The Mathematical Association of Victoria

Trial Examination 2024

MATHEMATICAL METHODS

Trial Written Examination 2 - SOLUTIONS

SECTION A: Multiple Choice

Question	Answer	Question	Answer
1	C	11	A
2	C	12	D
3	A	13	C
4	B	14	A
5	B	15	B
6	D	16	D
7	B	17	A
8	D	18	C
9	C	19	B
10	D	20	A

Question 1 **Answer C**

$f(x) = -\frac{3}{2}\sin\left(2x - \pi\right)$

Amplitude: $A = \frac{3}{2}$

Period: $P = \frac{2\pi}{2} = \pi$

Question 2 **Answer C**

$f(x) = \sqrt{x+2}$ and $g(x) = e^{2x}$

Test range of $f \subseteq$ domain of g

$\left[0, \infty\right) \subset R$

domain of $g \circ f =$ domain of $f = \left[-2, \infty\right)$

Question 3 **Answer A**

$0 = ax^2 + 4x + c$

two unique solutions if $\Delta > 0$

$4^2 - 4ac > 0$

$4ac < 16$

$ac < 4$

Question 4 **Answer B**

$x+(m-1)y=2 \Rightarrow y=\left(\dfrac{-1}{m-1}\right)x+\dfrac{2}{m-1}$

$(m+1)x+3y=8-m \Rightarrow y=-\left(\dfrac{m+1}{3}\right)x+\dfrac{8-m}{3}$

Equate gradients

$-\dfrac{1}{m-1}=-\dfrac{m+1}{3}$

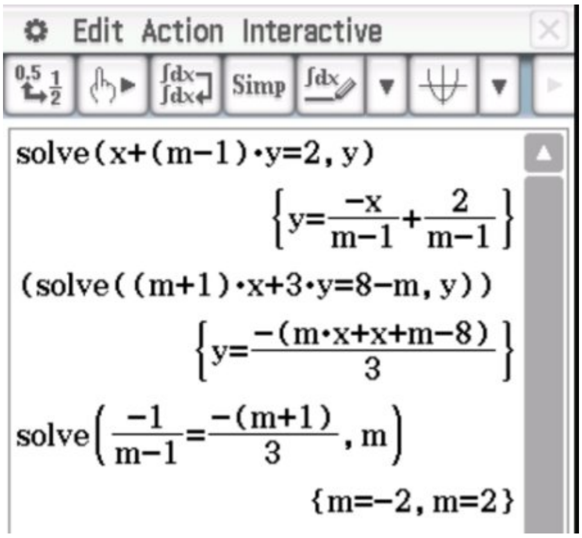
Gives $m=\pm 2$

Test for infinite number of solutions

$m=2$ $\quad \begin{array}{l} x+y=2 \\ 3x+3y=6 \end{array}$

$m=-2$ $\quad \begin{array}{l} x-3y=2 \\ -x+3y=10 \end{array}$

Answer: $m=2$



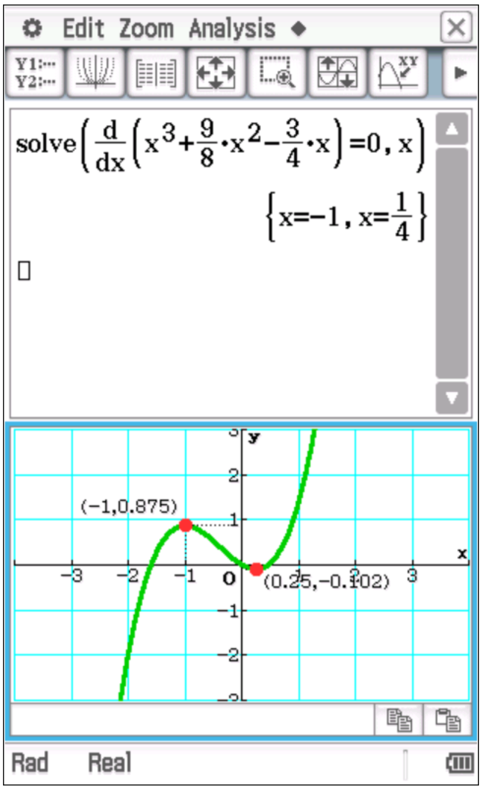
Question 5 **Answer B**

$g(x)=x^3+\frac{9}{8}x^2-\frac{3}{4}x$

$g'(x)=3x^2+\frac{9}{4}x-\frac{3}{4}=0$

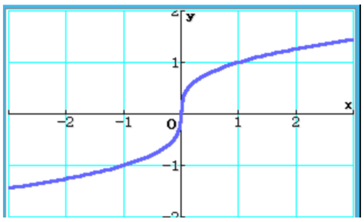
Gives $x=-1,\ x=\frac{1}{4}$

strictly decreasing for $\left[-1,\frac{1}{4}\right]$



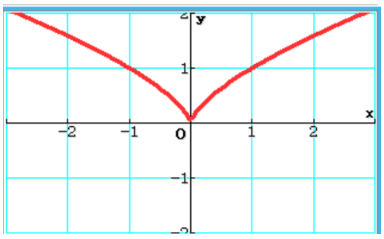
Question 6 **Answer D**

Option A $y = x^{\frac{1}{3}}$, $\dfrac{dy}{dx} = \dfrac{1}{3}x^{-\frac{2}{3}} \neq 0$



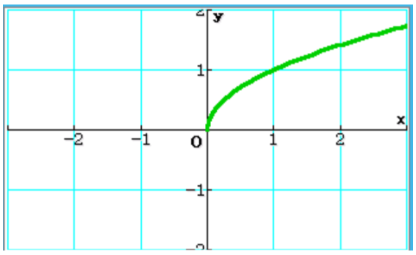
Gradient undefined at $x = 0$

Option B $y = x^{\frac{2}{3}}$, $\dfrac{dy}{dx} = \dfrac{2}{3}x^{-\frac{1}{3}} \neq 0$



Sharp point at $x = 0$

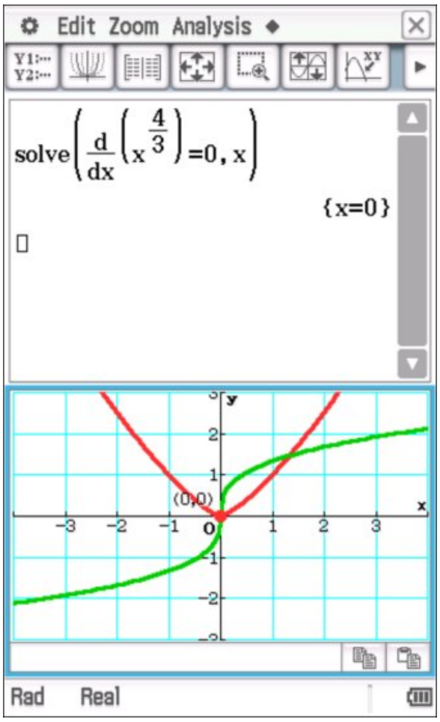
Option C $y = x^{\frac{1}{2}}$, $\dfrac{dy}{dx} = \dfrac{1}{2}x^{-\frac{1}{2}} \neq 0$



Endpoint at $(0,0)$

Option D $y=x^{\frac{4}{3}}$, $\frac{dy}{dx}=\frac{4}{3}x^{\frac{1}{3}}=0$ for $x=0$

Differentiable for all values over its maximal domain.
Gradient graph exists for all $x \in R$.



Question 7 **Answer B**

Given $\int_1^3 f(x)dx=4$ and $\int_3^1 g(x)dx=-2$.

Simplify $-\int_1^2 g(x)dx+\int_1^3 (2f(x)+3)dx-\int_2^3 g(x)dx$

$-\left(\int_1^2 g(x)dx+\int_2^3 g(x)dx\right)+2\int_1^3 f(x)dx+\int_1^3 (3)dx$

$=-\int_1^3 g(x)dx+2(4)+[3x]_1^3$

$=-2+8+6$

$=12$

Question 8 **Answer D**

Two balls of the same colour selected without replacement.

Box A: $\left(\frac{1}{2}\times\frac{4}{7}\times\frac{3}{6}\right)+\left(\frac{1}{2}\times\frac{3}{7}\times\frac{2}{6}\right)=\frac{3}{14}$

Box B: $\left(\frac{1}{2}\times\frac{4}{7}\times\frac{3}{6}\right)+\left(\frac{1}{2}\times\frac{3}{7}\times\frac{2}{6}\right)=\frac{3}{14}$

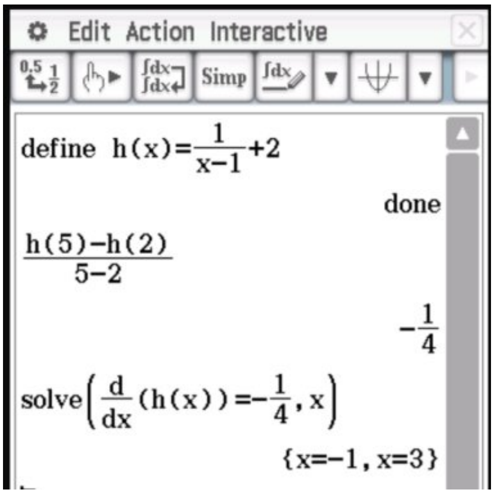
Answer: $\frac{3}{14}+\frac{3}{14}=\frac{3}{7}$

Question 9 **Answer C**

$h: R\setminus\{1\}\to R,\ h(x)=\frac{1}{x-1}+2$.

Average rate of change $=\frac{h(5)-h(2)}{5-2}=-\frac{1}{4}$

$h'(x)=-\frac{1}{4}$ at $x=-1$ or $x=3$

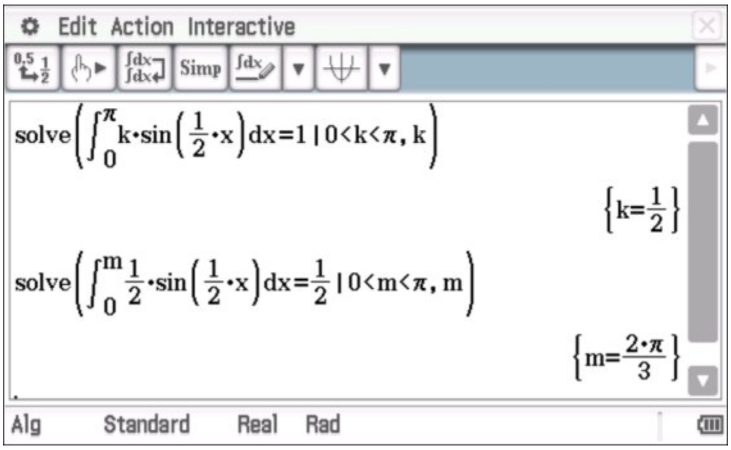


Question 10 **Answer D**

$$f(x)=\begin{cases} k\sin\left(\frac{1}{2}x\right) & 0<x<\pi \\ 0 & \text{otherwise}\end{cases}$$

$\int_0^{\pi} k\sin\left(\frac{1}{2}x\right)=1$ gives $k=\frac{1}{2}$

$\int_0^{m} \frac{1}{2}\sin\left(\frac{1}{2}x\right)=0.5$ gives $m=\frac{2\pi}{3}$



Question 11 **Answer A**

$f: R \setminus \{1\} \to R,\ f(x) = \dfrac{1}{(x-1)^2} - 2$

The tangent line at $x = 0$ is $y = 2x - 1$.

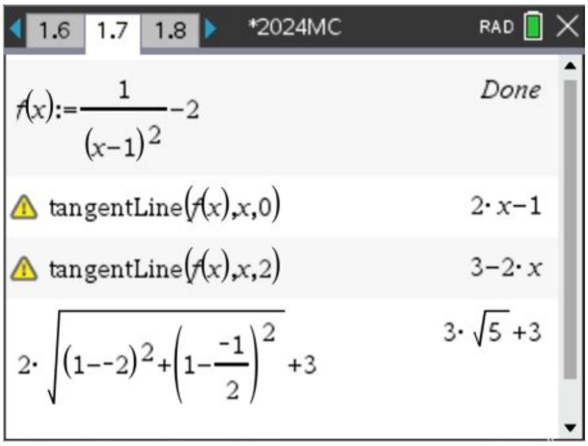
The tangent line at $x = 2$ is $y = 3 - 2x$.

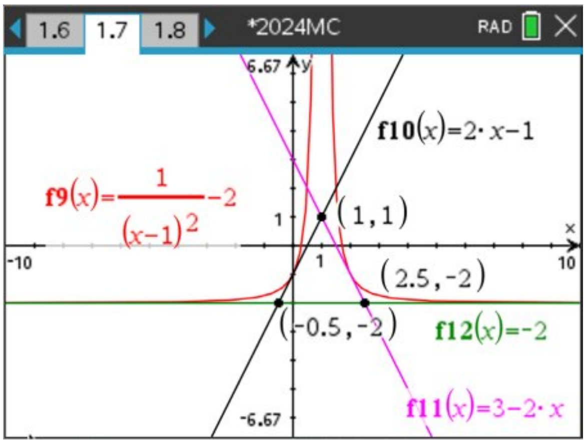
The coordinates of the vertices of the triangle are $(1,1)$, $\left(-\dfrac{1}{2}, -2\right)$ and $\left(\dfrac{5}{2}, -2\right)$.

The length of the base is 3.

The length of each of the other two sides is $\sqrt{\left(1+\dfrac{1}{2}\right)^2 + (1+2)^2} = \dfrac{3\sqrt{5}}{2}$.

Perimeter $= 3\sqrt{5} + 3$





Question 12 **Answer D**

$y = g(x) = 4\log_2(3x+5)$, $h = 1$

Area of the trapeziums $= \dfrac{1}{2}\left(g(0) + 2g(1) + 2g(2) + g(3)\right)$

$= 2\left(\log_2(5) + \log_2(14) + \log_2(64) + \log_2(121)\right)$

$= 2\left(\log_2(5 \times 14 \times 121) + \log_2(64)\right)$

$= 2\left(\log_2(5 \times 14 \times 121) + 6\right)$

$=\log_2(5\times14\times121)^2+12$

$\neq\log_2(5\times14\times121)^2+6^2$

Question 13 **Answer C**

$f(x)=3\tan\left(\frac{1}{2}\left(\frac{\pi}{3}x-1\right)\right)+5$

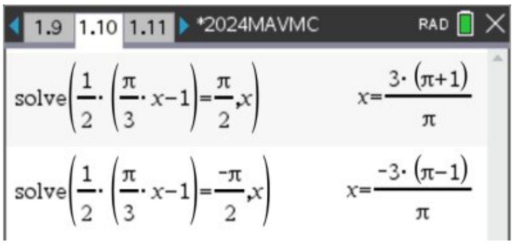
Solve $\frac{1}{2}\left(\frac{\pi}{3}x-1\right)=\frac{\pi}{2}$, $x=\frac{3(\pi+1)}{\pi}$

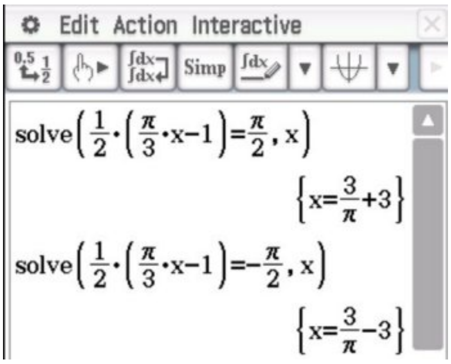
OR

$\frac{1}{2}\left(\frac{\pi}{3}x-1\right)=-\frac{\pi}{2}$, $x=\frac{-3(\pi-1)}{\pi}$

The period $=\frac{\pi}{\frac{\pi}{6}}=6$

A general solution is $x=\frac{-3(\pi-1)}{\pi}+6k,\ k\in Z$





Question 14 **Answer A**

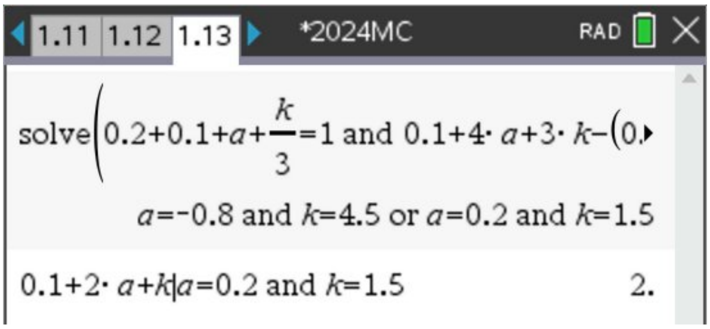
x	0	1	2	3
$\Pr(X=x)$	0.2	0.1	a	$\frac{k}{3}$

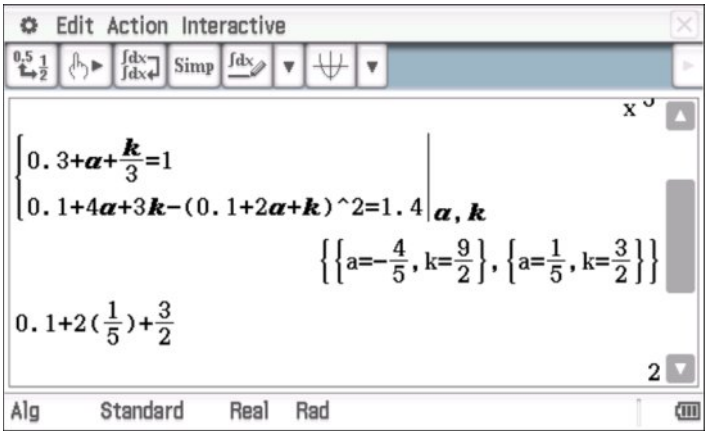
$\text{Var}(X)=0.1+4a+3k-(0.1+2a+k)^2=1.4$ …(1)

$0.3+a+\frac{k}{3}=1$…(2)

$a=0.2$, $k=1.5$

$\text{E}(X)=0.1+2a+k=2$





Question 15 **Answer B**

The domain of $s(x)=1-\log_e(1-x)$ is $(-\infty,-1)$.

The range $t(x)=3\cos(2x-1)+1$ is $[-2,4]$.

The domain of $s(x)+t^{-1}(x)$ is the intersection of $(-\infty,-1)$ and $[-2,4]$ which is $[-2,1)$.

Question 16 **Answer D**

$f:R\setminus\left\{\frac{a}{4}\right\}\to R, f(x)=\frac{2}{4x-a}+3$

$x_0=\frac{a}{4}$ will fail as $x=\frac{a}{4}$ is an asymptote.

etwons method will also fail if the x-intercept of the tangent line at x_n is undefined.

ind the equation of the tangent line at any point on the curve. Let the x-coordinate be *b*.

$$y=\frac{3a^2-2a(12b+1)+16b(3b+1)}{(a-4b)^2}-\frac{8x}{(a-4b)^2}$$

Solve $y=\frac{3a^2-2a(12b+1)+16b(3b+1)}{(a-4b)^2}-\frac{8x}{(a-4b)^2}=0$ when $x=\frac{a}{4}$

$b=\frac{3a-4}{12}$, hence $x_0=\frac{3a-4}{12}$ will fail.

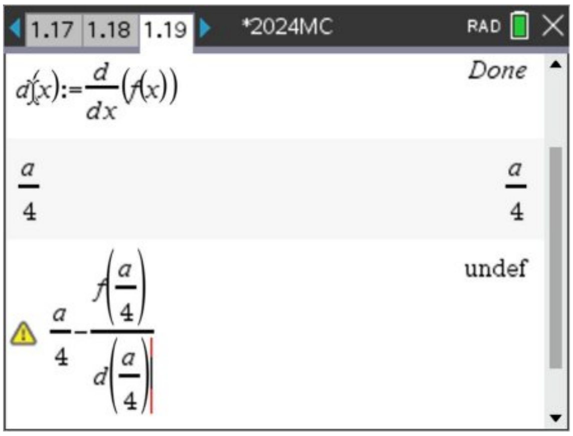
The x_0 values for any point on the S branch will fail as none of the x-intercepts of the tangent lines are less than $\frac{a}{4}$.

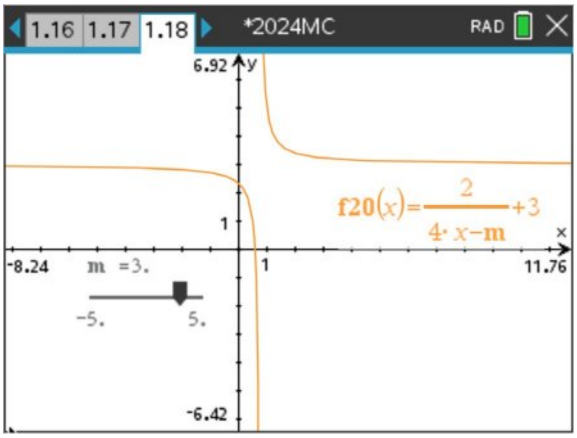
$x_0<\frac{3a-4}{12}$ will also fail as the x-intercept of the tangent lines are all greater than $\frac{a}{4}$.

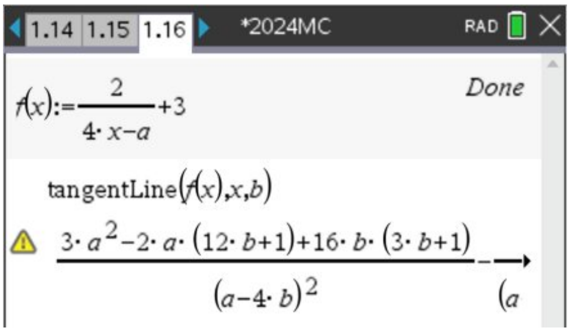
So convergence will only occur if $\frac{3a-4}{12} < x_0 < \frac{a}{4}$.

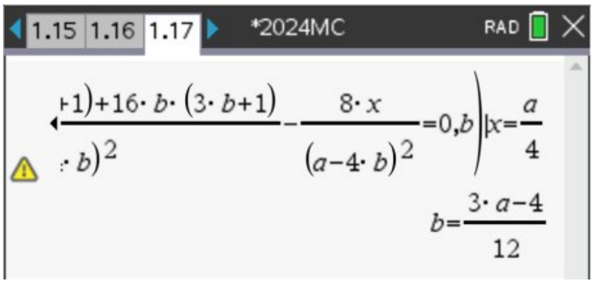
ewtons method fails if $x_0 \in R \setminus \left(\frac{3a-4}{12}, \frac{a}{4}\right)$.

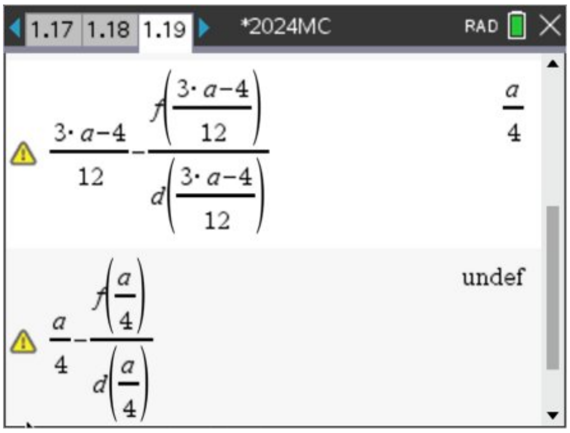
The answer can also be found by checking the values in the options to see if they fail when using ewtons method.











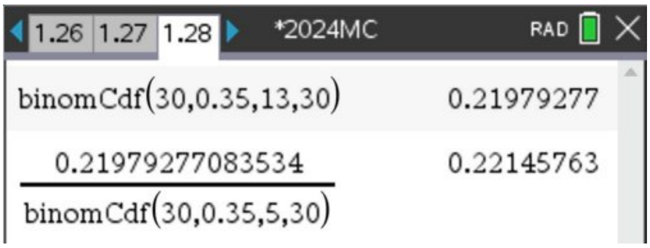
Question 17 **Answer A**

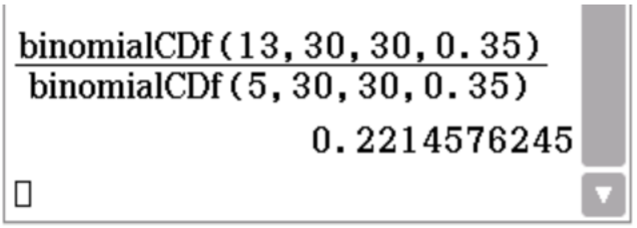
$X \sim \mathrm{Bi}(30, 0.35)$

$\Pr(X > 12 \setminus X \geq 5)$

$\frac{\Pr(X \geq 13)}{\Pr(X \geq 5)}$

$= \dfrac{0.2197...}{0.9925...}$
$= 0.2215$ correct to four decimal places





Question 18 **Answer C**

Let A_v be the average value of $f(x) = x^3 + x^2 - x + 1$ for the interval $[a, 1]$, where $a \in (-\infty, 1)$.

$$A_v = \frac{1}{1-a}\int_a^1 f(x)dx = \frac{3a^3 + 7a^2 + a + 13}{12}$$

A_v is a cubic function. $y = A_v$ will have 3 solutions between the two turning points.

Solve $\dfrac{d}{da}\left(\dfrac{3a^3 + 7a^2 + a + 13}{12}\right) = 0$ for *a*.

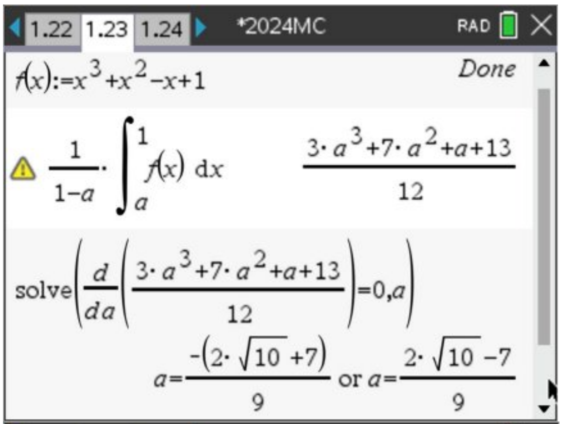
$$a = \frac{2\sqrt{10} - 7}{9},\ a = \frac{-2\sqrt{10} - 7}{9}$$

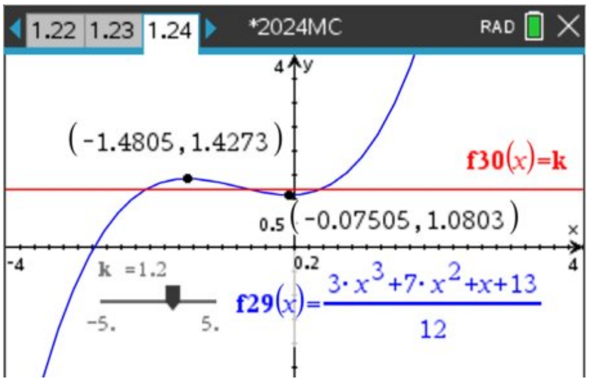
$$A_v\left(\frac{2\sqrt{10} - 7}{9}\right) = \frac{-2\left(20\sqrt{10} - 457\right)}{729}$$

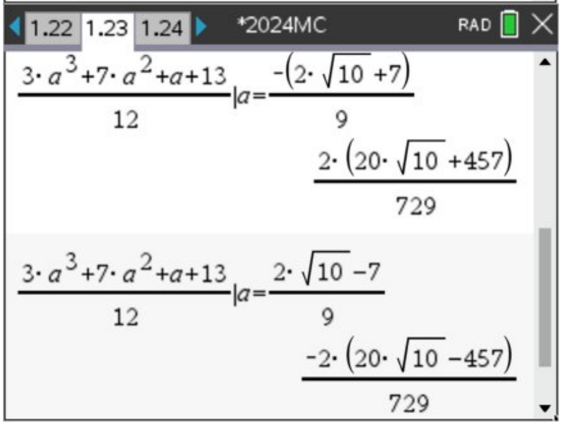
$$A_v\left(\frac{-2\sqrt{10} - 7}{9}\right) = \frac{2\left(20\sqrt{10} + 457\right)}{729}$$

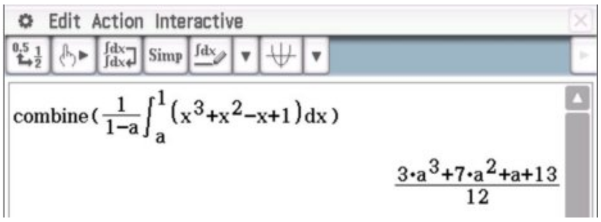
$$A_v \in \left(\frac{-2\left(20\sqrt{10} - 457\right)}{729}, \frac{2\left(20\sqrt{10} + 457\right)}{729}\right)$$

The answer can also be found by checking the values in the options to see if they give three a values.









Question 19 **Answer B**

$$f(x)=\frac{1}{\sqrt{18\pi}}e^{-\frac{1}{2}\left(\frac{2x-3}{6}\right)^2}=\frac{1}{3\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{x-\frac{3}{2}}{3}\right)^2}$$

$$X\sim \mathrm{N}\left(\frac{3}{2},3^2\right)$$

- a dilation by a factor of 3 from the x-axis

$$f_1(x)=\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{2x-3}{6}\right)^2}$$

- a dilation by a factor of $\frac{1}{3}$ from the y-axis

$$f_2(x)=\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{6x-3}{6}\right)^2}=\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}\left(x-\frac{1}{2}\right)^2}$$

- a translation of $\frac{1}{2}$ a unit left.

$$f_3(x)=\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}x^2}$$

Check: $(x,y)\to(x,3y)\to\left(\frac{x}{3},3y\right)\to\left(\frac{x}{3}-\frac{1}{2},3y\right)$

$$x'=\frac{x}{3}-\frac{1}{2},\ x=3x'+\frac{3}{2}$$

$y' = 3y,\ y = \dfrac{y'}{3}$

$\dfrac{y'}{3} = \dfrac{1}{3\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{3x' + \frac{3}{2} - \frac{3}{2}}{3}\right)^2}$

$y' = \dfrac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(x')^2}$

Question 20 **Answer A**

$f: R \to R, f(x) = e^{x^3 + bx}$

Solve $f''(x) = 0$ but does not work directly on the T or the CASO .

So find $f''(x) = \left(9b^2x^4 + 6bx^2 + 6bx + 1\right)e^{x^3 + bx}$.

There will be no points of inflection when $f''(x) \geq 0$ for all x.

Solve $9x^4 + 6bx^2 + 6x + b^2 = 0$ and $\dfrac{d}{dx}\left(9x^4 + 6bx^2 + 6x + b^2\right) = 0$

O

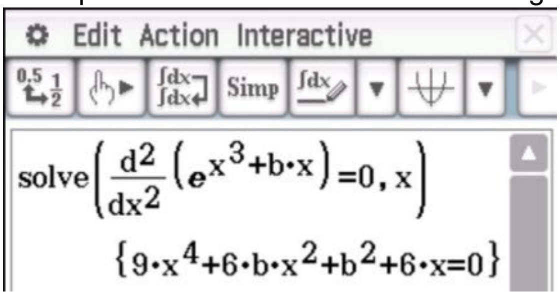
Solve $\left(9b^2x^4 + 6bx^2 + 6bx + 1\right)e^{x^3 + bx} = 0$ and $\dfrac{d}{dx}\left(\left(9b^2x^4 + 6bx^2 + 6bx + 1\right)e^{x^3 + bx}\right) = 0$

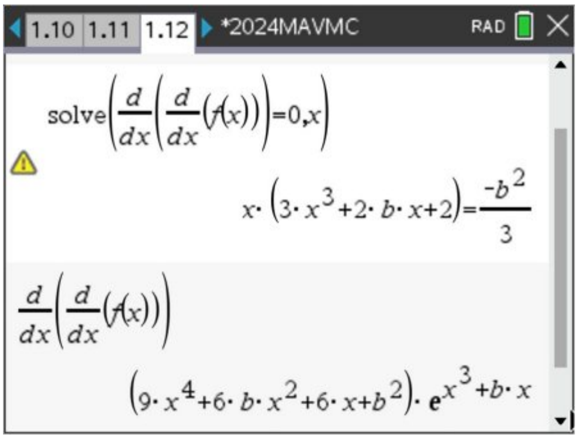
$b = \dfrac{3^{\frac{4}{3}}}{4}$

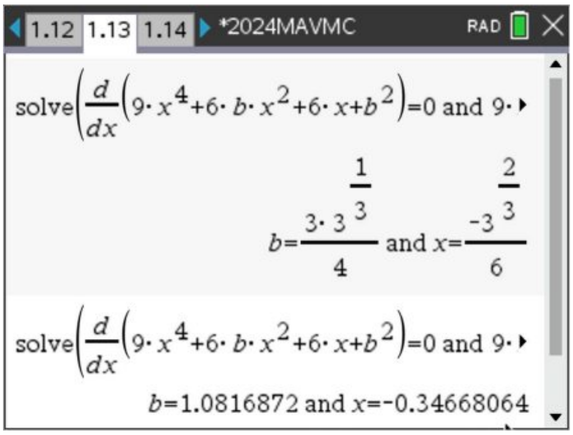
There will be no points of inflection when $b \geq \dfrac{3^{\frac{4}{3}}}{4}$.

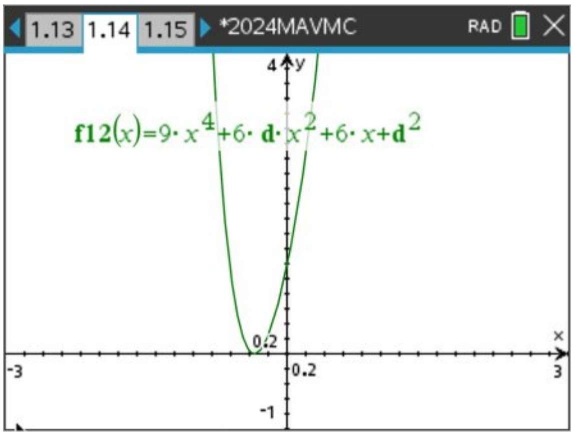
The answer can also be found by checking the values in the options. The easiest way to do this is to graph the function and use a slider. Choose a value of b that gives two points of inflection and label them with their coordinates. Then use the slider to see when they disappear.

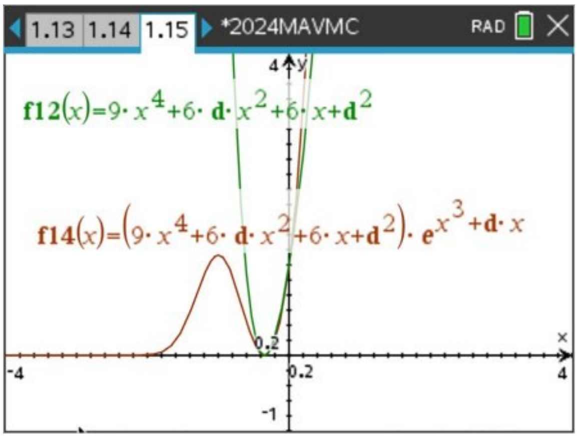
Example of second derivative not working on the CASO .







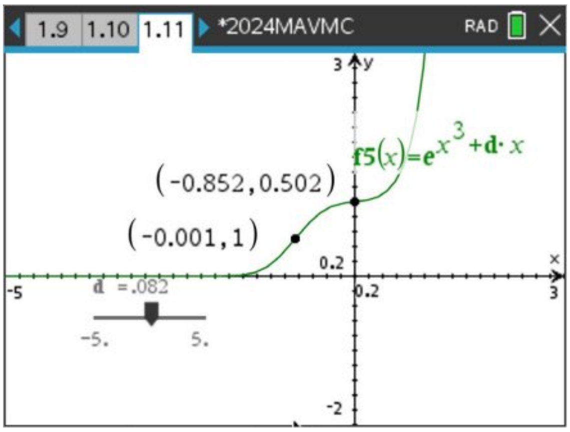


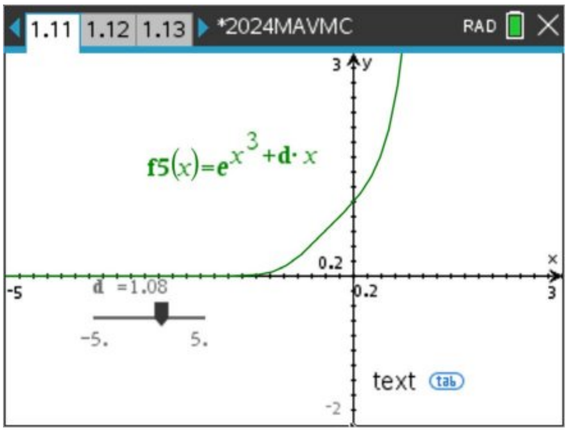


Examples

$b < \frac{3^{\frac{4}{3}}}{4}$ (2 points of inflection)

$b = \frac{3^{\frac{4}{3}}}{4}$ (no points of inflection)





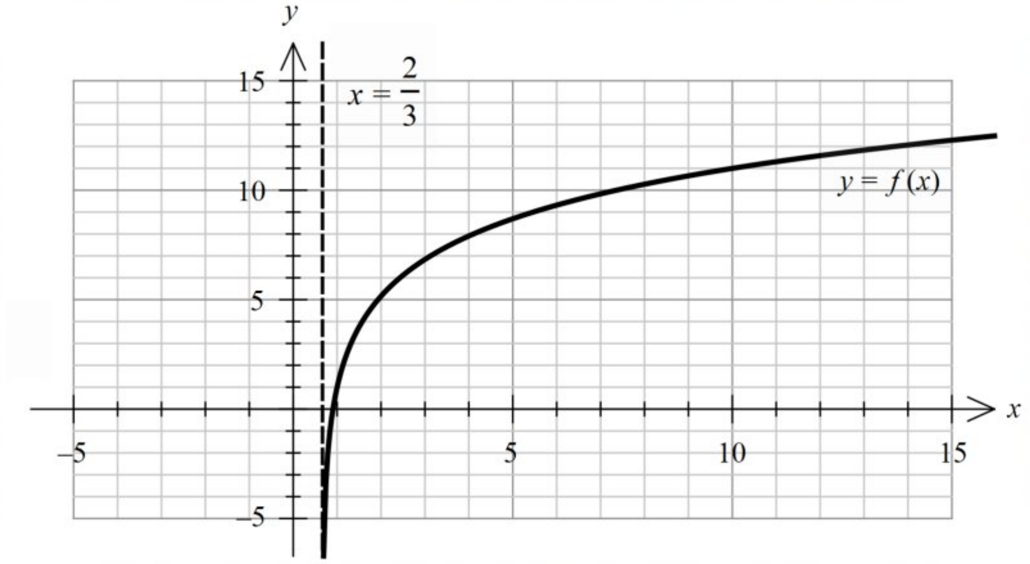
END OF SECTION A SOLUTIONS

SECTION B

Question 1

$f:\left(\frac{2}{3},\infty\right)\to R,\ f(x)=3\log_e(3x-2)+1$

a. Sketch and label asymptote $x=\frac{2}{3}$ **1A**



b. $f^{-1}(x)=\frac{1}{3}e^{\frac{x-1}{3}}+\frac{2}{3}$ **1A**

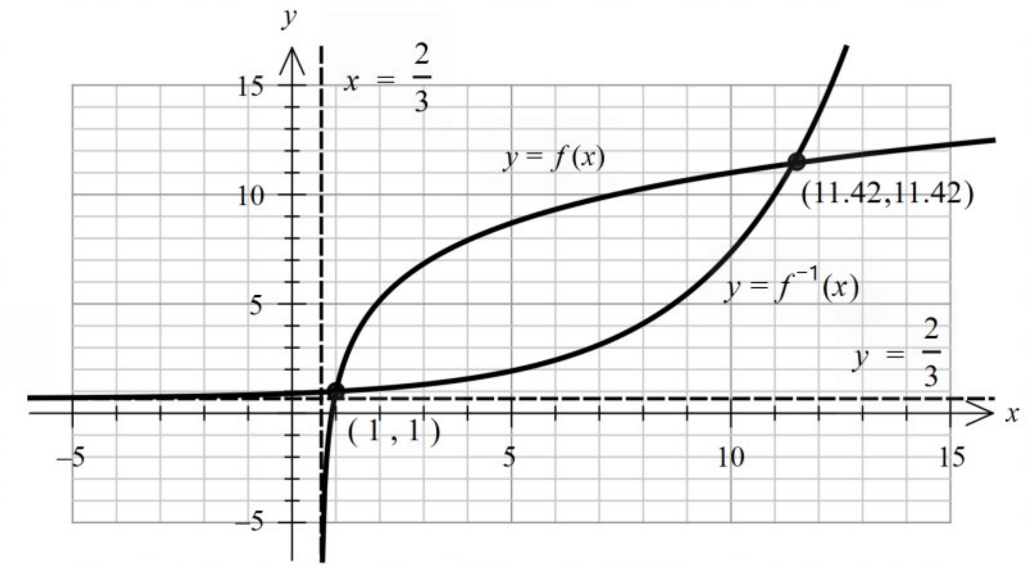
Dom: $x\in R$ **1A**

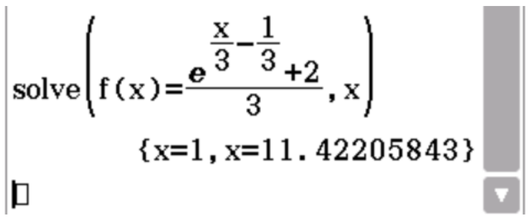
[redacted]

c. Sketch $y=f^{-1}(x)$. Shape and asymptote $y=\frac{2}{3}$ **1A**

Points of intersection between $y=f(x)$ and $y=f^{-1}(x)$

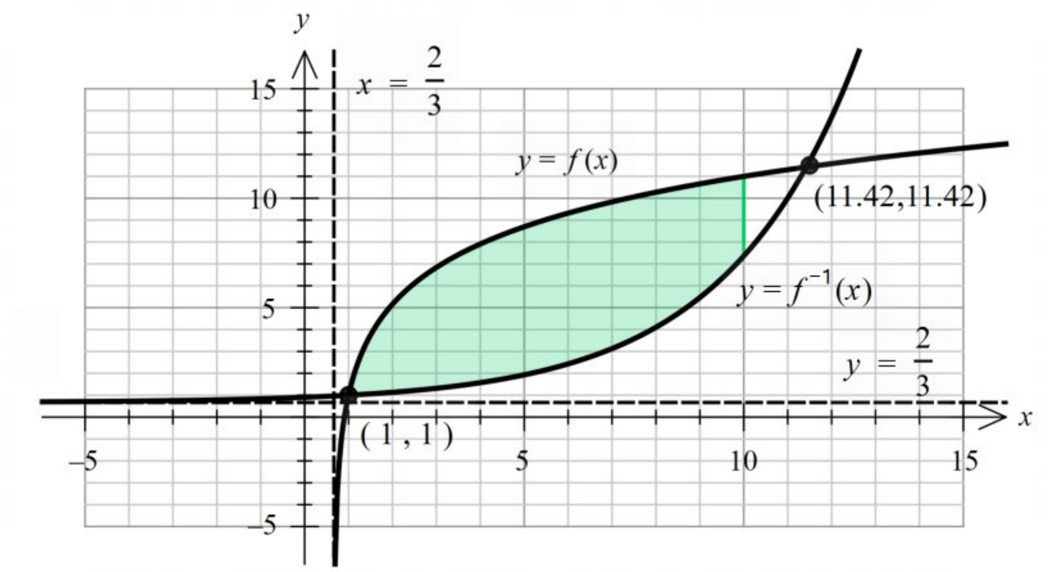
$(1,1),\ (11.42,11.42)$ **1A**

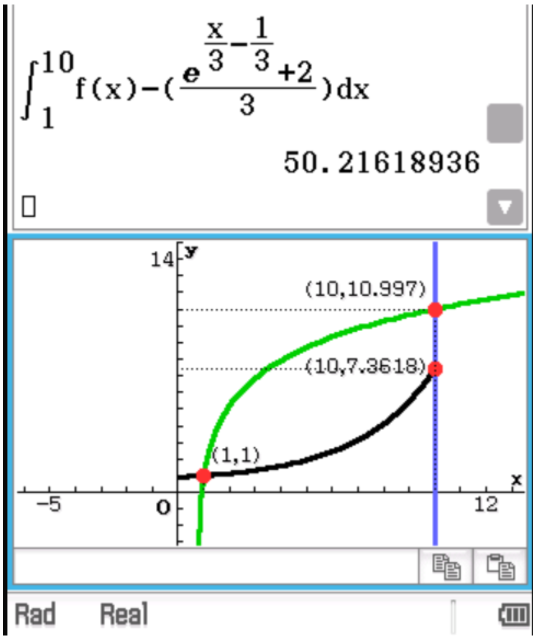


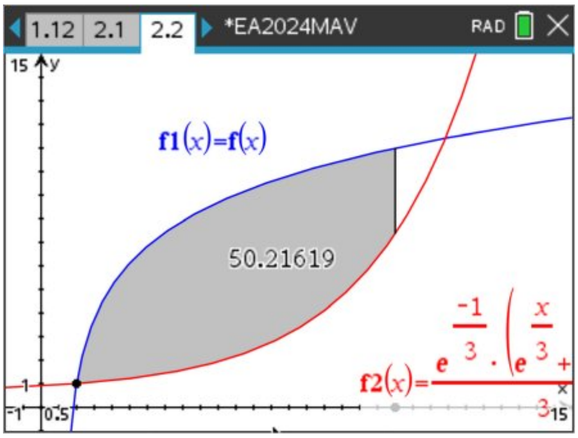


d.i. $\int_1^{10} \left(f(x) - f^{-1}(x) \right) dx$ **1A**

d.ii. Area $= 50.22$ sq units **1A**
Shading **1A**

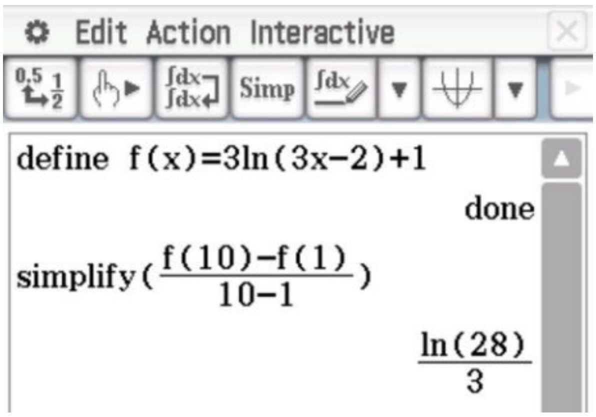






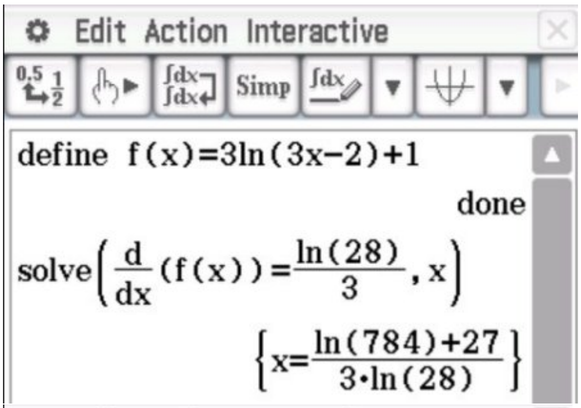
e.i. Average rate of change $= \dfrac{f(10)-f(1)}{10-1}$

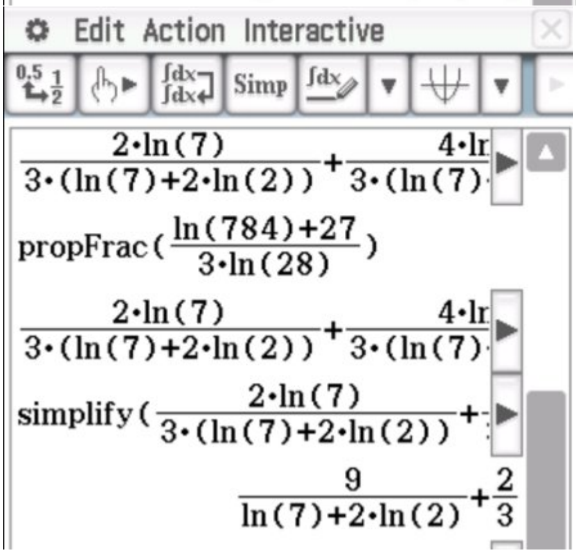
$= \dfrac{\log_e(28)}{3}$. n correct form $\dfrac{\log_e(a)}{b}$ **1A**

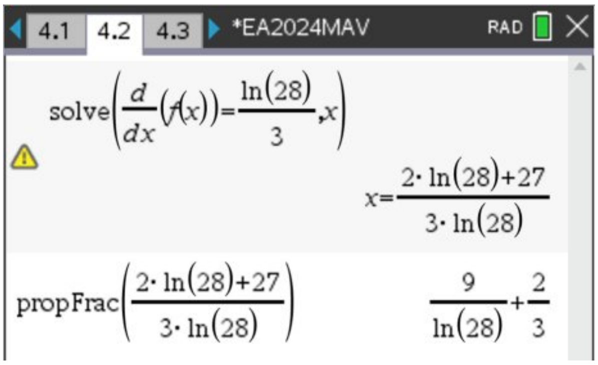


e.ii. Solve $f'(x) = \dfrac{\log_e(28)}{3}$ for x.

$x = \dfrac{2}{3} + \dfrac{9}{\log_e(28)}$ **1A** (other forms)





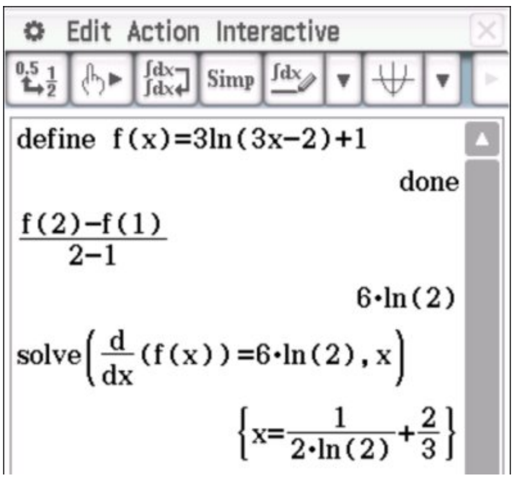


e.iii. The maximum value of the average rate of change will occur when the gradient of the line passing through $(a, f(a))$ and $(b, f(b))$ is steepest. This will occur when $a=1$ and $b=2$.

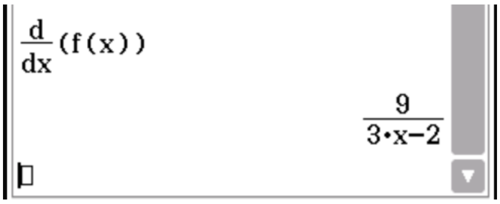
Maximum average rate of change $= \dfrac{f(2)-f(1)}{2-1} = 6\log_e(2)$ **1M**

Solve $f'(x) = 6\log_e(2)$ for x.

$x = \dfrac{2}{3} + \dfrac{1}{2\log_e(2)}$ **1A** (other forms)



e.iv. f is continuous over the interval $[a,b]$ and smooth over the interval (a,b) but $f'(x)=\dfrac{9}{3x-2}\neq 0$ for any x. ence, $f(a)\neq f(b)$. or the average value to equal zero, $f(a)$ must equal $f(b)$. **1A**



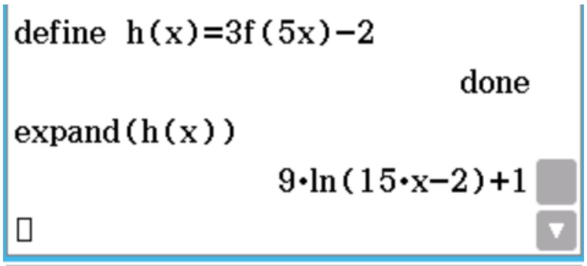
f.i. $h:\left(\dfrac{2}{b},\infty\right)\to R,\ h(x)=a\log_e\left(bx-2\right)+1$ where $h(x)=3f(5x)-2$

given $f(x)=3\log_e\left(3x-2\right)+1$

$h(x)=3f(5x)-2$

$h(x)=9\log_e(15x-2)+1$

$a=9,\ b=15$ **1A**



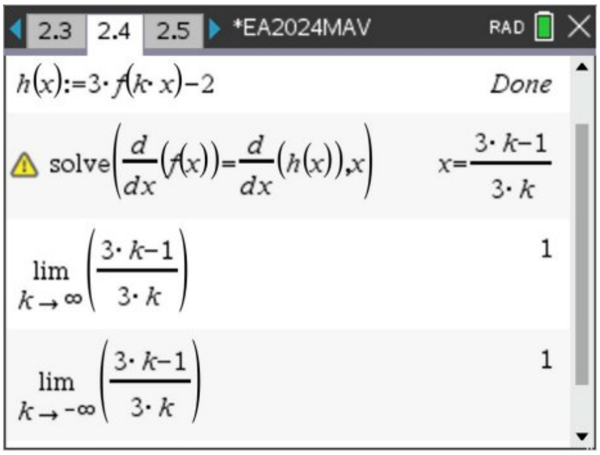
f.ii. Solve $h_1'(x)=f'(x)$

$x=\dfrac{3k-1}{3k}$

As $k\to\infty$, $x\to 1$

As $k\to-\infty$, $x\to 1$

$x=1$ **1A**



Question 2

$h(t) = a\sin\left(b(t-18)\right) + c$

a. max 50, min 10, amp $=20$

translation vertically: $-20+30=10$

$a=20,\ c=30$ **1M** (explanation)

b. One cycle $=18$ hours

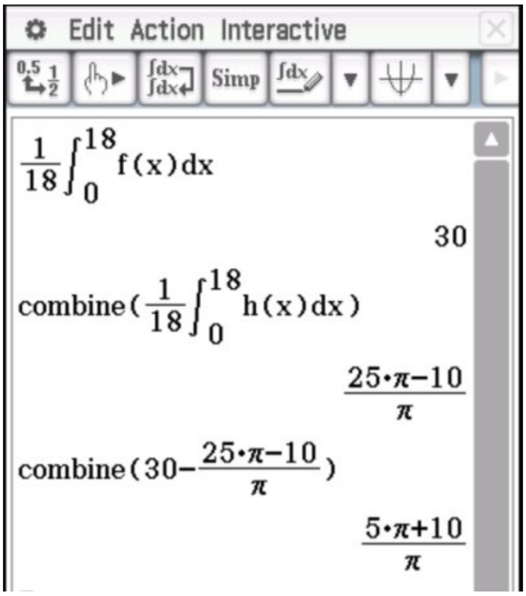
$\frac{2\pi}{b} = 18$

$2\pi = 18b$

Gives $b = \frac{\pi}{9}$ **1M** (show that)

c. $\frac{1}{18}\int_0^{18} h_A dx - \frac{1}{18}\int_0^{18} h_B dx$ **1M**

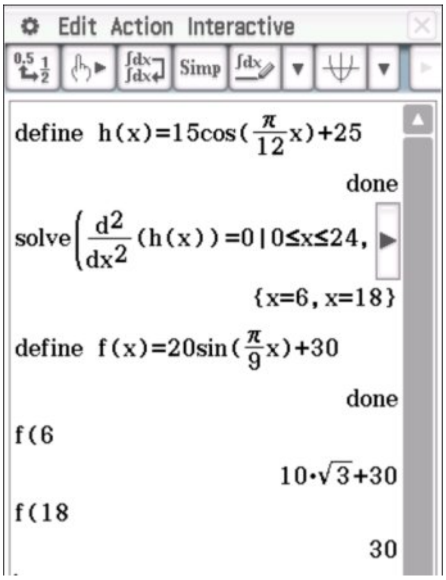
$= \frac{5(\pi+2)}{\pi}$ m **1A**

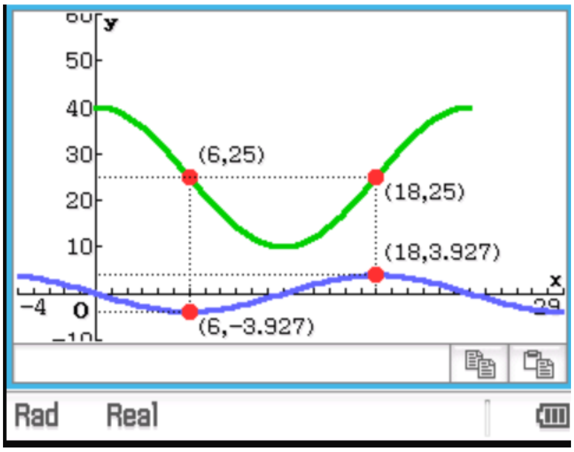


d. $h_B(t) = 15\cos\left(\frac{\pi t}{12}\right) + 25$

The height of the river would be changing fastest at the points of inflection of the graphs of h_B. So when $t=6$ and $t=18$. **1M**

$h_A(6) = 10\sqrt{3}+30$ and $h_A(18)=30$ **1A**

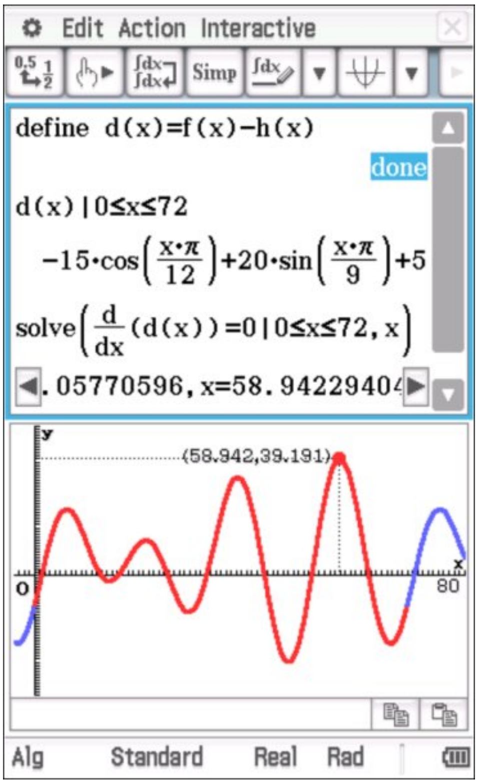


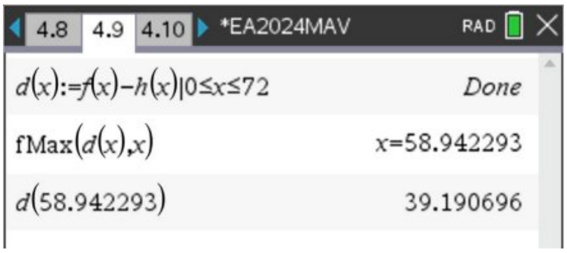


e. Let $d(x) = h_A(x) - h_B(x)$

The period of the graph of $d(x)$ is the lowest common multiple of 18 and 24 which is 72 hours. **1A**

The maximum difference is 39.19 m. **1A**

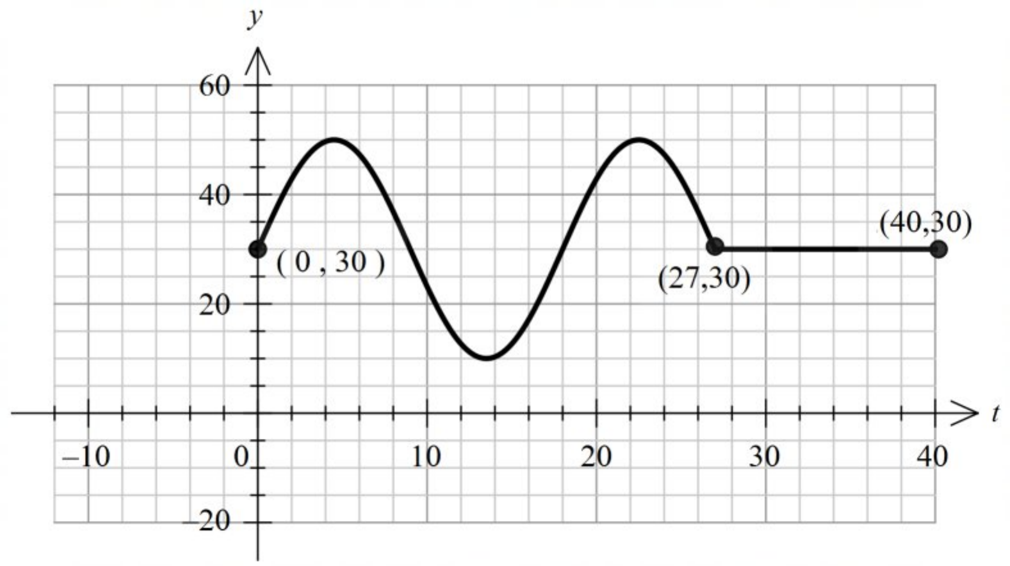


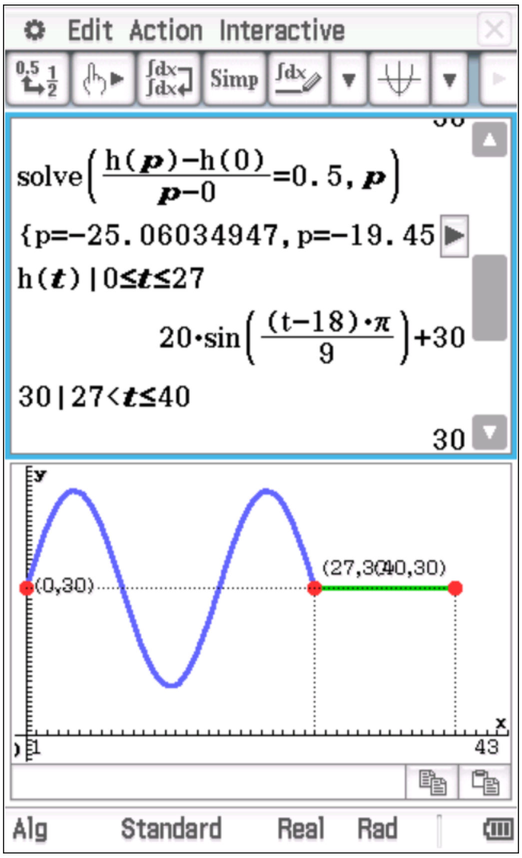


f. $w(t) = \begin{cases} h_A(t) & 0 \le t \le 27 \\ 30 & 27 < t \le 40 \end{cases}$

Graph of piecewise function w **1A**

Coordinates $(0,30)$, $(40,30)$, $(27,30)$ **1A**





g. $t \in (0,27) \cup (27,40)$ **1A**

h. $p(t) = \begin{cases} w(t) & 0 \le t \le 40 \\ m\cos(n(t-r)) + s & 40 < t \le k \end{cases}$

am Sunday to pm Tuesday is 0 hours.

$k = 60$ **1A**

i. Continuous and smooth at $t = 40$. So there is a turning point at $t = 40$.

$p = m\cos\left(n(t-r)\right)+s$ completes two cycles before recording capacities break, reaching zero height twice. So the range is $[0,30]$.

Amplitude $=15$, $m=15$

Period $=10$, $n=\frac{2\pi}{10}=\frac{\pi}{5}$ **1H**

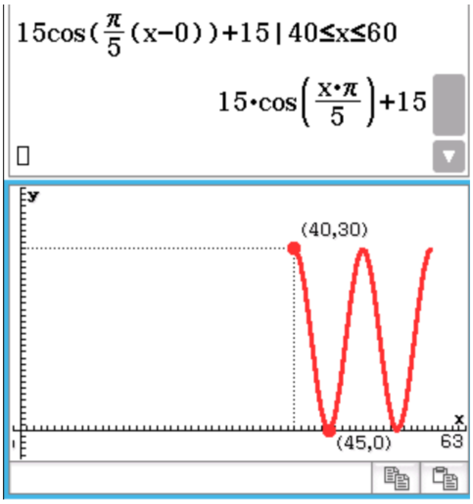
$m=15,\ s=15$. **1A**

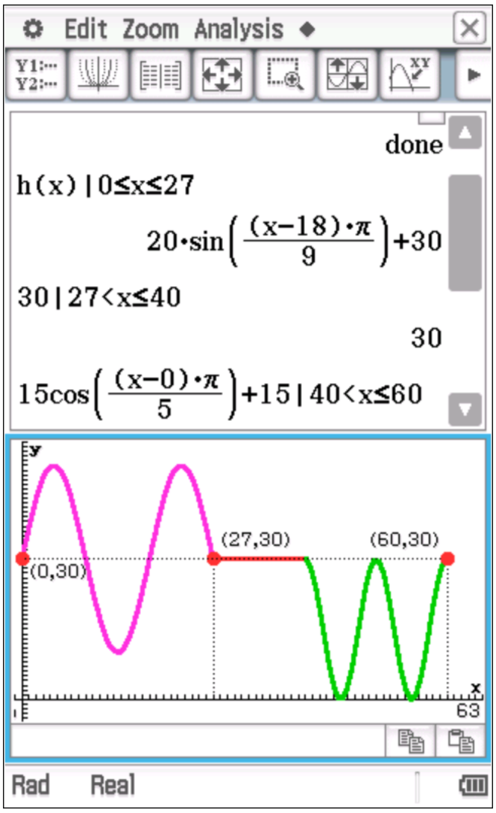
$r=10q$ where $q \in Z$ **1A**

OR

$m=-15,\ s=15$. **1A**

$r=5q$ where $q \in Z$ **1A**



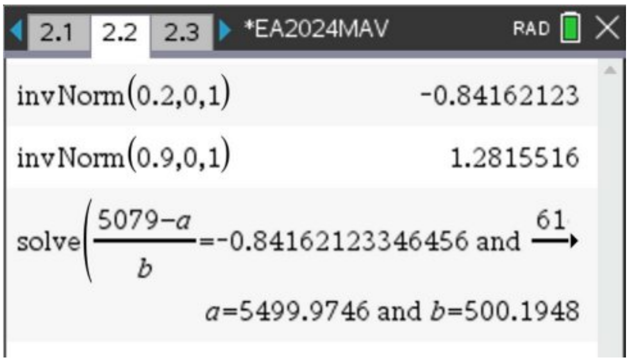


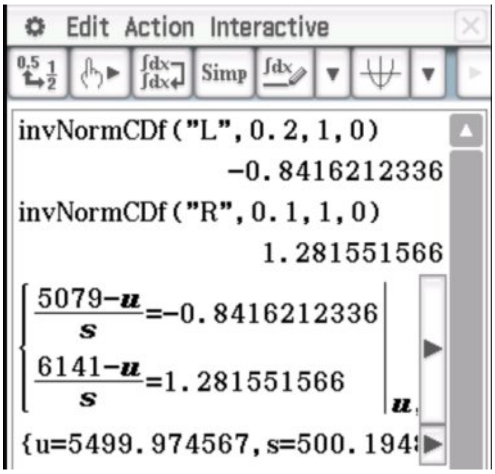
Question 3

a. $X_{AF} \sim \mathrm{N}\left(\mu,\sigma^2\right)$

Solve $\frac{5079-\mu}{\sigma}=-0.841...$ and $\frac{6141-\mu}{\sigma}=1.281...$ **1M**

$\mu = 5500.0$ kg and $\sigma = 500.2$ kg **1A**





b. $X_A \sim \mathrm{N}\left(4085, 445^2\right)$, $X_{AB} \sim \mathrm{N}\left(5375, 225^2\right)$

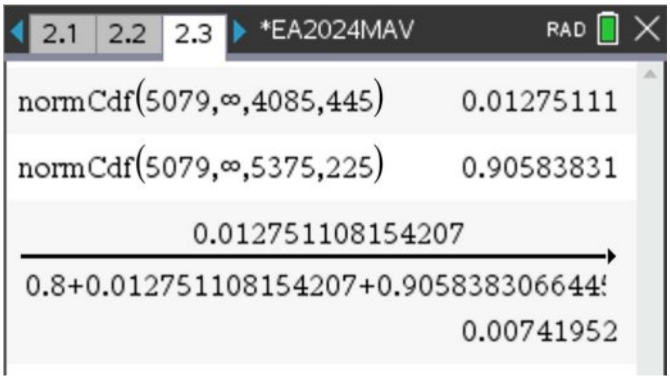
$\Pr\left(X_A > 5079\right) = 0.0127\ldots$, $\Pr\left(X_{AB} > 5079\right) = 0.9058\ldots$, $\Pr\left(X_{AF} > 5079\right) = 0.8$ **1M**

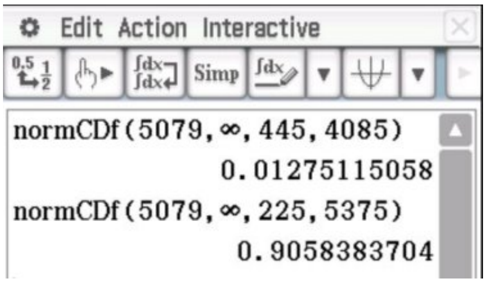
$\Pr\left(X_A > 5079 \mid \left(X_A > 5079 + X_{AB} > 5079 + X_{AF} > 5079\right)\right)$

$$= \frac{\frac{1}{3} \times 0.0127\ldots}{\frac{1}{3} \times 0.0127\ldots + \frac{1}{3} \times 0.9058\ldots + \frac{1}{3} \times 0.8}$$

$$= \frac{0.0127\ldots}{0.0127\ldots + 0.9058\ldots + 0.8}$$

$= 0.0074$ **1A**



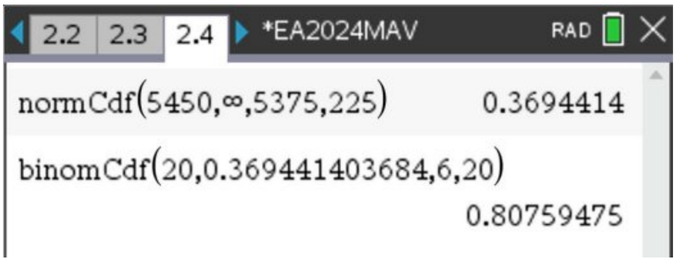


c. $X_{AB} \sim \mathrm{N}\left(5375, 225^2\right)$

$\Pr\left(X_{AB} > 5450\right) = 0.3694\ldots$

$X \sim \mathrm{Bi}\left(20, 0.3694..\right)$ **1M**

$\Pr\left(X > 5\right) = \Pr\left(X \geq 6\right) = 0.8076$ **1A**

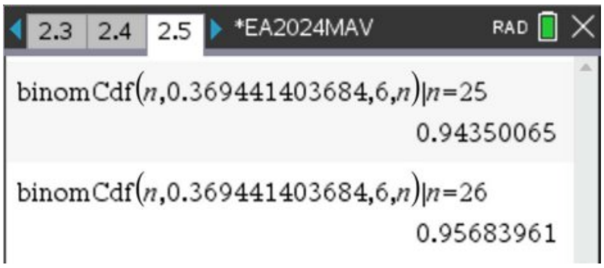


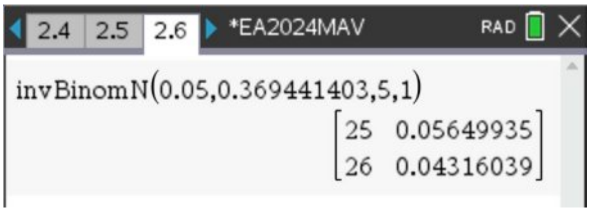
d. $X_2 \sim \text{Bi}(n, 0.3694..)$

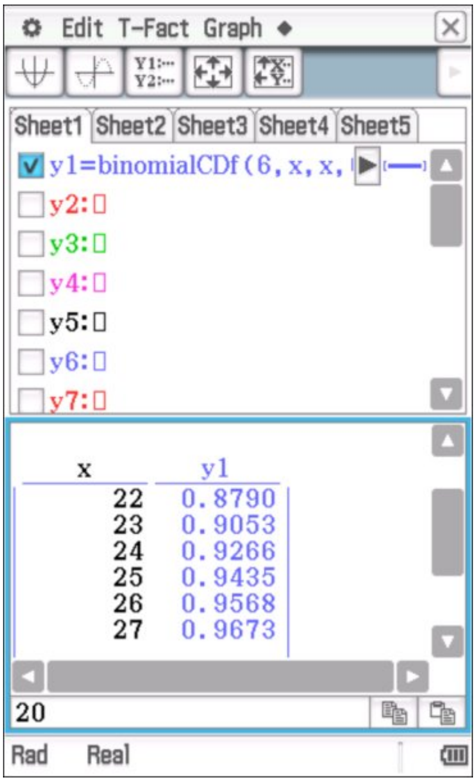
Trial and error **1M** (other methods)

n	$\Pr(X_2 \geq 6)$
25	0.9435...
26	0.9568...

$n = 26$ **1A**

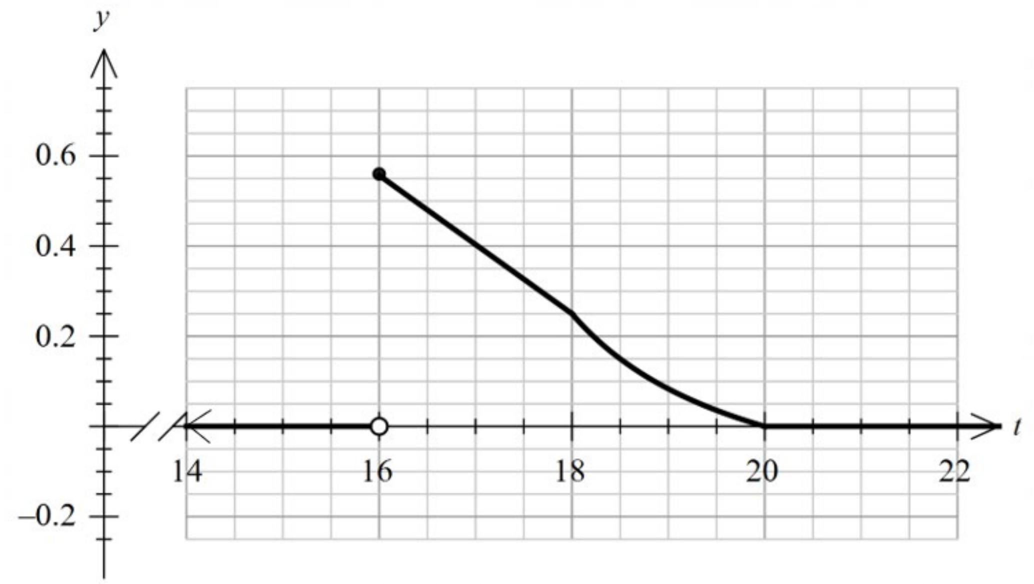






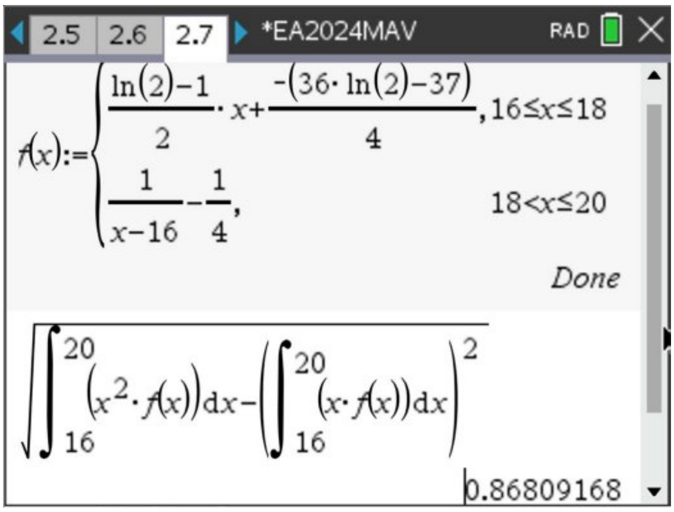
e. Shape, open circle, must draw along the x-axis **1A**

$$f(t)=\begin{cases}\dfrac{\log_e(2)-1}{2}t+\dfrac{37-36\log_e(2)}{4} & 16\le t\le 18\\[2ex] \dfrac{1}{t-16}-\dfrac{1}{4} & 18<t\le 20\\[2ex] 0 & \text{elsewhere}\end{cases}$$

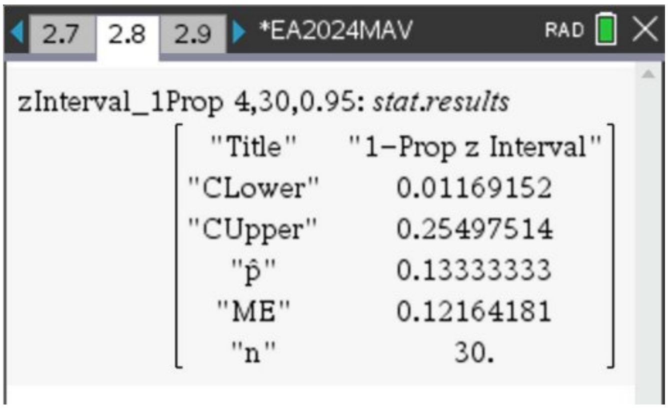


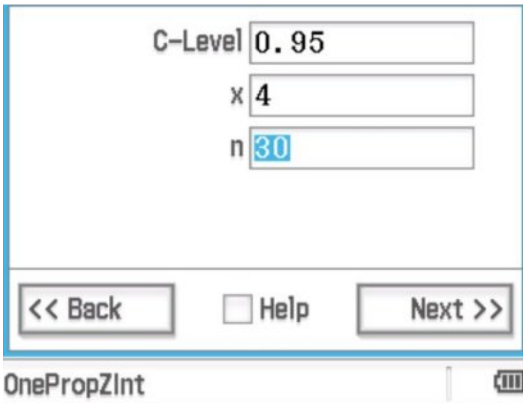
f. $\operatorname{sd}(T)=\sqrt{\int_{16}^{20}\left(t^2\times f(t)\right)dt-\left(\int_{16}^{20}\left(t\times f(t)\right)dt\right)^2}$ **1M**

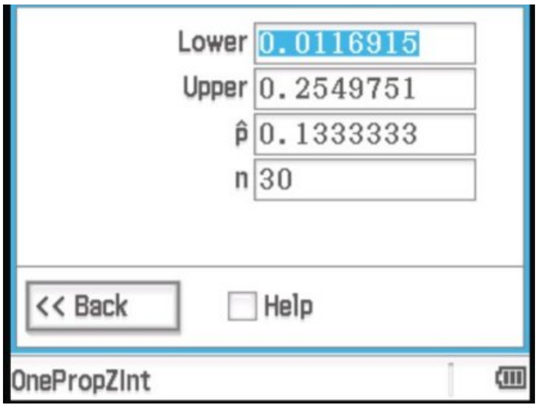
$=0.868$ **1A**



g. $(0.0117, 0.2550)$ **1A**

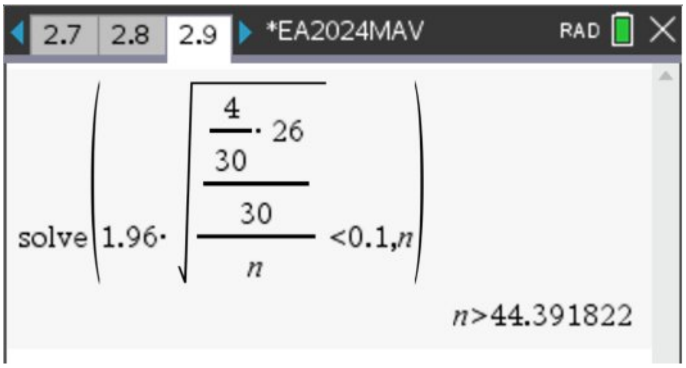






h. Solve $1.96\sqrt{\dfrac{\frac{4}{30}\times\frac{26}{30}}{n}}<0.1$ for n.

$n=45$ **1A**



i. Let AB be the African bush elephant and AF be the African forest elephant.

$\Pr(AB\cap AF)=\Pr(AB)\times\Pr(AF)=\dfrac{2-k^2}{2}$ independent events

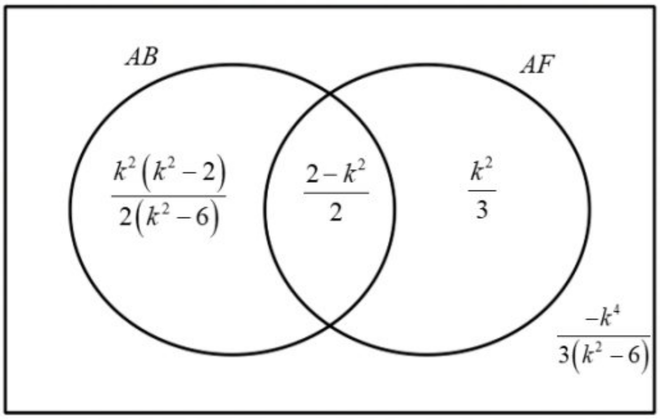
$\Pr(AB)\times\left(\dfrac{2-k^2}{2}+\dfrac{k^2}{3}\right)=\dfrac{2-k^2}{2}$ **1M**

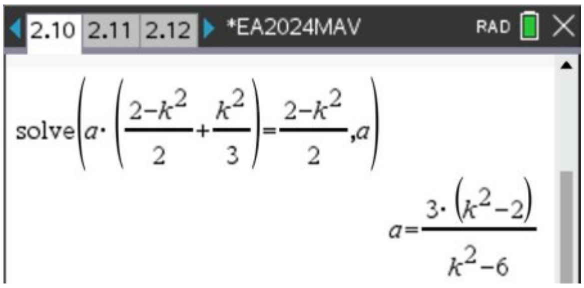
$$\Pr(AB)=\frac{3(k^2-2)}{k^2-6}$$

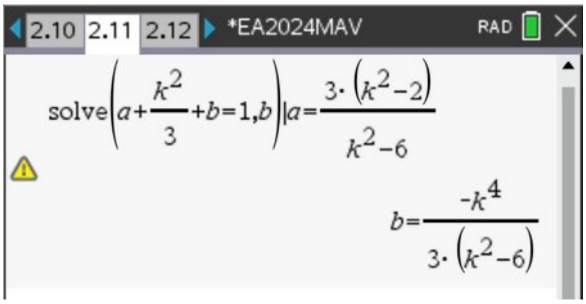
$$\Pr(AB\cup AF)+\Pr(AB'\cap AF')=1$$

$$\frac{3(k^2-2)}{k^2-6}+\frac{k^2}{3}+\Pr(AB'\cap AF')=1$$

$$\Pr(AB'\cap AF')=\frac{-k^4}{3(k^2-6)}$$ **1A**





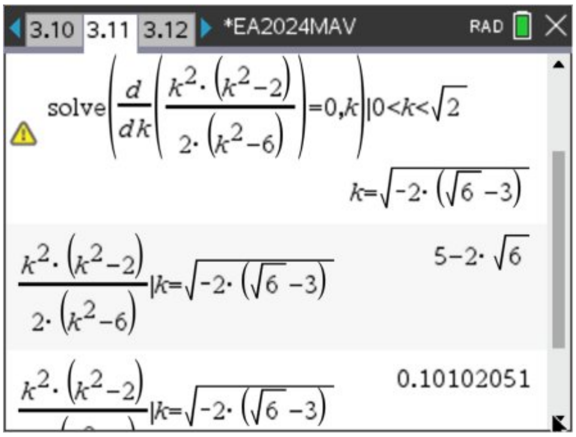


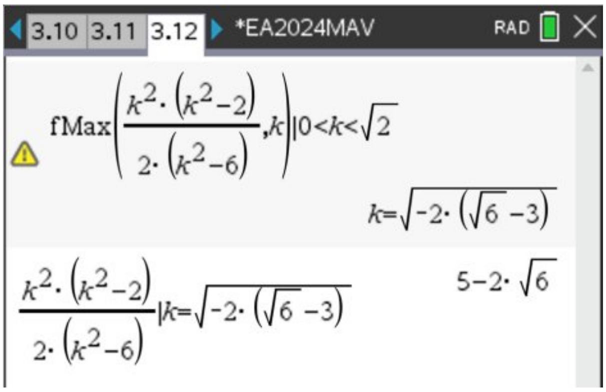
j. $\Pr(AB\cap AF')=\frac{3(k^2-2)}{k^2-6}-\frac{2-k^2}{2}=\frac{k^2(k^2-2)}{2(k^2-6)}$

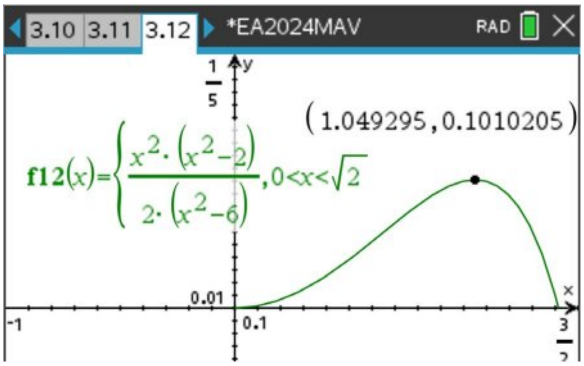
Solve $\frac{d}{dk}\left(\frac{k^2(k^2-2)}{2(k^2-6)}\right)=0$ or use fmax

$k=\sqrt{-2(\sqrt{6}-3)}$

Maximum probability is $5-2\sqrt{6}$ **1A**





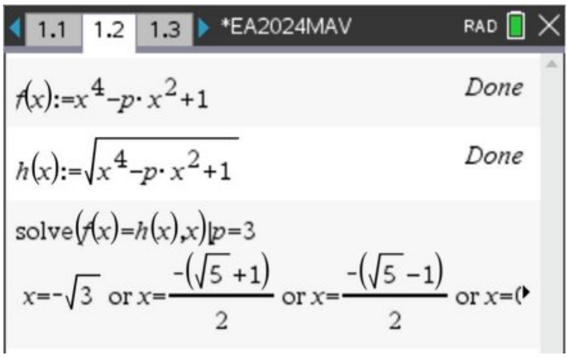


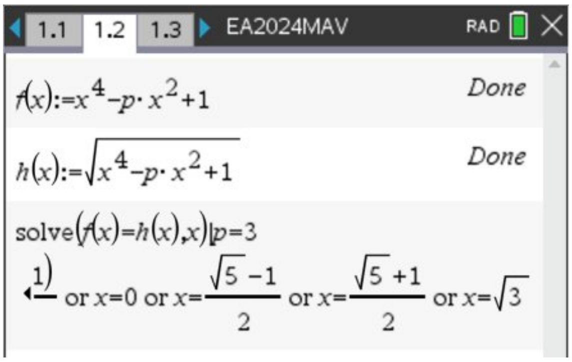
Question 4

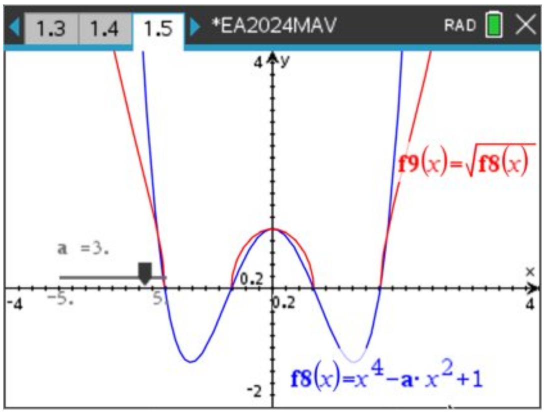
$h(x)=\sqrt{x^4-px^2+1}$ and $f(x)=x^4-px^2+1$, and $p\in R$

a. Solve $h(x)=f(x)$ when $p=3$

$x=\pm\sqrt{3},\ 0,\ \dfrac{-\sqrt{5}\pm1}{2},\ \dfrac{\sqrt{5}\pm1}{2}$ **1A**





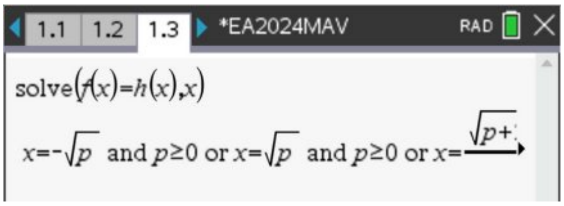


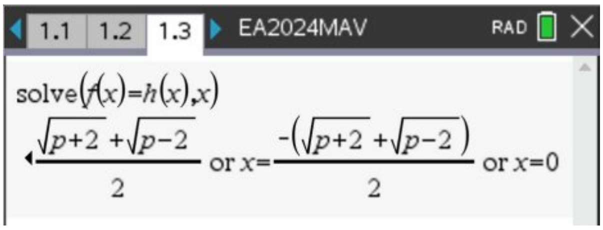
b. Solve $h(x) = f(x)$ for x.

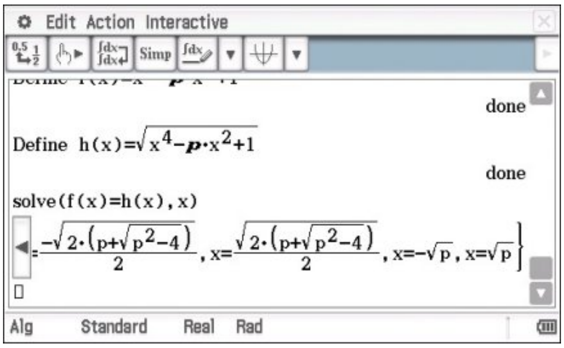
$$x = \pm\sqrt{p},\ 0,\ \frac{\pm\sqrt{p+2}+\sqrt{p-2}}{2},\ \frac{\pm\sqrt{p+2}-\sqrt{p-2}}{2} \qquad \textbf{1A}$$

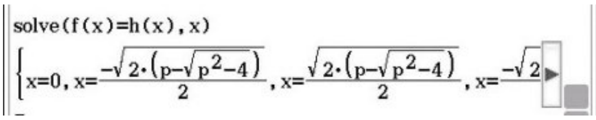
OR

$$x = \pm\sqrt{p},\ 0,\ \frac{\pm\sqrt{2\left(p+\sqrt{p^2-4}\right)}}{2},\ \frac{\pm\sqrt{2\left(p-\sqrt{p^2-4}\right)}}{2} \qquad \textbf{1A}$$







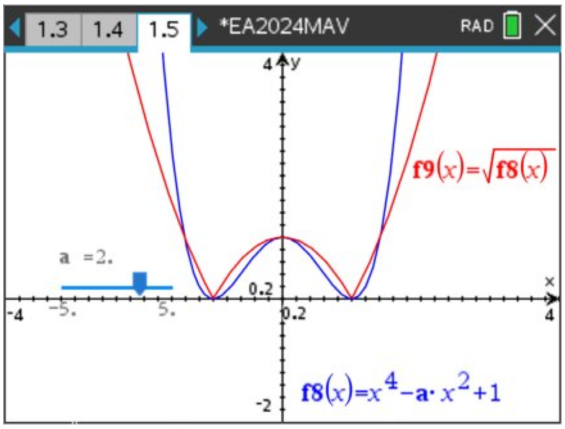


c. If $p = 2$, $\sqrt{p-2} = 0$, so $\dfrac{\pm\sqrt{p+2}+\sqrt{p-2}}{2} = \dfrac{\pm\sqrt{p+2}-\sqrt{p-2}}{2} = \dfrac{\pm\sqrt{p+2}}{2}$

OR

If $p = 2$, $p^2 - 4 = 0$, so $\dfrac{\pm\sqrt{2\left(p+\sqrt{p^2-4}\right)}}{2} = \dfrac{\pm\sqrt{2\left(p-\sqrt{p^2-4}\right)}}{2}$

ence onl five solutions. **1A**

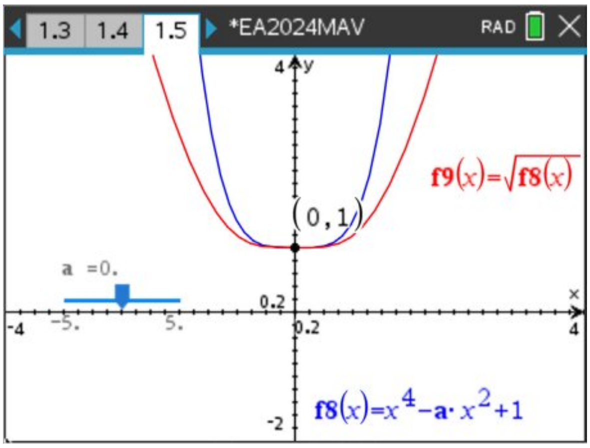


d. 2 correct **1A**
All correct **2A**

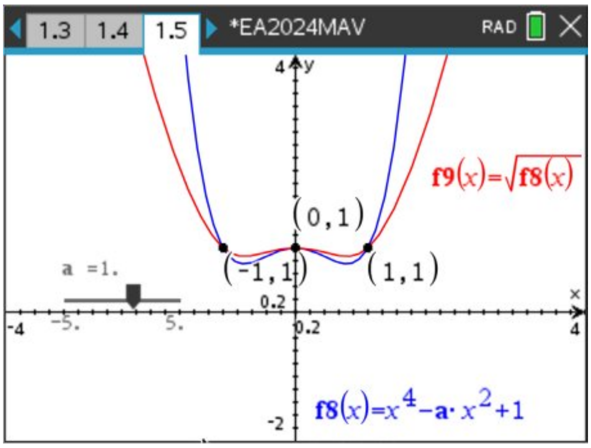
Number of points of intersection	1	3
p **values**	$p \le 0$	$0 < p < 2$
***x*-coordinates of points of intersection**	0	$0,\ \pm\sqrt{p}$

Examples

$p = 0$



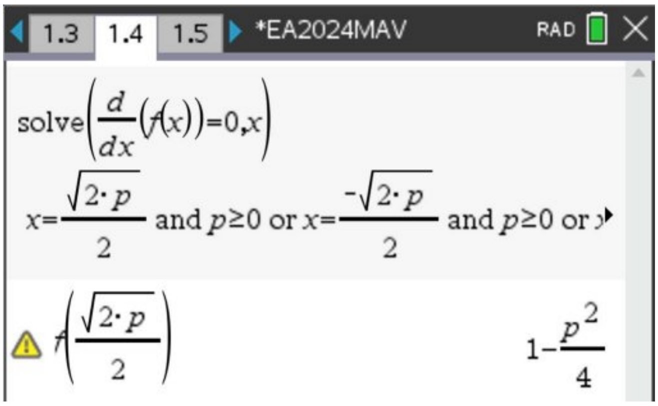
$p = 1$



e. $g(x) = \sqrt{x}$, $f(x) = x^4 - px^2 + 1$

or $g(f(x))$ to exist the range of f has to be a subset of, or equal to, the domain of g .
The domain of g is $[0,\infty)$, the range of f will be $[0,\infty)$ for $p = 2$ and a subset of $[0,\infty)$ for $p < 2$.

The range of f , $\left[1-\frac{p^2}{4},\infty\right)$, is not a subset of $[0,\infty)$ for $p > 2$. **1A**



f. Using the bounded area on the graph

Area = $2(0.0196238...+0.076381...)$ **1M**

= 0.192 **1A**

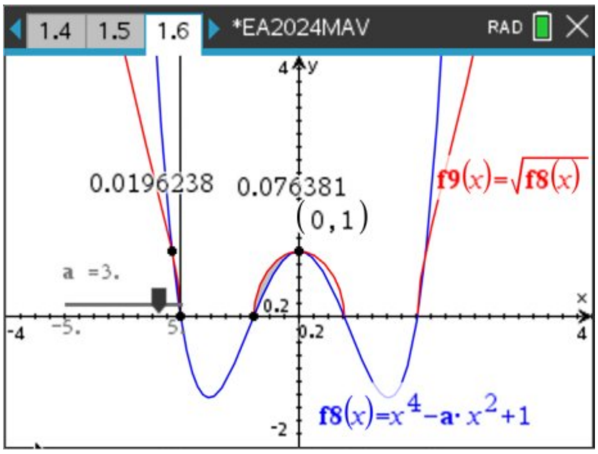
OR

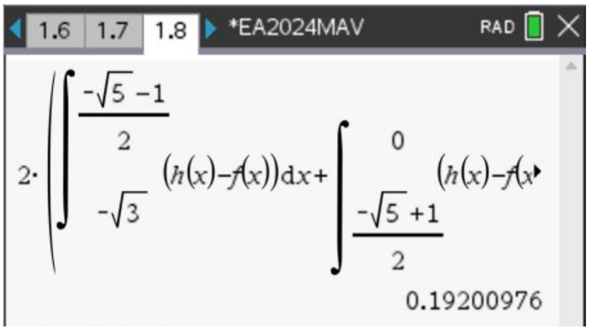
Using definite integrals

Area $=2\left(\int_{-\sqrt{3}}^{\frac{-\sqrt{5}-1}{2}}(h(x)-f(x))\,dx+\int_{\frac{-\sqrt{5}+1}{2}}^{0}(h(x)-f(x))\,dx\right)$ **OR**

$=2\left(\int_{0}^{\frac{\sqrt{5}-1}{2}}(h(x)-f(x))\,dx+\int_{\frac{\sqrt{5}+1}{2}}^{\sqrt{3}}(h(x)-f(x))\,dx\right)$ **1M** (either form)

$=0.192$ **1A**





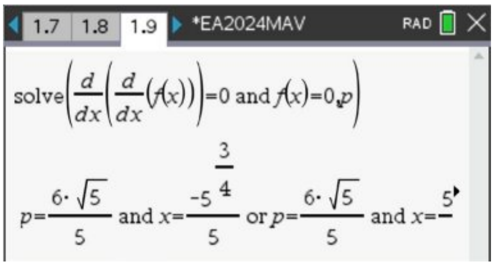
g. Area $=2\left(\int_{-\sqrt{p}}^{\frac{-\sqrt{p+2}-\sqrt{p-2}}{2}}(h(x)-f(x))\,dx+\int_{\frac{-\sqrt{p+2}+\sqrt{p-2}}{2}}^{0}(h(x)-f(x))\,dx\right)$ **OR**

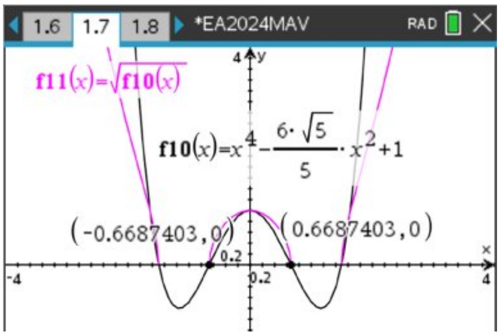
$=2\left(\int_{0}^{\frac{\sqrt{p+2}-\sqrt{p-2}}{2}}(h(x)-f(x))\,dx+\int_{\frac{\sqrt{p+2}+\sqrt{p-2}}{2}}^{\sqrt{p}}(h(x)-f(x))\,dx\right)$ **OR**

$$= 2\left(\int_0^{\frac{\sqrt{2\left(p-\sqrt{p^2-4}\right)}}{2}} \left(h(x)-f(x)\right)dx + \int_{\frac{\sqrt{2\left(p+\sqrt{p^2-4}\right)}}{2}}^{\sqrt{p}} \left(h(x)-f(x)\right)dx \right)$$ **1A**

h. Solve $f''(x)=0$ and $f(x)=0$ for p. **1M**

$p=\frac{6\sqrt{5}}{5}$ **1A**





i. $h_v(x)=\sqrt{x^4-3x^2+1}$ and $f_v(x)=x^4-3x^2+1$

Cross-sectional area $= \int_{-1.817...}^{1.817...} \left(2-f_v(x)\right)dx - 0.1920... = 7.517...$ **1M**

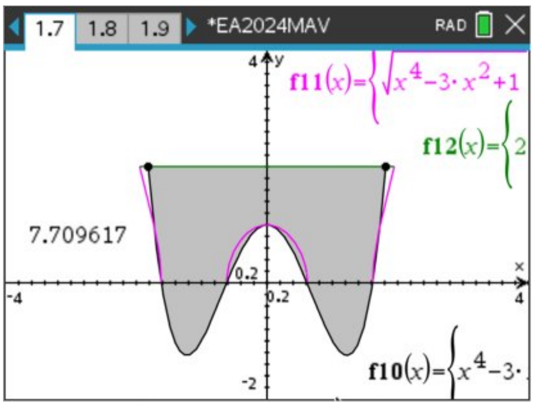
Volume = $= 7.517... \times 3 = 22.552...$

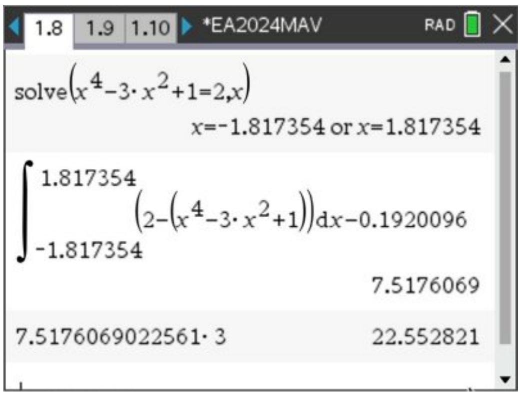
$\frac{dv}{dt}=2^t$

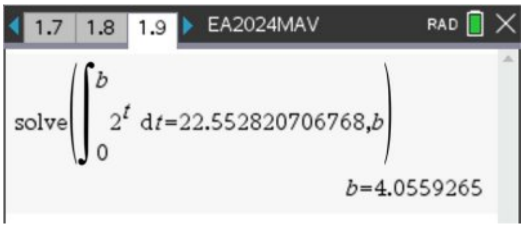
Solve $\int_0^b 2^t\,dt = 22.552...$ for b. **1H**

$b=4.06$ seconds **1A**

The shaded area on the graph is $\int_{-1.817...}^{1.817...} \left(2-f_v(x)\right)dx = 7.709...$ and then ou need to subtract the bound area found in **part f**.







END OF SOLUTIONS