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2024 Trial Examination

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STUDENT NUMBER									Letter

MATHEMATICAL METHODS

Units 3 & 4 – Written examination 1

Reading time: 15 minutes
Writing time: 1 hour

QUESTION & ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
9	9	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.
- No calculator is permitted in this examination.

Materials supplied

- Question and answer book of 12 pages.

Instructions

- Print your name in the space provided on the top of this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic communication devices into the examination room.

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Instructions

Answer **all** questions in the spaces provided.
A decimal approximation will not be accepted if an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working must be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (4 marks)

a. If $y = x^2e^{4x}$, find $\frac{dy}{dx}$.

1 mark

b. Let $f(x) = \frac{\sin(x)}{x^2+e^x}$. Find $f'(0)$

3 marks

Question 2 (5 marks)

Consider $f: R\backslash\{-1\} \to R, f(x) = \frac{3x+2}{x+1}$

a. Write $f(x)$ in the form $f(x) = a + \frac{b}{x+1}$ where $a, b \in Z$.

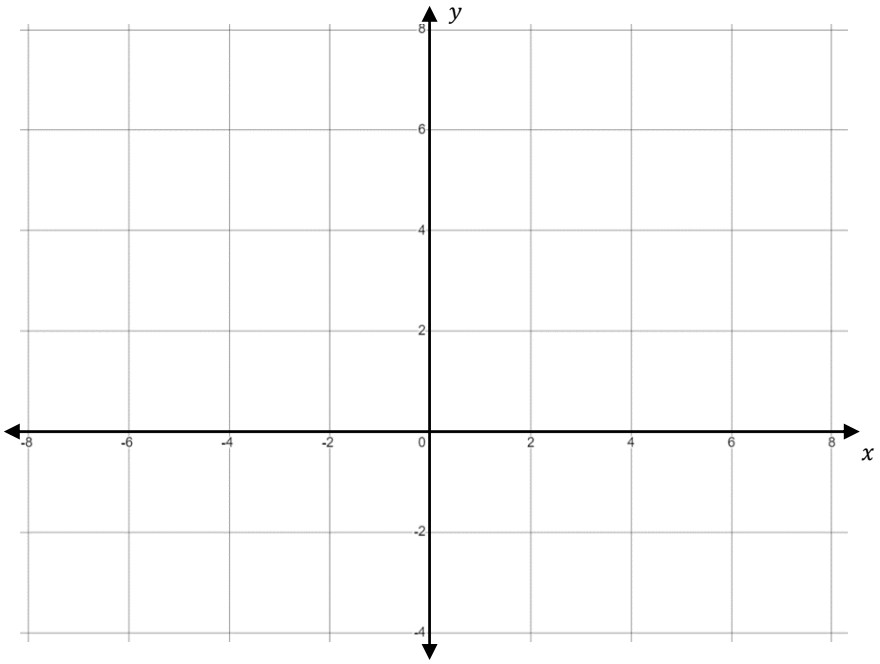
1 mark

b. State a series of transformations that take the graph of $y = \frac{1}{x}$ to the graph of $y = f(x)$.

2 marks

c. Graph $y = f(x)$ on the axes below, labelling all intercepts and asymptotes.

2 marks



Question 3 (4 marks)

a. Find the general solution to $2\tan(4x) - 2 = 0$, for $x \in R$

2 marks

b. Hence determine the sum of the first four positive solutions to $2\tan(4x) - 2 = 0$.

2 marks

TURN OVER

Question 4 (3 marks)

Consider the set of simultaneous equations, where m and n are real constants:

$$2x + (m + 1)y = 2$$
$$mx + 3y = 3n$$

Find the values of m and n for which the simultaneous equations have no solutions for x and y.

Question 5 (4 marks)

a. Show that $\frac{d}{dx}(x^2 \log_e(x)) = 2xlog_e(x) + x$

1 mark

b. Hence find $\int_1^e xlog_e(x)\,dx$. Give your answer in the form $\frac{1}{a}(e^b + c)$ where $a, b, c \in Z$.

3 marks

TURN OVER

Question 6 (4 marks)

Consider the functions g and h where:

$$g: R \to R, g(x) = x^2 + 3$$

$$h: (-\infty, 4] \to R, h(x) = \sqrt{4 - x}$$

a. Determine the rule and domain of $g \circ h$

2 marks

b. Let $g^*(x)$ be the function $g: A \to R, g(x) = x^2 + 3$.
Determine the maximal domain, A, such that $h \circ g^*$ exists.

2 marks

Question 7 (4 marks)

Hannah has 4 coins in each of her two pockets.
In her left pocket she has three $1 coins and one $2 coin.
In her right pocket she has two $1 coins and two $2 coins.

Hannah chooses a pocket at random and draws out a single coin.

a. What is the probability she draws out a $2 coin?

2 marks

b. If she has drawn out a $2 coin, what is the probability that it was drawn from her left pocket?

2 marks

TURN OVER

Question 8 (8 marks)

Consider the graph of $y(x) = 5 - 4x^2$

a. Calculate the average value of $y(x)$ between the two x intercepts

2 marks

b. Using the trapezium method, determine an estimate for the area bounded between $y(x)$ and the x axis using four trapeziums.

3 marks

c. Newton's method can be used to determine an estimate for the positive x intercept.
Find x_2 using $x_0 = 1$.

3 marks

TURN OVER

Question 9 (4 marks)

Consider the cubic $y = \frac{1}{3}mx^3 + 4x^2 + n$, where $m, n \in R$

a. Find the co-ordinates of the point of inflection in terms of m and n.

2 marks

b. Find the possible values of m and n such that the point of inflection is in the first quadrant.

2 marks

END OF QUESTION AND ANSWER BOOK