



2024 Trial Examination



STUDENT
NUMBER

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MATHEMATICAL METHODS

Units 3 & 4 – Written examination 2

Reading time: 15 minutes

Writing time: 2 hours

QUESTION & ANSWER BOOK

Structure of book

<i>Section</i>	<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
1	20	20	20
2	4	4	60
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, and rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved CAS calculator and if desired, one scientific calculator.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 20 pages.

Instructions

- Print your name in the space provided on the top of this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic communication devices into the examination room.

SECTION 1 – Multiple-choice questions**Instructions for Section 1**

A correct answer scores 1, an incorrect answer scores 0. Marks are not deducted for incorrect answers. If more than 1 answer is completed for any question, no mark will be given.

Question 1

The period and range of $y = 3 - 2 \tan\left(3x - \frac{\pi}{2}\right)$ are:

- A. $\frac{\pi}{3}, [1, 5]$
- B. $\frac{2\pi}{3}, [1, 5]$
- C. $\frac{2\pi}{3}, [-1, 5]$
- D. $\frac{\pi}{3}, R$

Question 2

Consider the system of simultaneous linear equations below containing the parameter p .

$$\begin{aligned} px + 2y &= 7 \\ 3x + (p - 1)y &= 2p \end{aligned}$$

The value(s) of p for which the system of equations has a unique solution are:

- A. $p \in \{-2\}$
- B. $p \in \{-2, 3\}$
- C. $p \in R \setminus \{-2, 3\}$
- D. $p \in [-2, 3]$

Question 3

Suppose that $\int_a^c f(x) dx = 10$ and $\int_b^c f(x) dx = 2$, where $a < b < c$. $\int_b^a f(x) dx$ is equal to:

- A. 8
- B. 12
- C. -8
- D. -12

SECTION 1- continued

Question 4

The direct distance between two successive stationary points of the graph $y = a \sin(bx)$ is:

- A. $2\sqrt{a^2 + \left(\frac{\pi}{b}\right)^2}$
- B. $\sqrt{a^2 + 4\left(\frac{\pi}{b}\right)^2}$
- C. $\sqrt{4a^2 + \left(\frac{\pi}{b}\right)^2}$
- D. $\sqrt{2a^2 + 2\left(\frac{\pi}{b}\right)^2}$

Question 5

Consider the functions $f(x) = \sqrt{x+5}$ and $g(x) = \frac{1}{x-2}$. The maximal domain of $f + g$ is:

- A. $[-5, \infty)$
- B. $\mathbb{R} \setminus \{2\}$
- C. $(-5, 2)$
- D. $[-5, 2) \cup (2, \infty)$

Question 6

The maximum number of points of inflection of the graph of $y = x^4 + ax^3 + 3x^2 + bx + c$, where $a, b, c \in \mathbb{R}$ is:

- A. 0
- B. 1
- C. 2
- D. 3

SECTION 1 - continued
TURN OVER

Question 7

A bag contains a identical gold coins and b identical silver coins. Two coins are selected at random, one at time, without replacement. The probability they are identical is:

- A. $\frac{ab}{a+b}$
B. $\frac{a(a-1)+b(b-1)}{(a+b-1)^2}$
C. $\frac{a+b}{(a+b)(a+b-1)}$
D. $1 - \frac{2ab}{(a+b)(a+b-1)}$

Question 8

Which one of the following is an even function?

- A. $f(x) = 4x^3$
B. $f(x) = \sin\left(x + \frac{\pi}{2}\right)$
C. $f(x) = 2(x-1)^2$
D. $f(x) = \cos\left(x - \frac{3\pi}{2}\right)$

Question 9

The tangent to $y = x^2 + 5x + 1$ passes through the origin for what value(s) of x ?

- A. $x = 1$ and $x = -1$
B. $x = 1$ only
C. $x = -1$ only
D. $x = -1, 0$ and 1

Question 10

Suppose that the survival rate in a given population of a particular disease is 70%. A sample of 80 people are randomly selected from the population. The probability that at least 60 survive given that at least 50 survive is closest to:

- A. 0.2102
B. 0.1978
C. 0.3687
D. 0.3917

SECTION 1- continued

Question 11

Consider the function:

$$f(x) = \begin{cases} kx^2 + 3, & 0 \leq x \leq 5 \\ 0, & \text{otherwise} \end{cases}$$

In order for $f(x)$ to be a probability density function k must be:

- A. -6
- B. $-\frac{1}{3}$
- C. $-\frac{1}{15}$
- D. $-\frac{42}{125}$

Question 12

The probability density function for a discrete random variable is shown below.

X	0	1	2	3
$\Pr(X = x)$	a	a^2	$2a$	$0.4 - a^2$

The expectation of X is

- A. 0.2
- B. 1.92
- C. 2.12
- D. 2.08

Question 13

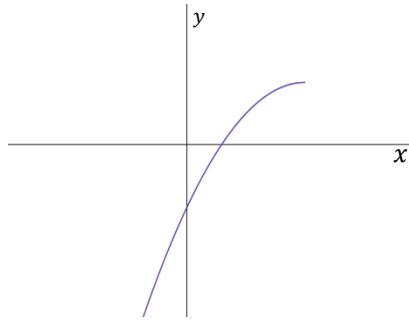
It is known that 65% of the population carries a particular genetic marker. A sample of size n of the population is taken. What is the minimum sample size required such that standard deviation of the sample proportion is below 0.1?

- A. 23
- B. 24
- C. 25
- D. 26

SECTION 1 - continued
TURN OVER

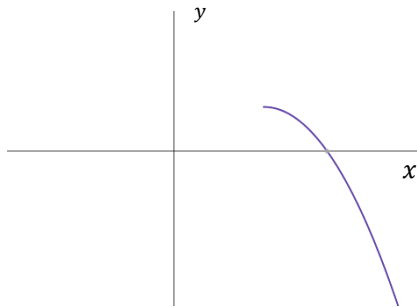
Question 14

Consider the graph below of $y = f(x)$

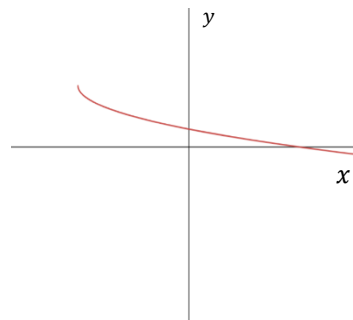


Which of the following could be the graph of $y = f^{-1}(x)$?

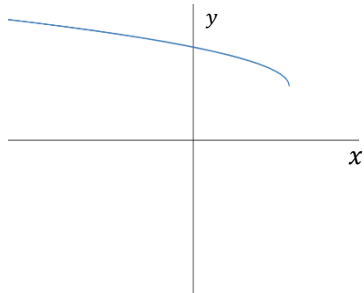
A.



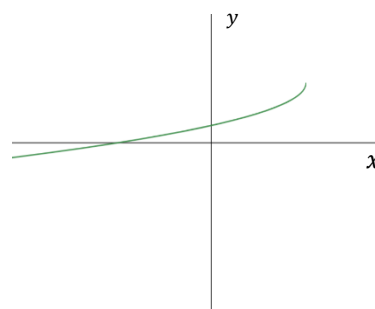
B.



C.



D.



SECTION 1- continued

Question 15

The average rate of change of $y = 2^{x+3} - 5$ between $x = 0$ and $x = 4$ is:

- A. 30
- B. 120
- C. $\frac{1}{30}$
- D. $\frac{1}{120}$

Question 16

The average value of $y = x + \sqrt{x}$ between $x = 1$ and $x = 5$ is:

- A. $\frac{3+\sqrt{5}}{4}$
- B. $3 + \sqrt{5}$
- C. $5\sqrt{5} + 17$
- D. $\frac{5\sqrt{5}+17}{6}$

Question 17

The function f is given by:

$$f(x) = \begin{cases} x^3 - 27, & x \leq 2 \\ bx^2 - 15x - 1, & x > 2 \end{cases}$$

The value of b for which f is continuous over the entire domain:

- A. 1
- B. 2
- C. 3
- D. 4

SECTION 1 - continued
TURN OVER

Question 18

Consider the function $f: (0, n] \rightarrow R, f(x) = \frac{m}{\sqrt{nx}}$ where $n \in R^+$ and $m \in R^+$. The range of f is:

- A. $\left[\frac{m}{n}, \infty\right)$
- B. $\left(0, \frac{m}{n}\right]$
- C. $\left[\frac{m}{\sqrt{n}}, \infty\right)$
- D. $(0, n]$

Question 19

Consider the following algorithm for Newton's method using a loop with 5 iterations

Inputs:

$f(x)$, a function of x

$df(x)$, the derivative of $f(x)$

x_0 , the initial estimate

Define newmet($f(x)$, $df(x)$, x_0)

For i **from** 1 **to** 5

If $df(x_0) = 0$ **Then**

Return "Error. Division by zero"

Else

$x_0 \leftarrow x_0 - f(x_0) / df(x_0)$

End For

Return x_0

The result for the function newmet($x^2 - 8x + 2, 2x - 8, 5$) is closest to:

- A. 8.8683
- B. 7.8363
- C. 7.7428
- D. 7.7417

SECTION 1- continued

Question 20

Consider the function $f(x) = x^3 - 2x^2 - x + 2$. An approximation can be found for the area bounded between $f(x)$ and the x axis using trapeziums of width 0.5. This approximation is closest to what percentage of the actual area?

- A. 90%
- B. 91%
- C. 92%
- D. 93%

END OF SECTION 1
TURN OVER

SECTION 2- Extended response questions**Instructions for Section 2**

A decimal approximation will not be accepted if the question specifically asks for an **exact** answer. In questions worth more than one mark, appropriate working **must** be shown.

Marks are given as specified for each question.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (13 marks)

Consider the function f with a rule given by: $f(x) = x^4 e^{-x}$

a. Find $f'(x)$

1 mark

b. Find the coordinates of, and classify, the stationary point(s) of $f(x)$.

2 marks

c. Find the coordinates of the point(s) of inflection of $f(x)$.

2 marks

SECTION 2- continued

d. Find $\{x: f'(x) < 0\}$

1 mark

e. State the values of c for which the graph of $y = f(x) + c$ has exactly one x intercept.

1 mark

f. Find the equation of the tangent to $f(x)$ at $x = 3$

1 mark

g. Determine the area, to three decimal places, bounded between $f(x)$ and the tangent found in **part f**.

2 marks

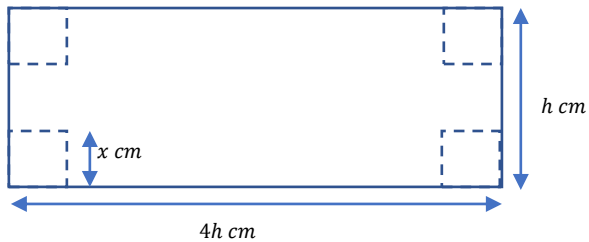
h. Let $P(x, y)$ be a point on the tangent found in **part f**, where $x \in [0, 3]$. Find the coordinates of P , correct to two decimal places, such that the vertical distance between P and the $f(x)$ is a maximum.

3 marks

SECTION 2 – continued
TURN OVER

Question 2 (10 marks)

A rectangular sheet of cardboard has a width of h cm and a length four times its width. Squares of side length x , where $x > 0$ are cut from each corner so that the four sides can be folded up to create an open box as seen in the diagrams below.



- a. Find a rule for V , the volume of the box, in terms of x and h .

2 marks

- b. State the domain of x in terms of h .

2 marks

- c. Find $\frac{dV}{dx}$ in terms of h .

1 mark

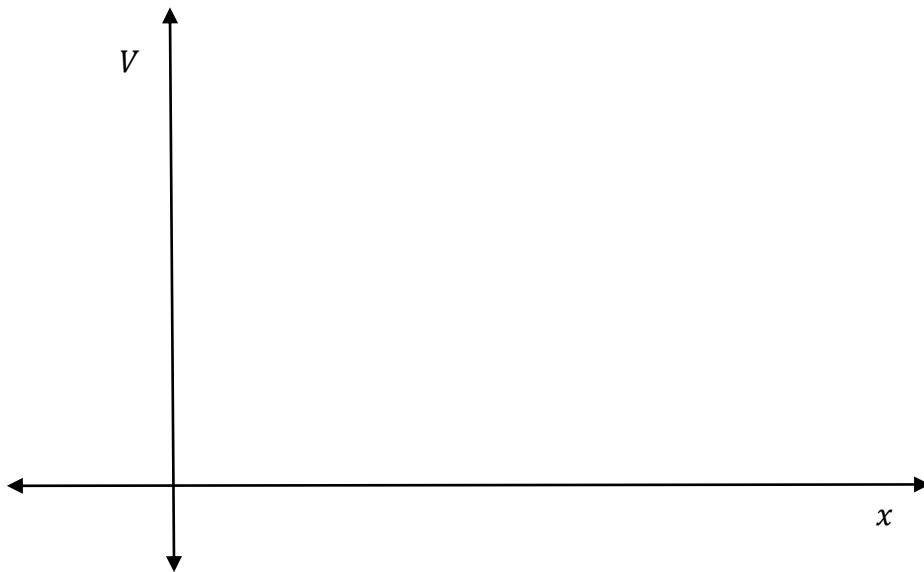
SECTION 2 - continued

- d. In terms of h , find the maximum volume of the box, and dimensions of the box for which this maximum volume occurs.

3 marks

- e. Graph $y = V(x)$ on the axes below, labelling all intercepts and stationary points.

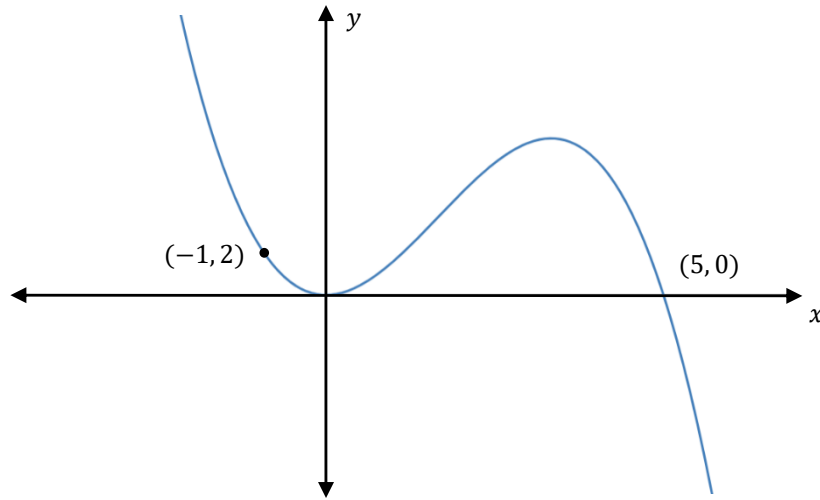
2 marks



SECTION 2 - continued
TURN OVER

Question 3 (16 marks)

Part of the graph of $y = g(x)$ is given below where $g(x) = ax^2(b - x)$, $a, b \in \mathbb{R}$



- a. Find the values of a and b .

2 marks

- b. Find $g'(x)$

1 mark

- c. State the value(s) of x for which $g(x)$ is strictly decreasing.

2 marks

SECTION 2 - continued

- d. Using trapeziums of width one unit, find an approximation for the area bound between $g(x)$ and the x axis.

2 marks

- e. Find the exact area bound between $g(x)$ and the x axis.

2 marks

Let the point $P(c, g(c))$ be the local maximum of $g(x)$.

A new function $h(x)$ is created such that:

$$h(x) = \begin{cases} g(x), & x \leq c \\ mx(n-x), & x > c \end{cases}$$

where $m, n \in R$.

It is known that $h(x)$ is continuous at $x = c$ and passes through the point $\left(4, \frac{50}{9}\right)$

- f. Find the exact values of m and n .

2 marks

SECTION 2 - continued
TURN OVER

- g.** Find the area bound between $h(x)$ and the x axis, correct to one decimal place.

2 marks

- h.** Find the co-ordinates of $h(x)$ where $(c < x < n)$ such that the gradient of $h(x)$ is equal to the average gradient of $h(x)$ between $x = c$ and $x = n$.

3 marks

SECTION 2 - continued

Question 4 (21 marks)

For a particular brand of trading cards, it is known that the value of any particular card is normally distributed with a mean of μ and a standard deviation of σ .

It is known that 1.5% of cards have a value less than \$2.90 and 9.1% of cards have a value greater than \$5.00.

Let the value of a trading card be the normal random variable X .

- a. Find the mean and standard deviation of X correct to two decimal places.

3 marks

- b. Find the probability that a randomly selected card has a value greater than \$3.50, correct to four decimal places.

1 mark

- c. Find the probability that a randomly selected card has a value greater than \$3.50, given it is known to have a value less than $E(X)$. Give your answer correct to four decimal places.

2 marks

SECTION 2 - continued
TURN OVER

- d. If it is known that $\Pr(n_1 < X < n_2) = 0.4$, find the values of n_1 and n_2 correct to two decimal places. (Assume n_1 and n_2 are symmetrical about the mean.)

2 marks

A particular collector of these cards refuses to trade a card if he knows the value is greater than \$5.50. He is also known to place any cards he owns worth more than \$6.00 in sealed bags.

- e. Of the cards this collector refuses to trade, what percentage, to the nearest whole percent, does he keep in sealed bags?

2 marks

Currently the collector has 2800 trading cards.

- f. What is the expected number of cards worth more than \$5.50? Give your answer correct to two decimal places.

1 mark

SECTION 2- continued

- g. What is the minimum number of trading cards the collector will need in order to ensure the probability of having at least 100 cards with a value of greater than \$5.50 is greater than 99%?

2 marks

The collector also likes to collect coins. The coins he collects can either be gold or silver. Of his collection of 500 coins, 220 of these are gold.

- h. Construct a 95% confidence interval, correct to four decimal places, for p the proportion of gold coins being circulated, and interpret this interval for this context.

2 marks

- i. If the confidence interval were changed to be a 99% confidence interval, by what factor, correct to two decimal places, would the width of the interval found in **part h.** be increased or decreased?

2 marks

SECTION 2 - continued
TURN OVER

A particular coin is known to have a value that is distributed with the following probability distribution.

$$P(v) = \begin{cases} kv^3e^{-v}, & v \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

Where v is the value of the coin in dollars.

- j.** Find the value of k such that $P(v)$ is a probability density function.

2 marks

- k.** Find the expected value of the coin.

2 marks

END OF QUESTION AND ANSWER BOOK