mark)

$\Pr(\text{sum to } 3) = \Pr(0 \text{ and } 3) + \Pr(1 \text{ and } 2) + \Pr(2 \text{ and } 1) + \Pr(3 \text{ and } 0)$ (1 mark)

$= (0.1)(0.15) + (0.15)(0.1) + (0.2)(0.4) + (0.4)(0.2) = 0.19$ (19/100) (if arrived at answer, award all 2 marks)

6.

$$\frac{dV}{dt} = \frac{-2T}{5}$$

$$V = \frac{4}{3}\pi r^3 \quad \frac{dV}{dr} = 4\pi r^2 \quad (1 \text{ mark})$$

$$\frac{dr}{dt} = \frac{dr}{dV} * \frac{dV}{dt} = \left(\frac{1}{4\pi r^2}\right) * \left(\frac{-2T}{5}\right) = \frac{-T}{10\pi r^2} \quad (1 \text{ mark})$$

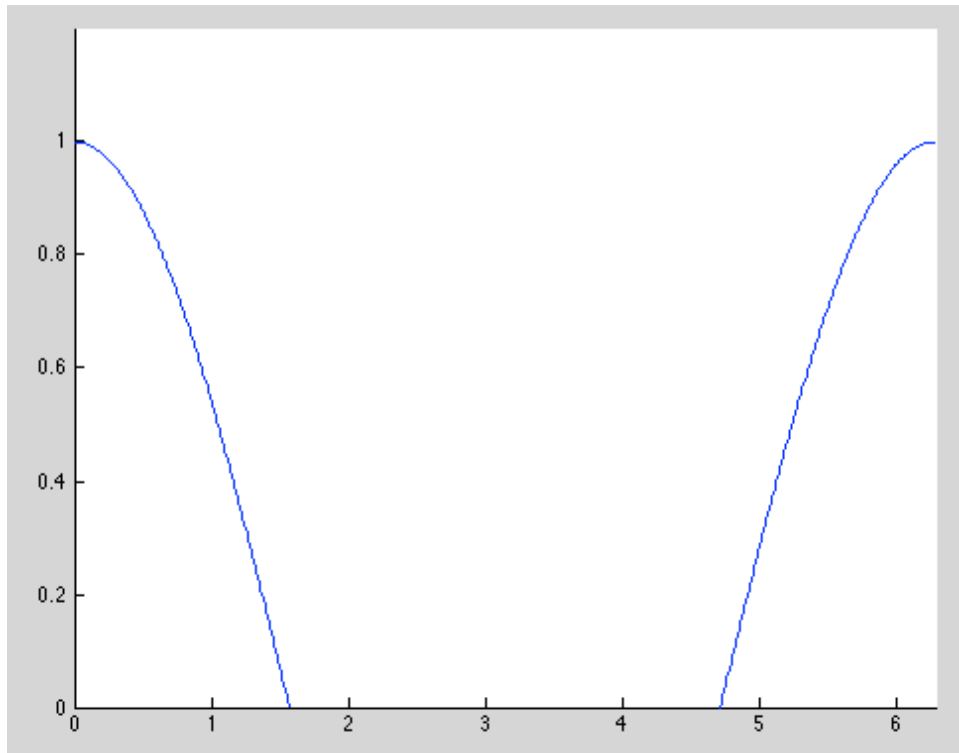
b.

When $V = 32\pi/3$, $r = 2$ (1 mark)

$$\frac{dr}{dt}(r = 2) = -\frac{25}{10\pi * 4} = -\frac{5}{8\pi}$$

7.

(1 mark, remember the y-axis is not 1, it is k).



b.

$$\int_{-\infty}^{\infty} f(x) dx = 1$$

Since the total area under the graph is equal to 2 (1 mark for showing this using integrals), k has to equal $\frac{1}{2}$ in order to have $f(x)$ define a probability distribution function. (1 mark for stating why k has to equal $\frac{1}{2}$, by referring to the total area under the graph equaling 1).

$$c. \Pr\left(X < \frac{\pi}{2} \mid X < \frac{7\pi}{4}\right) = \frac{\Pr\left(X < \frac{\pi}{2} \cap X < \frac{7\pi}{4}\right)}{\Pr\left(X < \frac{7\pi}{4}\right)} = \frac{\Pr\left(X < \frac{\pi}{2}\right)}{\Pr\left(X < \frac{7\pi}{4}\right)} \quad (1 \text{ mark})$$

Remember that

$$\Pr\left(X < \frac{\pi}{2}\right) = 0.5, \text{ and } \Pr\left(X < \frac{7\pi}{2}\right) =$$

$$0.5 + \int_{\frac{3\pi}{2}}^{\frac{7\pi}{2}} \cos(x) dx = 0.5 + \frac{1}{\sqrt{2}} \quad (1 \text{ mark for correct probabilities})$$

Therefore $\frac{\Pr\left(X < \frac{\pi}{2}\right)}{\Pr\left(X < \frac{7\pi}{2}\right)} = \frac{\frac{1}{2}}{\frac{1}{2} + \frac{1}{\sqrt{2}}} = \frac{1}{1 + \sqrt{2}}$ or $\frac{1}{1 + \frac{2}{\sqrt{2}}}$ (1 mark, for equivalent answer, award 1 mark)

8.

$$f'(x) = \frac{1}{2\sqrt{x}} \quad (1 \text{ mark})$$

Need to find where $f'(x) = 1/8$ (getting this from the normal equation).

This occurs when $x = 16$. Therefore $c = 16$ (1 mark)

Therefore, $f(16) = 9$, so therefore $b = 5$. (1 mark)

When now know that the point $x = 16, y = 9$ lies on the normal equation.

Therefore, $a = y + 8x = 9 + 16*8 = 137$ (1 mark)

9.

$$\Pr(3T) = (1-p)^3 \quad (1 \text{ mark})$$

b.

$$\Pr(X = 2) = nCr(3,2)*p^2(1-p) = 3p^2(1 - p) \quad (1 \text{ mark})$$

c.

$$\text{Therefore let, } P = 3p^2(1 - p)$$

$$P = 3p^2 - 3p^3$$

$$\frac{dP}{dt} = 6p - 9p^2 = 0 \quad (1 \text{ mark})$$

This occurs at $p = 2/3$. (1 mark)

10.

Since the cubic passes through the point $(0,0)$ $d = 0$. (1 mark)

Integrating from -1 to 0 will yield an area of $19/12$. Therefore $a = 12$. (1 mark for correct a and 1 mark for correct integration evaluation)

We know have the cubic $12(x^3 + 5x^2 + 6x)$. The x-intercepts are $x = -2$ and $x = -3$. Therefore $b = -2$ $c = -3$. (these values can be swapped). (1 mark).