

MATHEMATICAL METHODS

Units 3 & 4 – Written examination 1



2024 Trial Examination

SOLUTIONS

Question 1 (4 marks)

a. $\frac{dy}{dx} = 2xe^{4x} + 4x^2e^{4x}$

1 mark

b. $f'(x) = \frac{(x^2+e^x)(\cos(x)) - (2x+e^x)\sin(x)}{(x^2+e^x)^2}$

2 marks

$$\begin{aligned} f'(0) &= \frac{(0+1)\cos(0) - (0+1)\sin(0)}{(0+1)^2} \\ &= \frac{1-0}{1} = 1 \end{aligned}$$

1 mark

Question 2 (4 marks)

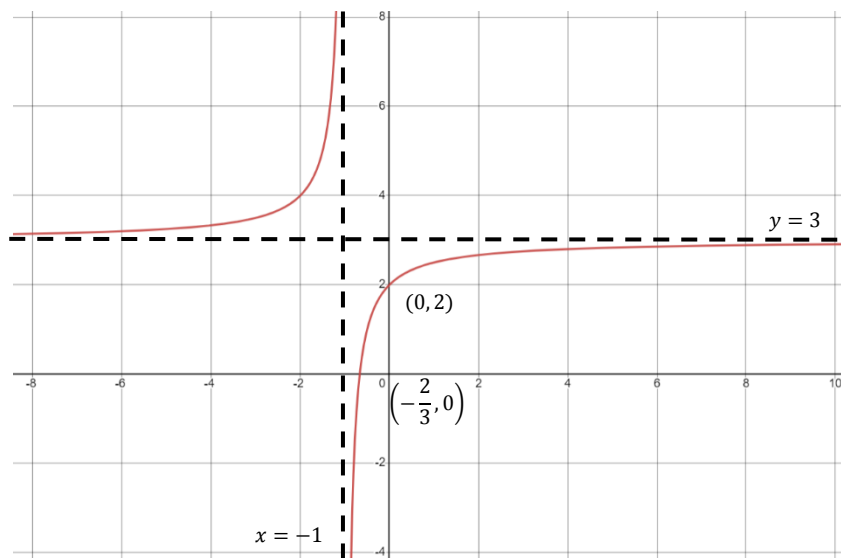
a.
$$\begin{aligned} f(x) &= \frac{3(x+1)-1}{x+1} \\ &= 3 - \frac{1}{x+1} = 3 + \frac{-1}{x+1} \end{aligned}$$

1 mark

- b. Reflection in the y-axis (or x-axis) followed by a translation of 3 units up and 1 unit to the left.

2 marks

c.



2 marks

Question 3 (4 marks)

a. $\tan(4x) = 1$

$$\theta_R = \frac{\pi}{4}$$

$$4x = \frac{\pi}{4} + n\pi, \quad n \in \mathbb{Z}$$

$$x = \frac{\pi}{16} + \frac{1}{4}n\pi, \quad n \in \mathbb{Z}$$

$$= \frac{\pi}{16}(1 + 4n), \quad n \in \mathbb{Z}$$

2 marks

b. $\frac{\pi}{16} + \frac{5\pi}{16} + \frac{9\pi}{16} + \frac{13\pi}{16}$

$$= \frac{28\pi}{16} = \frac{7\pi}{4}$$

2 marks

Question 4 (3 marks)

No solutions means that the linear equations have the same gradient but different y-intercepts.

Same gradient: $\frac{2}{m} = \frac{m+1}{3}$

$$\begin{aligned} m^2 + m - 6 &= 0 \\ (m + 3)(m - 2) &= 0 \\ m &= -3, 2 \end{aligned}$$

2 marks

For $m = -3$. $\frac{2}{-3} = -\frac{2}{3}$ $\frac{2}{3n} \neq -\frac{2}{3}$, $n \neq -1$

For $m = 2$, $\frac{2}{2} = 1$, $\frac{2}{3n} \neq 1$, $n \neq \frac{2}{3}$

So $m = -3$ and $n \in \mathbb{R} \setminus \{-1\}$ or $m = 2$ and $n \in \mathbb{R} \setminus \{\frac{2}{3}\}$

1 mark

Question 5 (4 marks)

a. $\frac{d}{dx}(x^2 \log_e(x)) = (2x)(\log_e(x)) + (x^2) \left(\frac{1}{x}\right)$
 $= 2x \log_e(x) + x$

1 mark

b. $\int (2x \log_e(x) + x) dx = x^2 \log_e(x)$
 $\int x \log_e(x) dx = \frac{1}{2} x^2 \log_e(x) - \frac{1}{4} x^2$

1 mark

So $\int_1^e x \log_e(x) dx = \left[\frac{1}{2} x^2 \log_e(x) - \frac{1}{4} x^2 \right]_1^e$
 $= \left(\frac{1}{2} e^2 \log_e(e) - \frac{1}{4} e^2 \right) - \left(\frac{1}{2} \log_e(1) - \frac{1}{4} \right)$
 $= \frac{1}{2} e^2 - \frac{1}{4} e^2 + \frac{1}{4}$
 $= \frac{1}{4} (e^2 + 1)$

2 marks

Question 6 (4 marks)

a. $g \circ h(x) = (\sqrt{4-x})^2 + 3 = 7 - x$
 $\text{dom } g \circ h = \text{dom } h = (-\infty, 4]$

2 marks

b. Need to restrict $\text{ran } g^*$ from $[3, \infty)$ to $[3, 4]$
 $x^2 + 3 \leq 4$
 $x^2 - 1 \leq 0$
 $x \in [-1, 1]$

2 marks

Question 7 (4 marks)

a. $\Pr(\$2) = \Pr(L \cap \$2) + \Pr(R \cap \$2)$
 $= \frac{1}{2} \times \frac{1}{4} + \frac{1}{2} \times \frac{1}{2}$
 $= \frac{1}{8} + \frac{1}{4} = \frac{3}{8}$

2 marks

b. $\Pr(L|\$2) = \frac{\Pr(L \cap \$2)}{\Pr(\$2)}$
 $= \frac{\frac{1}{8}}{\frac{3}{8}} = \frac{1}{3}$

2 marks

Question 8 (8 marks)

a. $5 - 4x^2 = 0, x = \pm \frac{\sqrt{5}}{2}$

$$\begin{aligned} AV &= \frac{1}{\frac{\sqrt{5}}{2} - -\frac{\sqrt{5}}{2}} \int_{-\frac{\sqrt{5}}{2}}^{\frac{\sqrt{5}}{2}} (5 - 4x^2) dx \\ &= \frac{1}{\sqrt{5}} \left[5x - \frac{4}{3}x^3 \right]_{-\frac{\sqrt{5}}{2}}^{\frac{\sqrt{5}}{2}} \\ &= \frac{1}{\sqrt{5}} \left(\left(\frac{5\sqrt{5}}{2} - \frac{4}{3} \times \frac{5\sqrt{5}}{8} \right) - \left(-\frac{5\sqrt{5}}{2} - \frac{4}{3} \times -\frac{5\sqrt{5}}{8} \right) \right) \\ &= \frac{1}{\sqrt{5}} \left(\frac{5\sqrt{5}}{2} - \frac{5\sqrt{5}}{6} + \frac{5\sqrt{5}}{2} - \frac{5\sqrt{5}}{6} \right) = \frac{1}{\sqrt{5}} \left(5\sqrt{5} - \frac{5\sqrt{5}}{3} \right) \\ &= \frac{1}{\sqrt{5}} \left(\frac{10\sqrt{5}}{3} \right) = \frac{10}{3} \end{aligned}$$

2 marks

b. $Area = \frac{\frac{\sqrt{5}}{2} - -\frac{\sqrt{5}}{2}}{2 \times 4} \left(y \left(-\frac{\sqrt{5}}{2} \right) + 2y \left(-\frac{\sqrt{5}}{4} \right) + 2y(0) + 2y \left(\frac{\sqrt{5}}{4} \right) + y \left(\frac{\sqrt{5}}{2} \right) \right)$

$$= \frac{\sqrt{5}}{8} \left(0 + \frac{15}{2} + 10 + \frac{15}{2} + 0 \right) = \frac{25\sqrt{5}}{8}$$

3 marks

c. $x_1 = x_0 - \frac{y(x_0)}{y'(x_0)}$

$y(1) = 1, y'(x) = -8x, \text{ so } y'(1) = -8$

$$x_1 = 1 - \frac{1}{-8} = \frac{9}{8}$$

$$x_2 = \frac{9}{8} - \frac{y\left(\frac{9}{8}\right)}{y'\left(\frac{9}{8}\right)} = \frac{9}{8} - \frac{5 - 4\left(\frac{9}{8}\right)^2}{-9} = 5 - \frac{81}{16} = \frac{-1}{16} = \frac{9}{8} - \frac{1}{9 \times 16} = \frac{161}{144}$$

3 marks

Question 9 (4 marks)

a. $y''(x) = 2mx + 8 = 0$

$$x = -\frac{4}{m}$$

$$y\left(-\frac{4}{m}\right) = n + \frac{128}{3m^2}$$

So, POI = $\left(-\frac{4}{m}, n + \frac{128}{3m^2}\right)$

2 marks

b. $-\frac{4}{m} > 0$ and $n + \frac{128}{3m^2} > 0$

$$m < 0, n > -\frac{128}{3m^2}$$

2 marks