

The Mathematical Association of Victoria

Trial Examination 2024

MATHEMATICAL METHODS

Trial Written Examination 2 - SOLUTIONS

SECTION A: Multiple Choice

Question	Answer	Question	Answer
1	C	11	A
2	C	12	D
3	A	13	C
4	B	14	A
5	B	15	B
6	D	16	D
7	B	17	A
8	D	18	C
9	C	19	B
10	D	20	A

Question 1 Answer C

$$f(x) = -\frac{3}{2}\sin(2x - \pi)$$

Amplitude: $A = \frac{3}{2}$

Period: $P = \frac{2\pi}{2} = \pi$

Question 2 Answer C

$$f(x) = \sqrt{x+2} \text{ and } g(x) = e^{2x}$$

Test range of $f \subseteq$ domain of g

$$[0, \infty) \subset \mathbb{R}$$

domain of $g \circ f =$ domain of $f = [-2, \infty)$

Question 3 Answer A

$$0 = ax^2 + 4x + c$$

two unique solutions if $\Delta > 0$

$$4^2 - 4ac > 0$$

$$4ac < 16$$

$$ac < 4$$

Question 4**Answer B**

$$x + (m-1)y = 2 \Rightarrow y = \left(\frac{-1}{m-1} \right)x + \frac{2}{m-1}$$

$$(m+1)x + 3y = 8 - m \Rightarrow y = -\left(\frac{m+1}{3} \right)x + \frac{8-m}{3}$$

Equate gradients

$$-\frac{1}{m-1} = -\frac{m+1}{3}$$

Gives $m = \pm 2$

Test for infinite number of solutions

$$\begin{array}{ll} m = 2 & \begin{array}{l} x + y = 2 \\ 3x + 3y = 6 \end{array} \\ m = -2 & \begin{array}{l} x - 3y = 2 \\ -x + 3y = 10 \end{array} \end{array}$$

Answer: $m = 2$

TI-Nspire calculator interface showing the solution to the system of equations. The input is $\text{solve}(x+(m-1) \cdot y=2, y)$ and the output is $\left\{ y = \frac{-x}{m-1} + \frac{2}{m-1} \right\}$. The next input is $(\text{solve}((m+1) \cdot x + 3 \cdot y = 8 - m, y))$ and the output is $\left\{ y = \frac{-(m \cdot x + x + m - 8)}{3} \right\}$. The final input is $\text{solve}\left(\frac{-1}{m-1} = -\frac{(m+1)}{3}, m\right)$ and the output is $\{m=-2, m=2\}$.

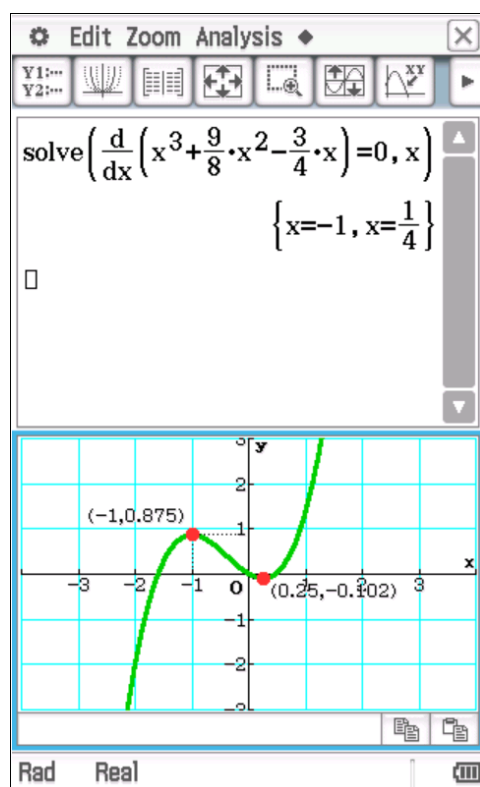
Question 5**Answer B**

$$g(x) = x^3 + \frac{9}{8}x^2 - \frac{3}{4}x$$

$$g'(x) = 3x^2 + \frac{9}{4}x - \frac{3}{4} = 0$$

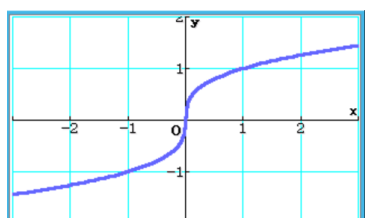
$$\text{Gives } x = -1, x = \frac{1}{4}$$

strictly decreasing for $\left[-1, \frac{1}{4} \right]$



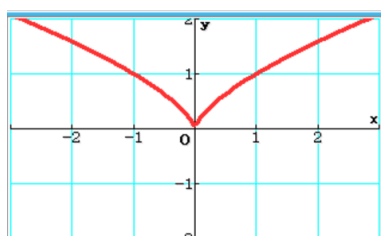
Question 6 **Answer D**

Option A $y = x^{\frac{1}{3}}, \frac{dy}{dx} = \frac{1}{3}x^{-\frac{2}{3}} \neq 0$



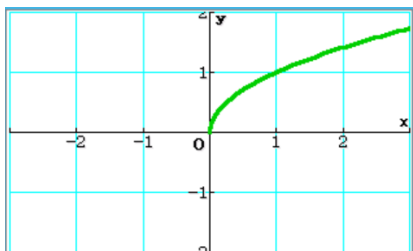
Gradient undefined at $x = 0$

Option B $y = x^{\frac{2}{3}}, \frac{dy}{dx} = \frac{2}{3}x^{-\frac{1}{3}} \neq 0$



Sharp point at $x = 0$

Option C $y = x^{\frac{1}{2}}, \frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}} \neq 0$

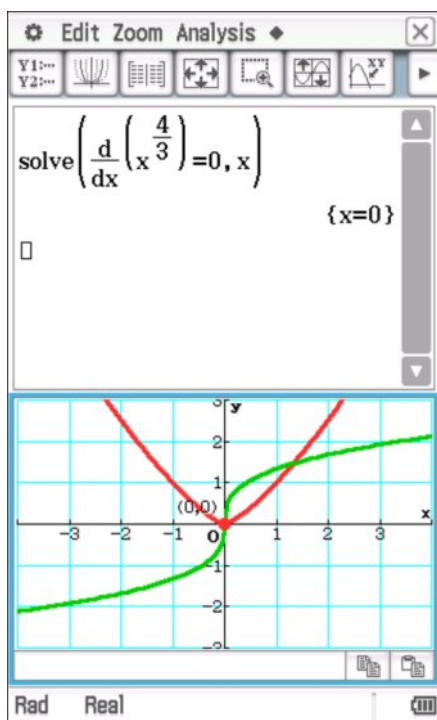


Endpoint at $(0,0)$

Option D $y = x^{\frac{4}{3}}$, $\frac{dy}{dx} = \frac{4}{3}x^{\frac{1}{3}} = 0$ for $x = 0$

Differentiable for all values over its maximal domain.

Gradient graph exists for all $x \in \mathbb{R}$.



Question 7

Answer B

Given $\int_1^3 f(x)dx = 4$ and $\int_3^1 g(x)dx = -2$.

Simplify $-\int_1^2 g(x)dx + \int_1^3 (2f(x) + 3)dx - \int_2^3 g(x)dx$

$-\left(\int_1^2 g(x)dx + \int_2^3 g(x)dx\right) + 2\int_1^3 f(x)dx + \int_1^3 (3)dx$

$= -\int_1^3 g(x)dx + 2(4) + [3x]_1^3$

$= -2 + 8 + 6$

$= 12$

Question 8 **Answer D**

Two balls of the same colour selected without replacement.

$$\text{Box A: } \left(\frac{1}{2} \times \frac{4}{7} \times \frac{3}{6}\right) + \left(\frac{1}{2} \times \frac{3}{7} \times \frac{2}{6}\right) = \frac{3}{14}$$

$$\text{Box B: } \left(\frac{1}{2} \times \frac{4}{7} \times \frac{3}{6}\right) + \left(\frac{1}{2} \times \frac{3}{7} \times \frac{2}{6}\right) = \frac{3}{14}$$

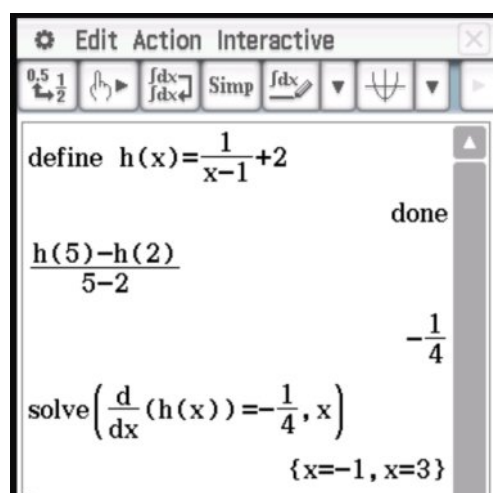
$$\text{Answer: } \frac{3}{14} + \frac{3}{14} = \frac{3}{7}$$

Question 9 **Answer C**

$$h: R \setminus \{1\} \rightarrow R, h(x) = \frac{1}{x-1} + 2.$$

$$\text{Average rate of change} = \frac{h(5) - h(2)}{5 - 2} = -\frac{1}{4}$$

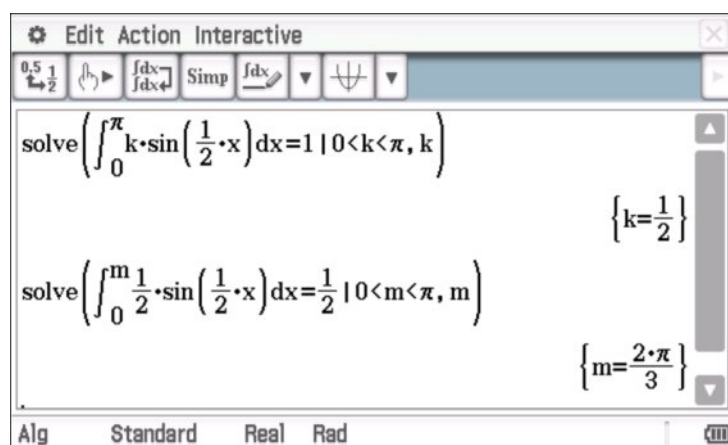
$$h'(x) = -\frac{1}{4} \text{ at } x = -1 \text{ or } x = 3$$

**Question 10** **Answer D**

$$f(x) = \begin{cases} k \sin\left(\frac{1}{2}x\right) & 0 < x < \pi \\ 0 & \text{otherwise} \end{cases}$$

$$\int_0^{\pi} k \sin\left(\frac{1}{2}x\right) = 1 \text{ gives } k = \frac{1}{2}$$

$$\int_0^m \frac{1}{2} \sin\left(\frac{1}{2}x\right) = 0.5 \text{ gives } m = \frac{2\pi}{3}$$

**Question 11** **Answer A**

$$f: \mathbb{R} \setminus \{1\} \rightarrow \mathbb{R}, f(x) = \frac{1}{(x-1)^2} - 2$$

The tangent line at $x=0$ is $y=2x-1$.

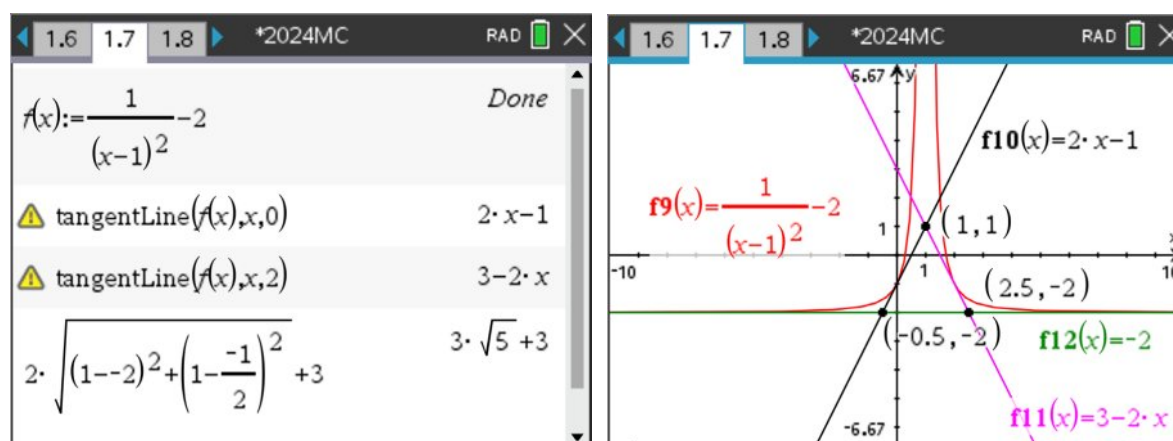
The tangent line at $x=2$ is $y=3-2x$.

The coordinates of the vertices of the triangle are $(1,1)$, $\left(-\frac{1}{2}, -2\right)$ and $\left(\frac{5}{2}, -2\right)$.

The length of the base is 3.

The length of each of the other two sides is $\sqrt{\left(1+\frac{1}{2}\right)^2 + (1+2)^2} = \frac{3\sqrt{5}}{2}$.

$$\text{Perimeter} = 3\sqrt{5} + 3$$

**Question 12** **Answer D**

$$y = g(x) = 4\log_2(3x+5), \quad h=1$$

$$\text{Area of the trapeziums} = \frac{1}{2}(g(0) + 2g(1) + 2g(2) + g(3))$$

$$= 2(\log_2(5) + \log_2(14) + \log_2(64) + \log_2(121))$$

$$= 2(\log_2(5 \times 14 \times 121) + \log_2(64))$$

$$= 2(\log_2(5 \times 14 \times 121) + 6)$$

$$= \log_2(5 \times 14 \times 121)^2 + 12$$

$$\neq \log_2(5 \times 14 \times 121)^2 + 6^2$$

Question 13 Answer C

$$f(x) = 3 \tan\left(\frac{1}{2}\left(\frac{\pi}{3}x - 1\right)\right) + 5$$

$$\text{Solve } \frac{1}{2}\left(\frac{\pi}{3}x - 1\right) = \frac{\pi}{2}, x = \frac{3(\pi + 1)}{\pi}$$

OR

$$\frac{1}{2}\left(\frac{\pi}{3}x - 1\right) = -\frac{\pi}{2}, x = \frac{-3(\pi - 1)}{\pi}$$

$$\text{The period} = \frac{\pi}{\frac{\pi}{6}} = 6$$

$$\text{A general solution is } x = \frac{-3(\pi - 1)}{\pi} + 6k, k \in \mathbb{Z}$$

The image shows two screenshots of a graphing calculator interface. The left screenshot shows the 'solve' function being used to solve the equation $\frac{1}{2} \cdot \left(\frac{\pi}{3} \cdot x - 1\right) = \frac{\pi}{2}$ for x , resulting in $x = \frac{3 \cdot (\pi + 1)}{\pi}$. The right screenshot shows the same equation solved for x , resulting in $x = \frac{3}{\pi} + 3$.

Question 14 Answer A

x	0	1	2	3
$\Pr(X = x)$	0.2	0.1	a	$\frac{k}{3}$

$$\text{Var}(X) = 0.1 + 4a + 3k - (0.1 + 2a + k)^2 = 1.4 \dots (1)$$

$$0.3 + a + \frac{k}{3} = 1 \dots (2)$$

$$a = 0.2, k = 1.5$$

$$\text{E}(X) = 0.1 + 2a + k = 2$$

1.11 1.12 1.13 *2024MC RAD

solve $\left(0.2 + 0.1 + a + \frac{k}{3} = 1 \text{ and } 0.1 + 4 \cdot a + 3 \cdot k - (0.1 + 2 \cdot a + k)^2 = 1.4 \right)$

$a = -0.8 \text{ and } k = 4.5 \text{ or } a = 0.2 \text{ and } k = 1.5$

$0.1 + 2 \cdot a + k \mid a = 0.2 \text{ and } k = 1.5$ 2.

Edit Action Interactive

$\left\{ \begin{array}{l} 0.3 + a + \frac{k}{3} = 1 \\ 0.1 + 4a + 3k - (0.1 + 2a + k)^2 = 1.4 \end{array} \right. \mid a, k$

$\left\{ \left\{ a = -\frac{4}{5}, k = \frac{9}{2} \right\}, \left\{ a = \frac{1}{5}, k = \frac{3}{2} \right\} \right\}$

$0.1 + 2 \left(\frac{1}{5} \right) + \frac{3}{2}$

Alg Standard Real Rad

Question 15 Answer B

The domain of $s(x) = 1 - \log_e(1 - x)$ is $(-\infty, -1)$.

The range $t(x) = 3 \cos(2x - 1) + 1$ is $[-2, 4]$.

The domain of $s(x) + t^{-1}(x)$ is the intersection of $(-\infty, -1)$ and $[-2, 4]$ which is $[-2, -1)$.

Question 16 Answer D

$$f: R \setminus \left\{ \frac{a}{4} \right\} \rightarrow R, f(x) = \frac{2}{4x - a} + 3$$

$x_0 = \frac{a}{4}$ will fail as $x = \frac{a}{4}$ is an asymptote.

etwons method will also fail if the x -intercept of the tangent line at x_n is undefined.

ind the equation of the tangent line at any point on the curve. Let the x -coordinate be b .

$$y = \frac{3a^2 - 2a(12b + 1) + 16b(3b + 1)}{(a - 4b)^2} - \frac{8x}{(a - 4b)^2}$$

$$\text{Solve } y = \frac{3a^2 - 2a(12b + 1) + 16b(3b + 1)}{(a - 4b)^2} - \frac{8x}{(a - 4b)^2} = 0 \text{ when } x = \frac{a}{4}$$

$$b = \frac{3a - 4}{12}, \text{ hence } x_0 = \frac{3a - 4}{12} \text{ will fail.}$$

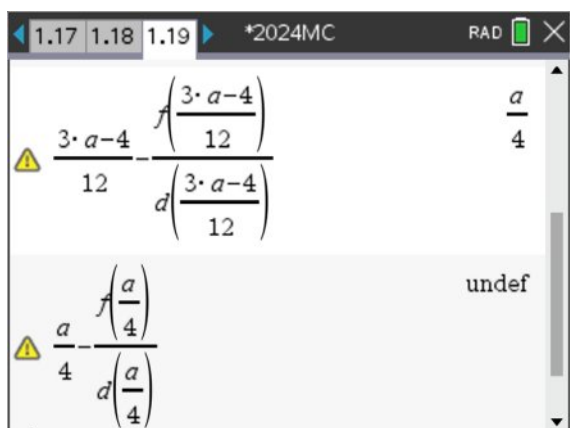
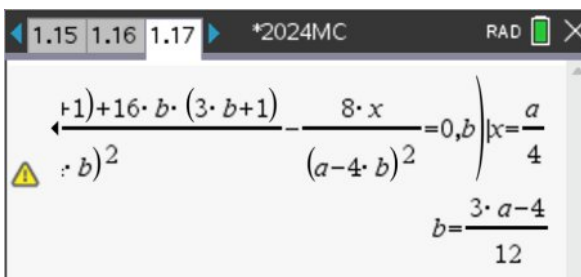
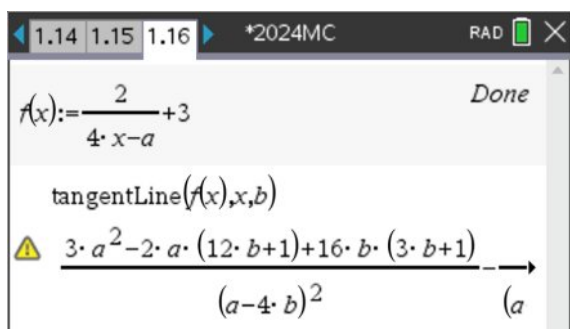
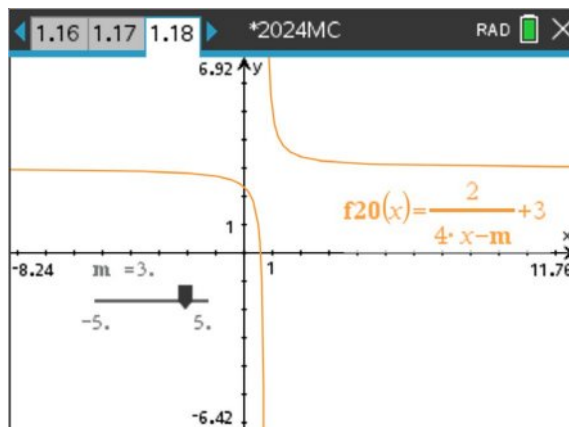
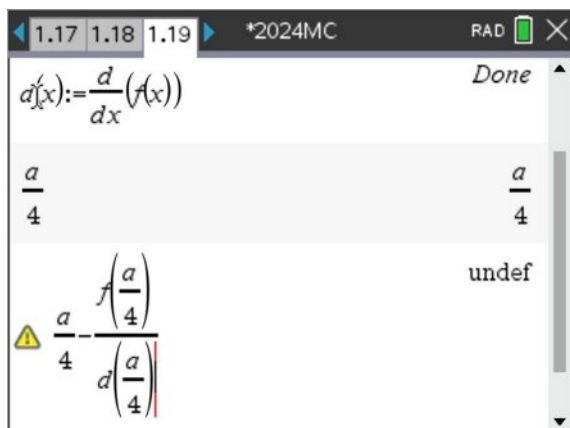
The x_0 values for any point on the S branch will fail as none of the x -intercepts of the tangent lines are less than $\frac{a}{4}$.

$$x_0 < \frac{3a - 4}{12} \text{ will also fail as the } x\text{-intercept of the tangent lines are all greater than } \frac{a}{4}.$$

So convergence will only occur if $\frac{3a-4}{12} < x_0 < \frac{a}{4}$.

ewtons method fails if $x_0 \in R \setminus \left(\frac{3a-4}{12}, \frac{a}{4}\right)$.

The answer can also be found by checking the values in the options to see if they fail when using ewtons method.



Question 17 Answer A

$$X \sim \text{Bi}(30, 0.35)$$

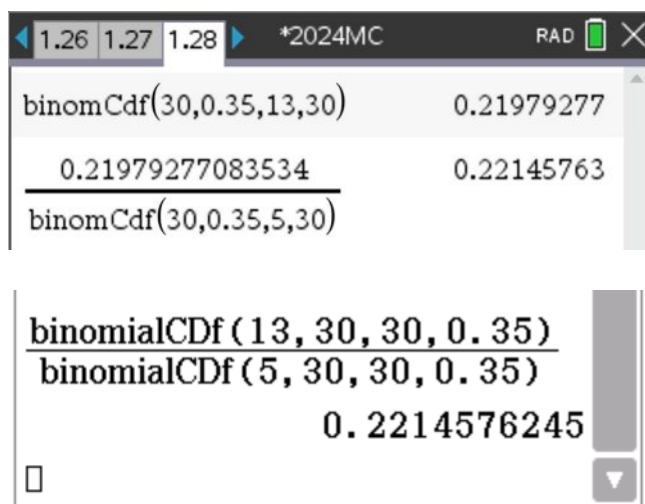
$$\Pr(X > 12 \mid X \geq 5)$$

$$\Pr(X \geq 13)$$

$$\Pr(X \geq 5)$$

$$= \frac{0.2197...}{0.9925...}$$

$$= 0.2215 \text{ correct to four decimal places}$$

**Question 18** **Answer C**

Let A_v be the average value of $f(x) = x^3 + x^2 - x + 1$ for the interval $[a, 1]$, where $a \in (-\infty, 1)$.

$$A_v = \frac{1}{1-a} \int_a^1 f(x) dx = \frac{3a^3 + 7a^2 + a + 13}{12}$$

A_v is a cubic function. $y = A_v$ will have 3 solutions between the two turning points.

Solve $\frac{d}{da} \left(\frac{3a^3 + 7a^2 + a + 13}{12} \right) = 0$ for a .

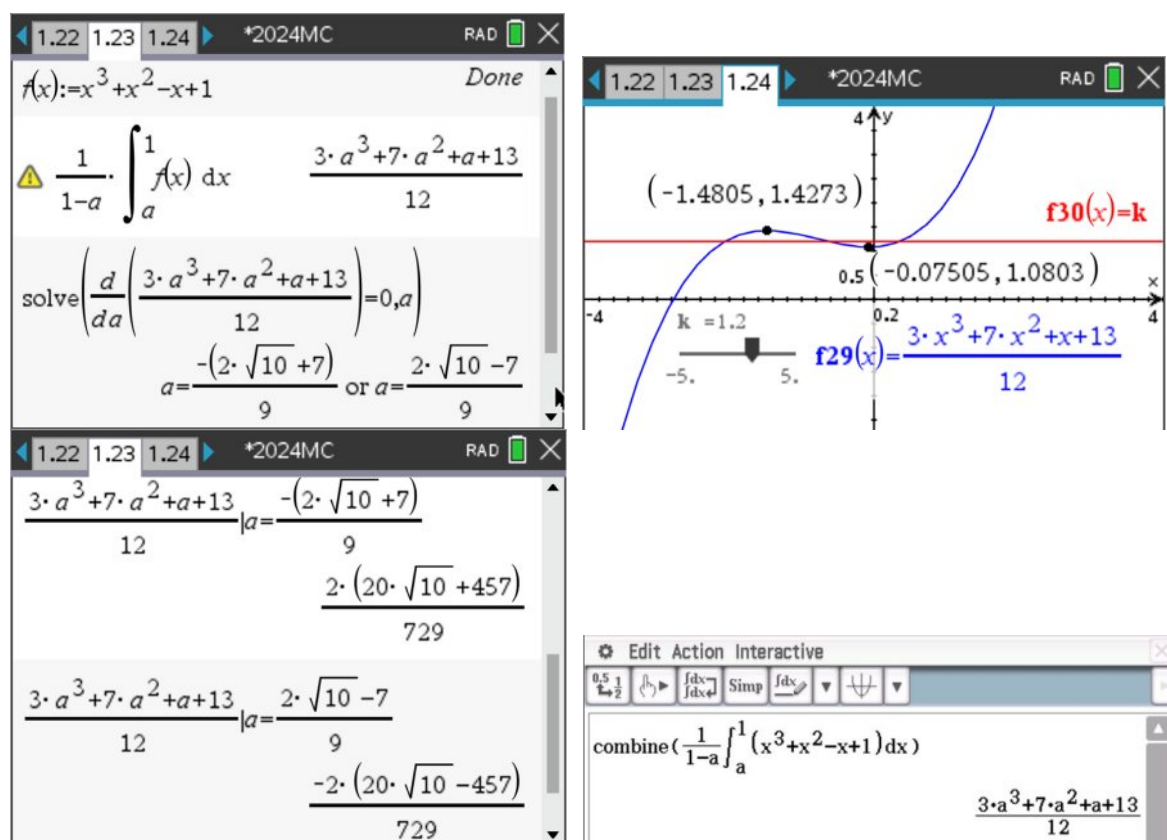
$$a = \frac{2\sqrt{10}-7}{9}, \quad a = \frac{-2\sqrt{10}-7}{9}$$

$$A_v \left(\frac{2\sqrt{10}-7}{9} \right) = \frac{-2(20\sqrt{10}-457)}{729}$$

$$A_v \left(\frac{-2\sqrt{10}-7}{9} \right) = \frac{2(20\sqrt{10}+457)}{729}$$

$$A_v \in \left(\frac{-2(20\sqrt{10}-457)}{729}, \frac{2(20\sqrt{10}+457)}{729} \right)$$

The answer can also be found by checking the values in the options to see if they give three a values.



Question 19

Answer B

$$f(x) = \frac{1}{\sqrt{18\pi}} e^{-\frac{1}{2} \left(\frac{2x-3}{6} \right)^2} = \frac{1}{3\sqrt{2\pi}} e^{-\frac{1}{2} \left(\frac{x-\frac{3}{2}}{3} \right)^2}$$

$$X \sim N\left(\frac{3}{2}, 3^2\right)$$

- a dilation by a factor of 3 from the x-axis

$$f_1(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2} \left(\frac{2x-3}{6} \right)^2}$$

- a dilation by a factor of $\frac{1}{3}$ from the y-axis

$$f_2(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2} \left(\frac{6x-3}{6} \right)^2} = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2} \left(x - \frac{1}{2} \right)^2}$$

- a translation of $\frac{1}{2}$ a unit left.

$$f_3(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2} x^2}$$

$$\text{Check: } (x, y) \rightarrow (x, 3y) \rightarrow \left(\frac{x}{3}, 3y\right) \rightarrow \left(\frac{x}{3} - \frac{1}{2}, 3y\right)$$

$$x' = \frac{x}{3} - \frac{1}{2}, \quad x = 3x' + \frac{3}{2}$$

$$y' = 3y, y = \frac{y'}{3}$$

$$\frac{y'}{3} = \frac{1}{3\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{3x' + \frac{3}{2} - \frac{3}{2}}{3}\right)^2}$$

$$y' = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(x')^2}$$

Question 20 Answer A

$$f: \mathbb{R} \rightarrow \mathbb{R}, f(x) = e^{x^3+bx}$$

Solve $f''(x) = 0$ but does not work directly on the T or the CASIO.

So find $f''(x) = (9b^2x^4 + 6bx^2 + 6bx + 1)e^{x^3+bx}$.

There will be no points of inflection when $f''(x) \geq 0$ for all x .

Solve $9x^4 + 6bx^2 + 6x + b^2 = 0$ and $\frac{d}{dx}(9x^4 + 6bx^2 + 6x + b^2) = 0$

O

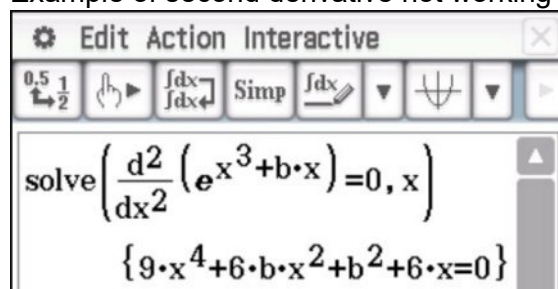
Solve $(9b^2x^4 + 6bx^2 + 6bx + 1)e^{x^3+bx} = 0$ and $\frac{d}{dx}((9b^2x^4 + 6bx^2 + 6bx + 1)e^{x^3+bx}) = 0$

$$b = \frac{3^{\frac{4}{3}}}{4}$$

There will be no points of inflection when $b \geq \frac{3^{\frac{4}{3}}}{4}$.

The answer can also be found by checking the values in the options. The easiest way to do this is to graph the function and use a slider. Choose a value of b that gives two points of inflection and label them with their coordinates. Then use the slider to see when they disappear.

Example of second derivative not working on the CASIO.



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solve $\left(\frac{d}{dx}\left(\frac{d}{dx}(f(x))\right)\right)=0, x$

$x \cdot (3 \cdot x^3 + 2 \cdot b \cdot x + 2) = \frac{-b^2}{3}$

$\frac{d}{dx}\left(\frac{d}{dx}(f(x))\right)$

$(9 \cdot x^4 + 6 \cdot b \cdot x^2 + 6 \cdot x + b^2) \cdot e^{x^3 + b \cdot x}$

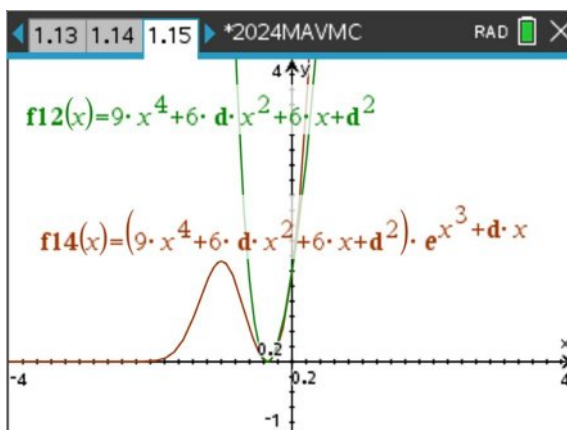
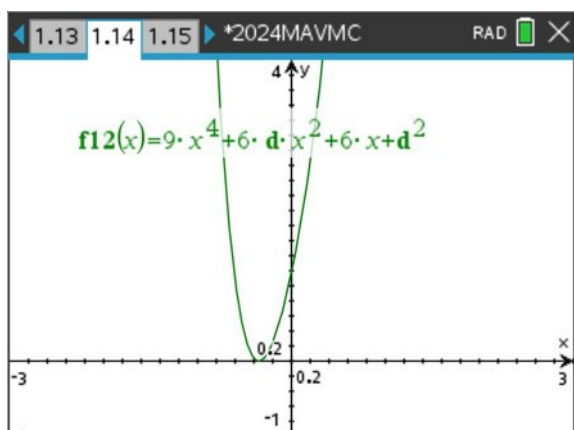
1.12 1.13 1.14 *2024MAVMC RAD

solve $\left(\frac{d}{dx}(9 \cdot x^4 + 6 \cdot b \cdot x^2 + 6 \cdot x + b^2)\right)=0$ and $9 \cdot$

$b = \frac{3 \cdot 3^{\frac{1}{3}}}{4}$ and $x = \frac{-3^{\frac{2}{3}}}{6}$

solve $\left(\frac{d}{dx}(9 \cdot x^4 + 6 \cdot b \cdot x^2 + 6 \cdot x + b^2)\right)=0$ and $9 \cdot$

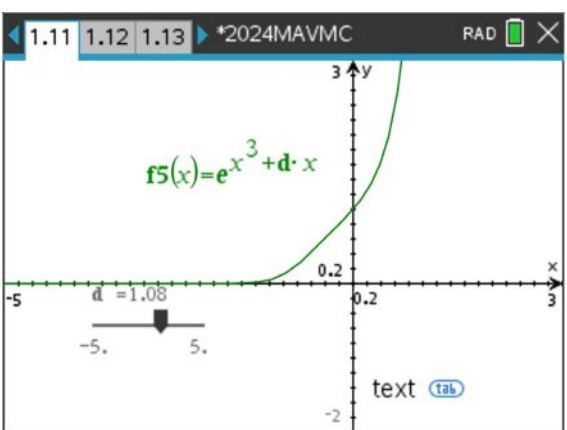
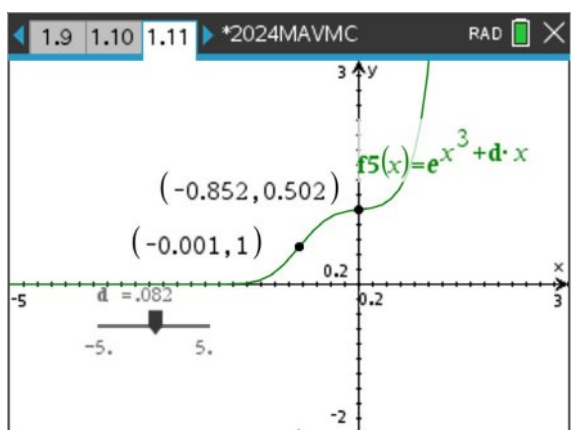
$b = 1.0816872$ and $x = -0.34668064$



Examples

$b < \frac{3^{\frac{4}{3}}}{4}$ (2 points of inflection)

$b = \frac{3^{\frac{4}{3}}}{4}$ (no points of inflection)

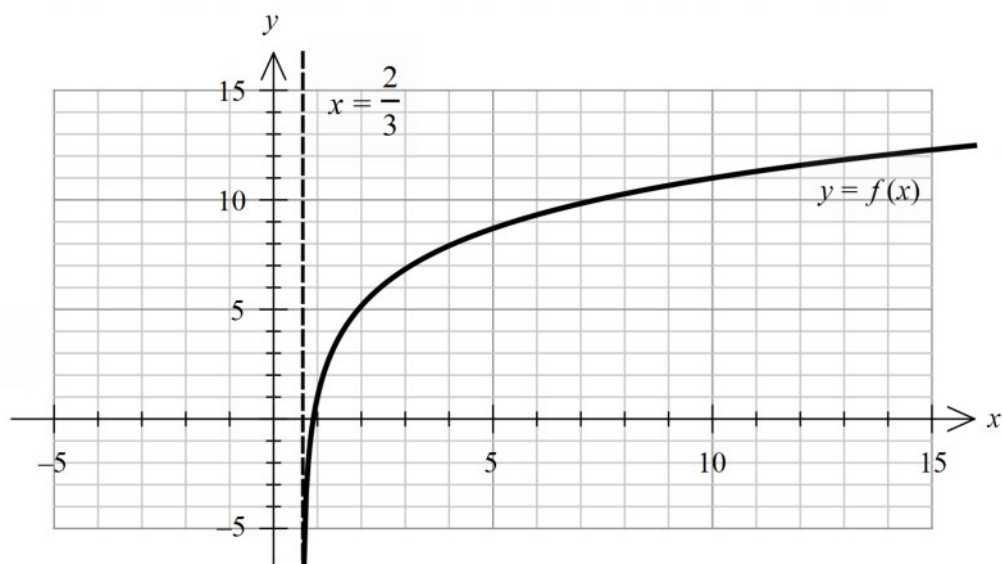


END OF SECTION A SOLUTIONS

SECTION B**Question 1**

$$f: \left(\frac{2}{3}, \infty\right) \rightarrow \mathbb{R}, f(x) = 3 \log_e(3x - 2) + 1$$

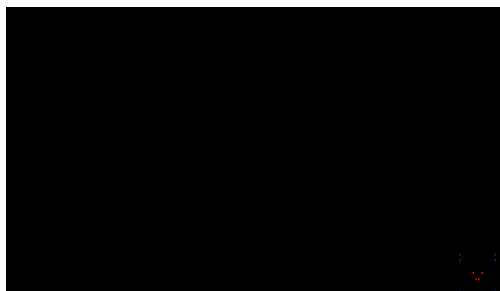
a. Sketch and label asymptote $x = \frac{2}{3}$

1A

b. $f^{-1}(x) = \frac{1}{3}e^{\frac{x-1}{3}} + \frac{2}{3}$

1A

Dom: $x \in \mathbb{R}$

1A

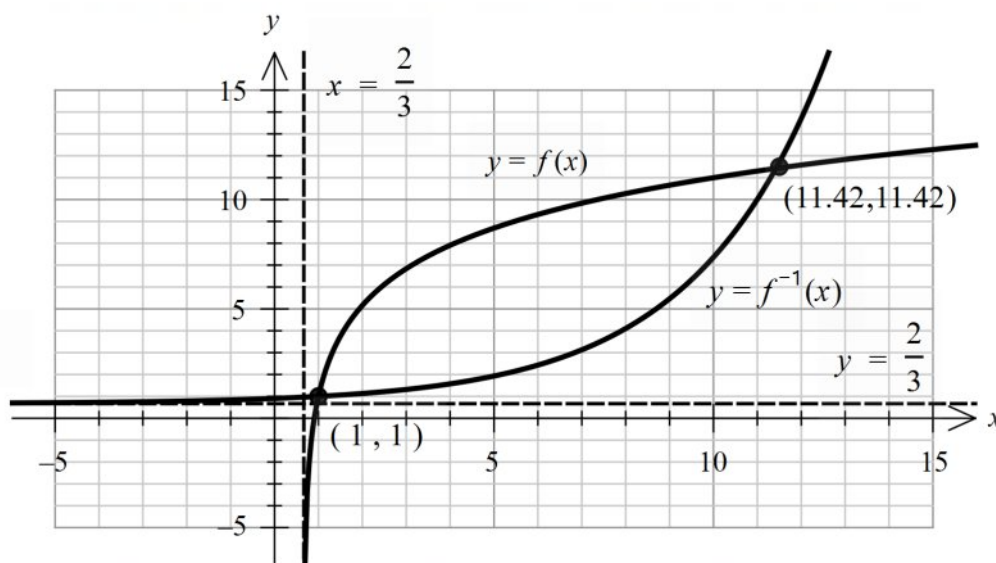
c. Sketch $y = f^{-1}(x)$. Shape and asymptote $y = \frac{2}{3}$

1A

Points of intersection between $y = f(x)$ and $y = f^{-1}(x)$

$(1, 1), (11.42, 11.42)$

1A



$$\text{solve}\left(f(x) = \frac{e^{\frac{x}{3} - \frac{1}{3}} + 2}{3}, x\right)$$

$$\{x=1, x=11.42205843\}$$

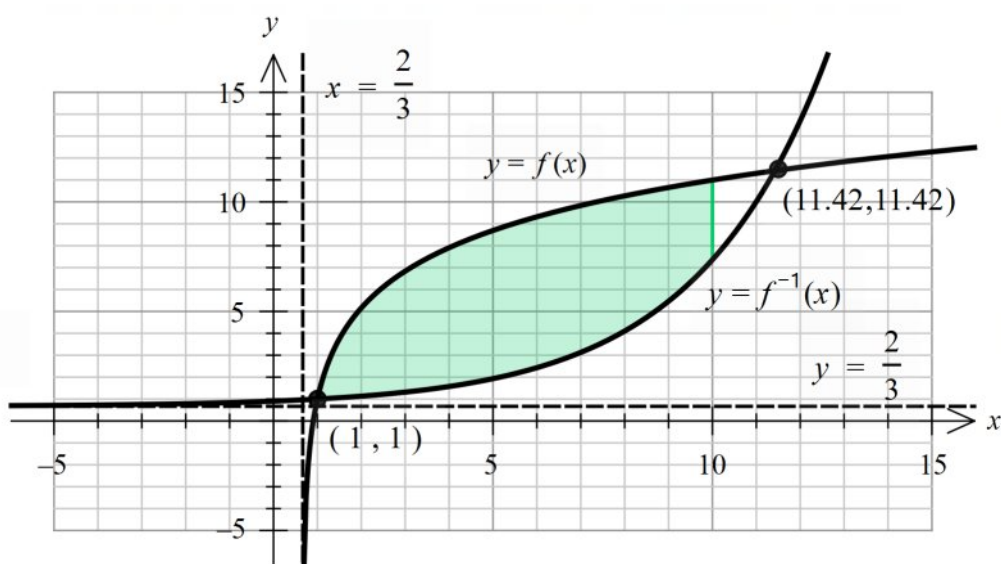
d.i. $\int_1^{10} (f(x) - f^{-1}(x)) dx$

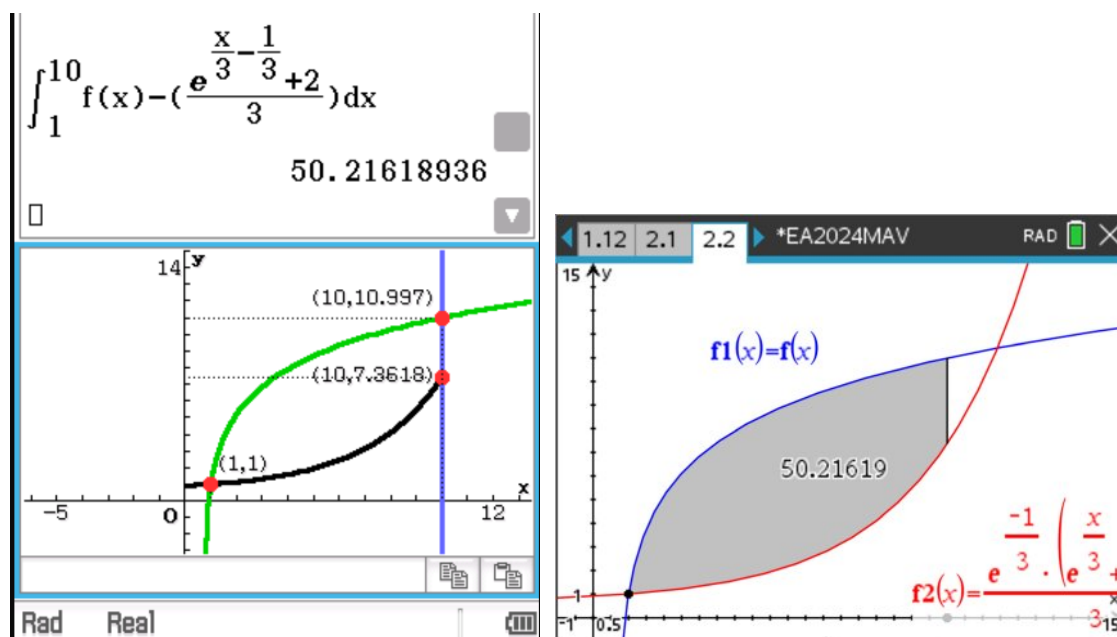
1A

d.ii. Area = 50.22 sq units
Shading

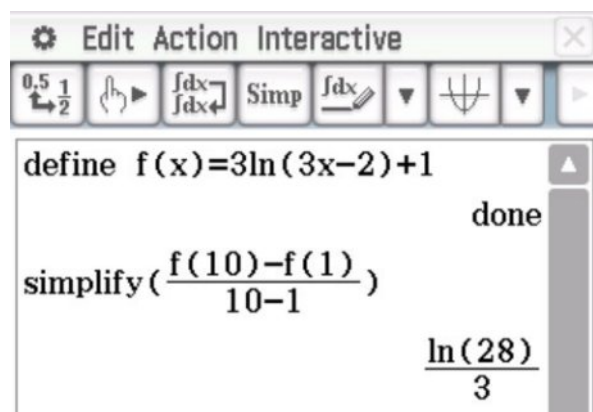
1A

1A





e.i. Average rate of change = $\frac{f(10) - f(1)}{10 - 1}$
 $= \frac{\log_e(28)}{3}$. n correct form $\frac{\log_e(a)}{b}$

1A

e.ii. Solve $f'(x) = \frac{\log_e(28)}{3}$ for x .

$$x = \frac{2}{3} + \frac{9}{\log_e(28)}$$

1A (other forms)

Edit Action Interactive

define $f(x) = 3\ln(3x-2)+1$ done

solve $\left(\frac{d}{dx}(f(x)) = \frac{\ln(28)}{3}, x\right)$

$$\left\{x = \frac{\ln(784)+27}{3 \cdot \ln(28)}\right\}$$

Edit Action Interactive

$\frac{2 \cdot \ln(7)}{3 \cdot (\ln(7)+2 \cdot \ln(2))} + \frac{4 \cdot \ln(7)}{3 \cdot (\ln(7)+2 \cdot \ln(2))}$

propFrac $\left(\frac{\ln(784)+27}{3 \cdot \ln(28)}\right)$

$\frac{2 \cdot \ln(7)}{3 \cdot (\ln(7)+2 \cdot \ln(2))} + \frac{4 \cdot \ln(7)}{3 \cdot (\ln(7)+2 \cdot \ln(2))}$

simplify $\left(\frac{2 \cdot \ln(7)}{3 \cdot (\ln(7)+2 \cdot \ln(2))} + \frac{4 \cdot \ln(7)}{3 \cdot (\ln(7)+2 \cdot \ln(2))}\right)$

$$\frac{9}{\ln(7)+2 \cdot \ln(2)} + \frac{2}{3}$$

4.1 4.2 4.3 *EA2024MAV RAD

solve $\left(\frac{d}{dx}(f(x)) = \frac{\ln(28)}{3}, x\right)$

$$x = \frac{2 \cdot \ln(28)+27}{3 \cdot \ln(28)}$$

propFrac $\left(\frac{2 \cdot \ln(28)+27}{3 \cdot \ln(28)}\right)$

$$\frac{9}{\ln(28)} + \frac{2}{3}$$

e.iii. The maximum value of the average rate of change will occur when the gradient of the line passing through $(a, f(a))$ and $(b, f(b))$ is steepest. This will occur when $a=1$ and $b=2$.

Maximum average rate of change $= \frac{f(2) - f(1)}{2 - 1} = 6 \log_e(2)$ **1M**

Solve $f'(x) = 6 \log_e(2)$ for x .

$x = \frac{2}{3} + \frac{1}{2 \log_e(2)}$ **1A (other forms)**

Edit Action Interactive

define $f(x) = 3\ln(3x-2)+1$ done

$\frac{f(2)-f(1)}{2-1}$

$6 \cdot \ln(2)$

solve $\left(\frac{d}{dx}(f(x)) = 6 \cdot \ln(2), x\right)$

$$\left\{x = \frac{1}{2 \cdot \ln(2)} + \frac{2}{3}\right\}$$

e.iv. f is continuous over the interval $[a, b]$ and smooth over the interval (a, b) but

$f'(x) = \frac{9}{3x-2} \neq 0$ for any x . Hence, $f(a) \neq f(b)$. or the average value to equal zero, $f(a)$ must equal $f(b)$. **1A**

$$\frac{d}{dx}(f(x)) = \frac{9}{3x-2}$$

f.i. $h: \left(\frac{2}{b}, \infty\right) \rightarrow \mathbb{R}$, $h(x) = a \log_e(bx - 2) + 1$ where $h(x) = 3f(5x) - 2$

given $f(x) = 3 \log_e(3x - 2) + 1$

$$h(x) = 3f(5x) - 2$$

$$h(x) = 9 \log_e(15x - 2) + 1$$

$$a = 9, b = 15$$

1A

$$\begin{aligned} \text{define } h(x) &= 3f(5x) - 2 \\ &\text{done} \\ \text{expand}(h(x)) &= 9 \cdot \ln(15x - 2) + 1 \end{aligned}$$

f.ii. Solve $h_1'(x) = f'(x)$

$$x = \frac{3k-1}{3k}$$

As $k \rightarrow \infty$, $x \rightarrow 1$

As $k \rightarrow -\infty$, $x \rightarrow 1$

$$x = 1$$

1A

$$\begin{aligned} h(x) &:= 3 \cdot f(k \cdot x) - 2 \\ \text{solve} \left(\frac{d}{dx}(f(x)) = \frac{d}{dx}(h(x)), x \right) &= \frac{3 \cdot k - 1}{3 \cdot k} \\ \lim_{k \rightarrow \infty} \left(\frac{3 \cdot k - 1}{3 \cdot k} \right) &= 1 \\ \lim_{k \rightarrow -\infty} \left(\frac{3 \cdot k - 1}{3 \cdot k} \right) &= 1 \end{aligned}$$

Question 2

$$h(t) = a \sin(b(t-18)) + c$$

a. max 50, min 10, amp = 20

translation vertically: $-20 + 30 = 10$

$a = 20, c = 30$ **1M** (explanation)

b. One cycle = 18 hours

$$\frac{2\pi}{b} = 18$$

$$2\pi = 18b$$

Gives $b = \frac{\pi}{9}$ **1M** (show that)

c. $\frac{1}{18} \int_0^{18} h_A dx - \frac{1}{18} \int_0^{18} h_B dx$ **1M**

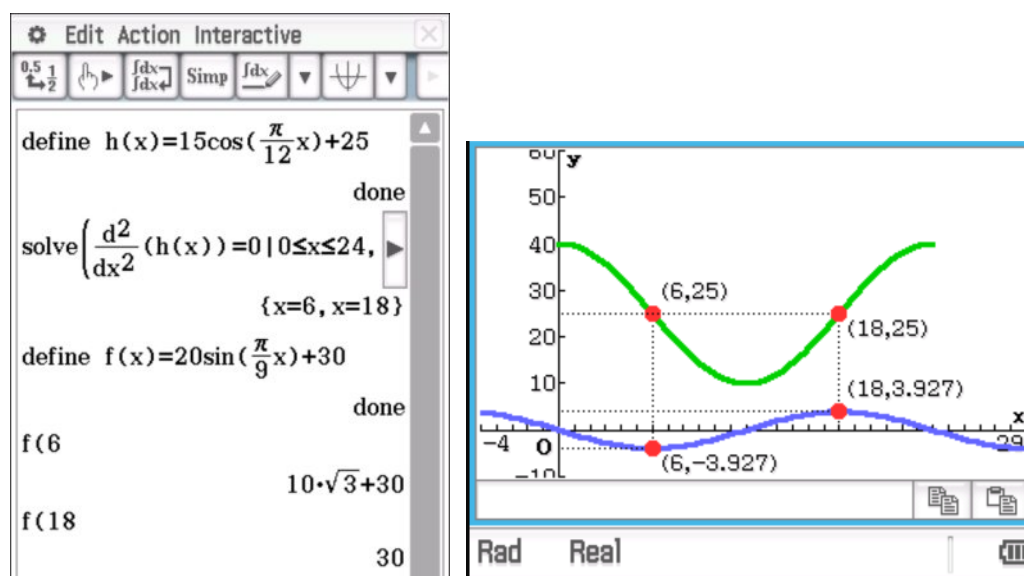
$$= \frac{5(\pi + 2)}{\pi} \text{ m} \quad \mathbf{1A}$$

$\frac{1}{18} \int_0^{18} f(x) dx$
 30
 $\text{combine}\left(\frac{1}{18} \int_0^{18} h(x) dx\right)$
 $\frac{25\pi - 10}{\pi}$
 $\text{combine}\left(30 - \frac{25\pi - 10}{\pi}\right)$
 $\frac{5\pi + 10}{\pi}$

d. $h_B(t) = 15 \cos\left(\frac{\pi t}{12}\right) + 25$

The height of the river would be changing fastest at the points of inflection of the graphs of h_B . So when $t = 6$ and $t = 18$. **1M**

$h_A(6) = 10\sqrt{3} + 30$ and $h_A(18) = 30$ **1A**



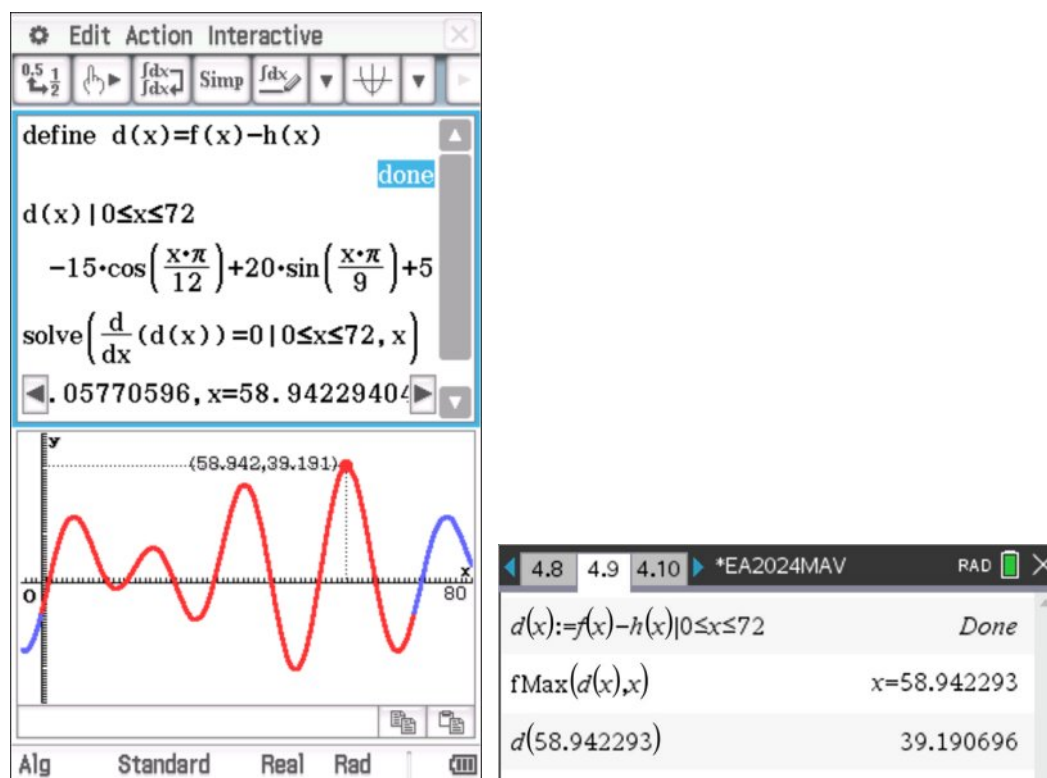
e. Let $d(x) = h_A(x) - h_B(x)$

The period of the graph of $d(x)$ is the lowest common multiple of 18 and 24 which is 72 hours.

1A

The maximum difference is 39.19 m.

1A



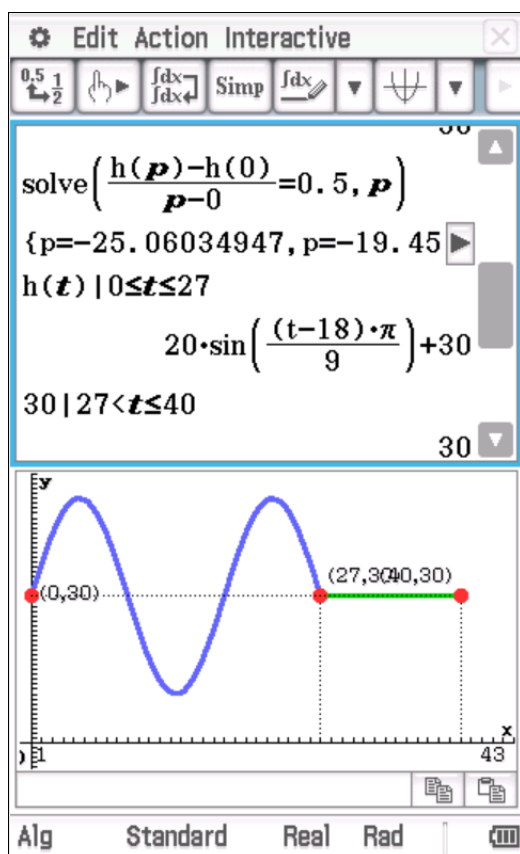
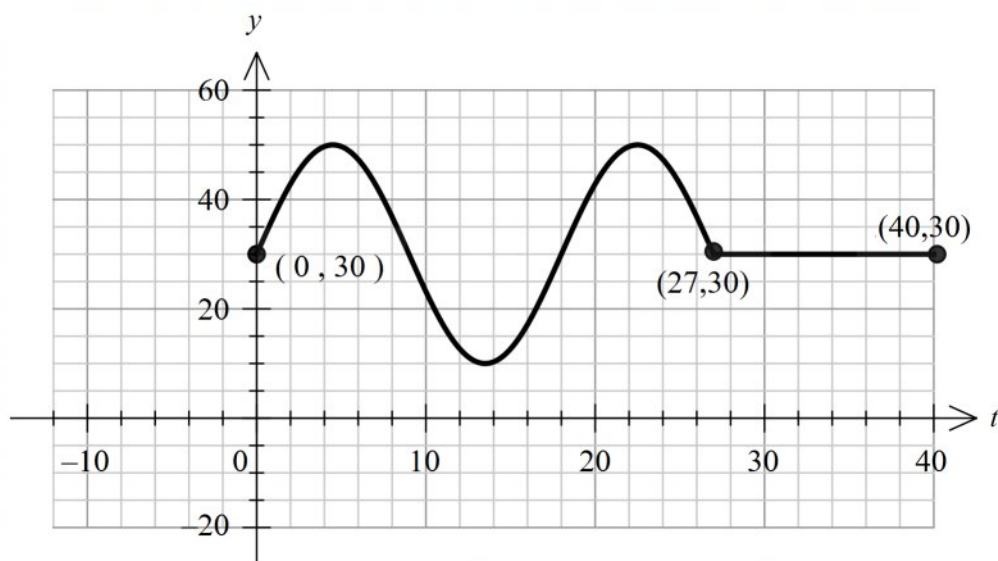
f. $w(t) = \begin{cases} h_A(t) & 0 \leq t \leq 27 \\ 30 & 27 < t \leq 40 \end{cases}$

Graph of piecewise function w

1A

Coordinates $(0, 30)$, $(40, 30)$, $(27, 30)$

1A



g. $t \in (0, 27) \cup (27, 40)$ **1A**

h.
$$p(t) = \begin{cases} w(t) & 0 \leq t \leq 40 \\ m \cos(n(t-r)) + s & 40 < t \leq k \end{cases}$$

am Sunday to pm Tuesday is 0 hours.
 $k = 60$ **1A**

i. Continuous and smooth at $t = 40$. So there is a turning point at $t = 40$.

$p = m \cos(n(t-r)) + s$ completes two cycles before recording capacities break, reaching zero height twice. So the range is $[0, 30]$.

Amplitude = 15, $m = 15$

Period = 10, $n = \frac{2\pi}{10} = \frac{\pi}{5}$

1H

$m = 15, s = 15$.

1A

$r = 10q$ where $q \in \mathbb{Z}$

1A

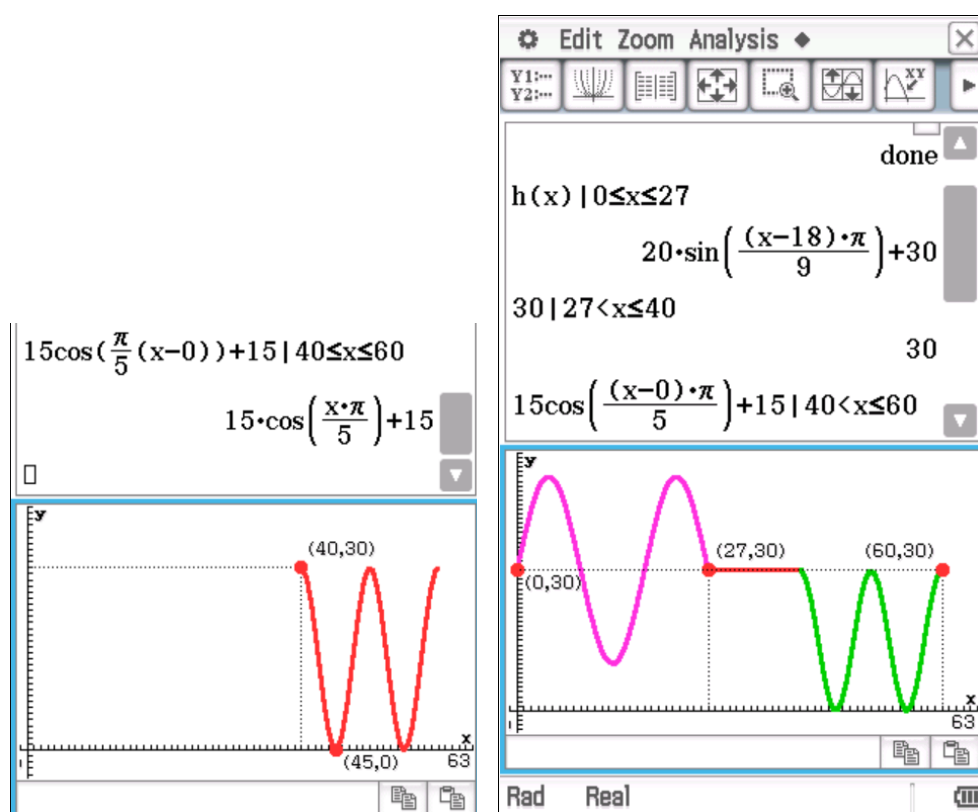
OR

$m = -15, s = 15$.

1A

$r = 5q$ where $q \in \mathbb{Z}$

1A

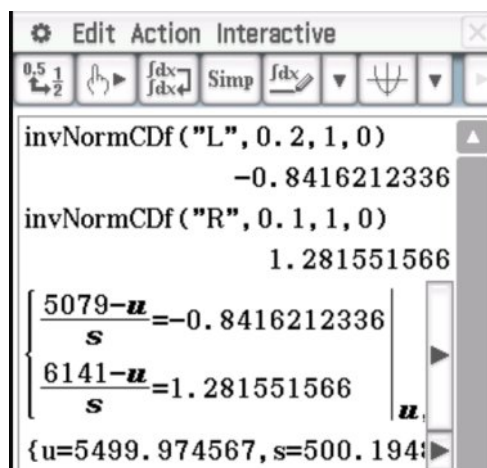
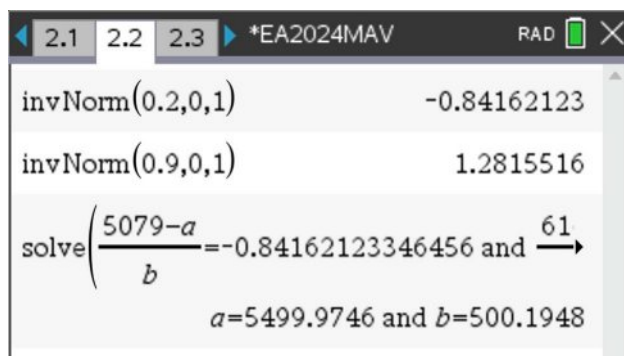


Question 3

a. $X_{AF} \sim N(\mu, \sigma^2)$

Solve $\frac{5079 - \mu}{\sigma} = -0.841\dots$ and $\frac{6141 - \mu}{\sigma} = 1.281\dots$ **1M**

$\mu = 5500.0$ kg and $\sigma = 500.2$ kg **1A**



b. $X_A \sim N(4085, 445^2)$, $X_{AB} \sim N(5375, 225^2)$

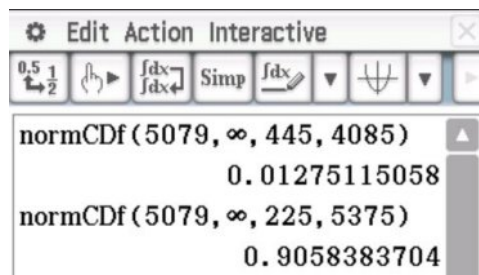
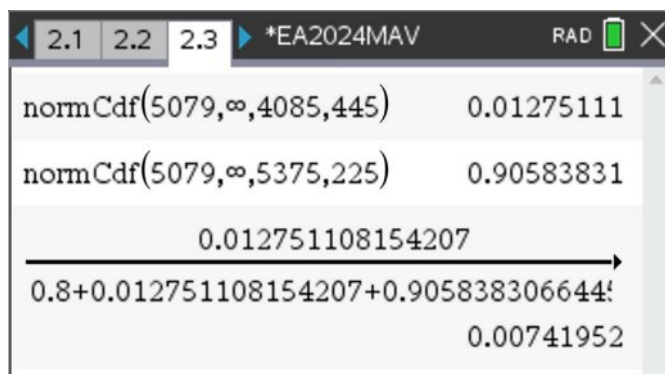
$\Pr(X_A > 5079) = 0.0127\dots$, $\Pr(X_{AB} > 5079) = 0.9058\dots$, $\Pr(X_{AF} > 5079) = 0.8$ **1M**

$\Pr(X_A > 5079 | (X_A > 5079 + X_{AB} > 5079 + X_{AF} > 5079))$

$$= \frac{\frac{1}{3} \times 0.0127\dots}{\frac{1}{3} \times 0.0127\dots + \frac{1}{3} \times 0.9058\dots + \frac{1}{3} \times 0.8}$$

$$= \frac{0.0127\dots}{0.0127\dots + 0.9058\dots + 0.8}$$

$$= 0.0074 \quad \mathbf{1A}$$

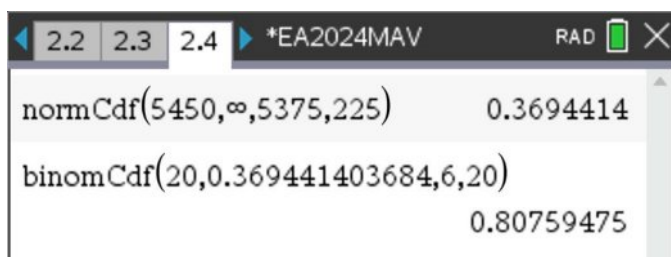


c. $X_{AB} \sim N(5375, 225^2)$

$\Pr(X_{AB} > 5450) = 0.3694\dots$

$X \sim \text{Bi}(20, 0.3694\dots)$ **1M**

$\Pr(X > 5) = \Pr(X \geq 6) = 0.8076$ **1A**



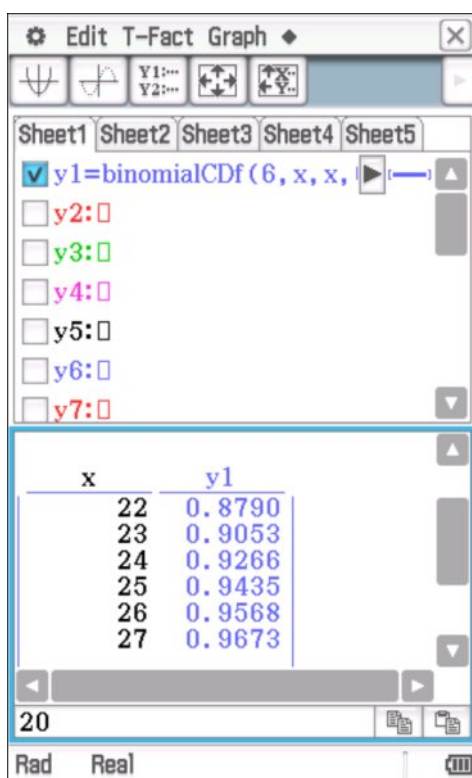
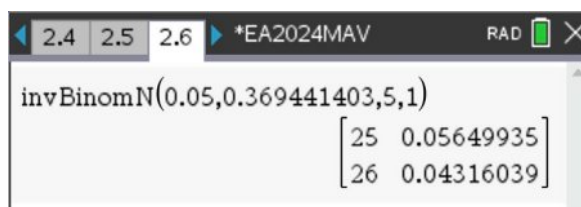
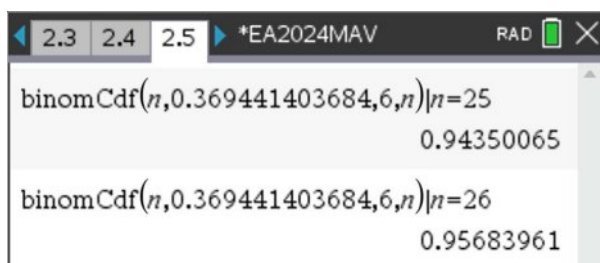
d. $X_2 \sim \text{Bi}(n, 0.3694\dots)$

Trial and error **1M** (other methods)

n	$\Pr(X_2 \geq 6)$
25	0.9435...
26	0.9568...

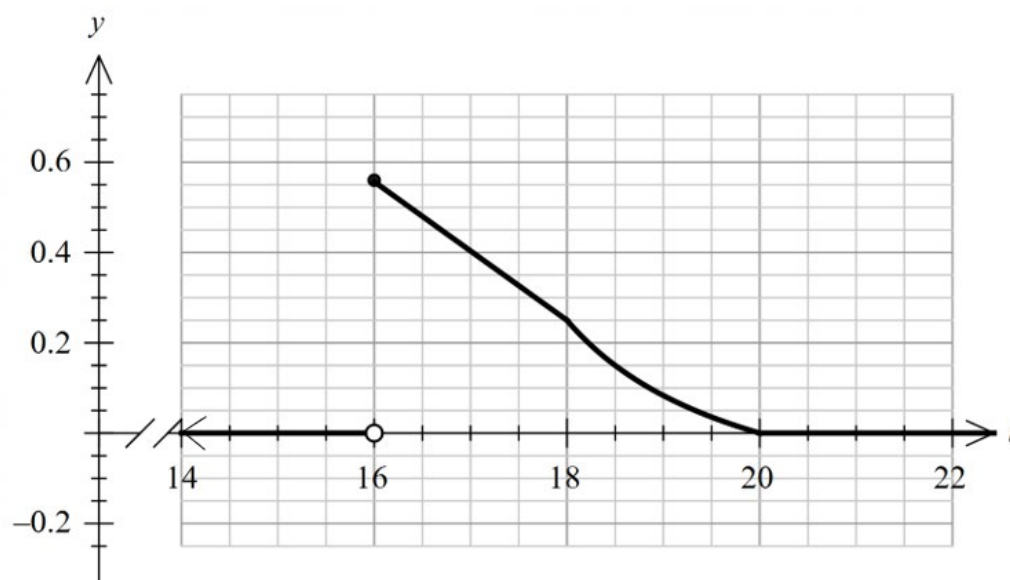
$n = 26$

1A



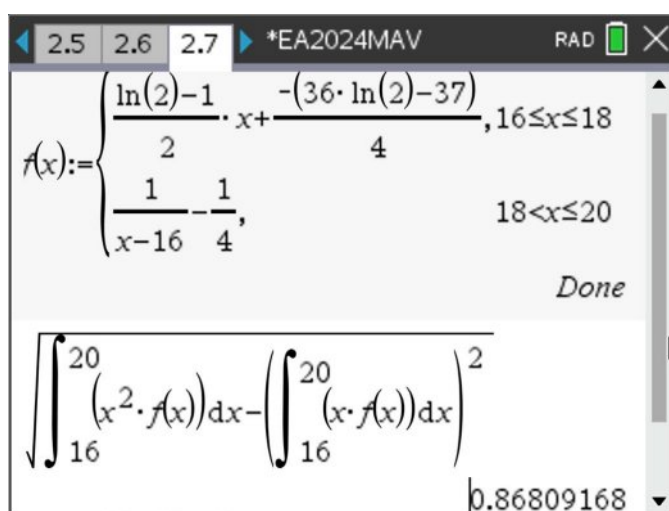
e. Shape, open circle, must draw along the x -axis **1A**

$$f(t) = \begin{cases} \frac{\log_e(2)-1}{2}t + \frac{37-36\log_e(2)}{4} & 16 \leq t \leq 18 \\ \frac{1}{t-16} - \frac{1}{4} & 18 < t \leq 20 \\ 0 & \text{elsewhere} \end{cases}$$



$$\text{f. } \text{sd}(T) = \sqrt{\int_{16}^{20} (t^2 \times f(t)) dt - \left(\int_{16}^{20} (t \times f(t)) dt \right)^2} \quad \mathbf{1M}$$

$$= 0.868 \quad \mathbf{1A}$$



g. (0.0117, 0.2550) 1A

2.7 2.8 2.9 ▶ *EA2024MAV RAD

zInterval_1Prop 4,30,0.95: stat.results

"Title"	"1-Prop z Interval"
"CLower"	0.01169152
"CUpper"	0.25497514
"p̂"	0.13333333
"ME"	0.12164181
"n"	30.

C-Level 0.95

x 4

n 30

<< Back Help Next >>

OnePropZInt

Lower 0.0116915

Upper 0.2549751

p̂ 0.1333333

n 30

<< Back Help

OnePropZInt

h. Solve $1.96\sqrt{\frac{\frac{4}{30} \times \frac{26}{30}}{n}} < 0.1$ for n .

$n = 45$ **1A**

2.7 2.8 2.9 ▶ *EA2024MAV RAD

solve $\left(1.96 \cdot \sqrt{\frac{\frac{4}{30} \cdot 26}{30}} < 0.1, n \right)$

$n > 44.391822$

i. Let AB be the African bush elephant and AF be the African forest elephant.

$$\Pr(AB \cap AF) = \Pr(AB) \times \Pr(AF) = \frac{2-k^2}{2} \text{ independent events}$$

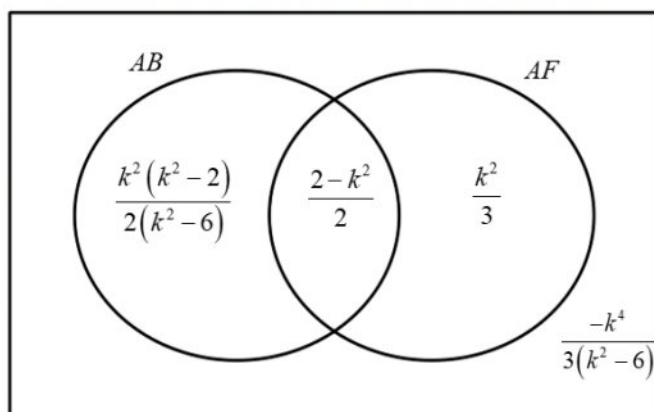
$$\Pr(AB) \times \left(\frac{2-k^2}{2} + \frac{k^2}{3} \right) = \frac{2-k^2}{2} \quad \mathbf{1M}$$

$$\Pr(AB) = \frac{3(k^2 - 2)}{k^2 - 6}$$

$$\Pr(AB \cup AF) + \Pr(AB' \cap AF') = 1$$

$$\frac{3(k^2 - 2)}{k^2 - 6} + \frac{k^2}{3} + \Pr(AB' \cap AF') = 1$$

$$\Pr(AB' \cap AF') = \frac{-k^4}{3(k^2 - 6)} \quad \mathbf{1A}$$



$$\mathbf{j.} \Pr(AB \cap AF') = \frac{3(k^2 - 2)}{k^2 - 6} - \frac{2 - k^2}{2} = \frac{k^2(k^2 - 2)}{2(k^2 - 6)}$$

$$\text{Solve } \frac{d}{dk} \left(\frac{k^2(k^2 - 2)}{2(k^2 - 6)} \right) = 0 \text{ or use fmax}$$

$$k = \sqrt{-2(\sqrt{6} - 3)}$$

$$\text{Maximum probability is } 5 - 2\sqrt{6} \quad \mathbf{1A}$$

3.10 3.11 3.12 *EA2024MAV RAD

solve $\left(\frac{d}{dk} \left(\frac{k^2 \cdot (k^2 - 2)}{2 \cdot (k^2 - 6)} \right) = 0, k \right) | 0 < k < \sqrt{2}$

$k = \sqrt{-2 \cdot (\sqrt{6} - 3)}$

$\frac{k^2 \cdot (k^2 - 2)}{2 \cdot (k^2 - 6)} | k = \sqrt{-2 \cdot (\sqrt{6} - 3)}$ $5 - 2 \cdot \sqrt{6}$

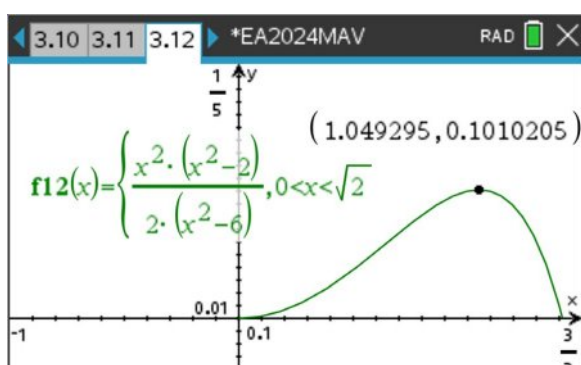
$\frac{k^2 \cdot (k^2 - 2)}{2 \cdot (k^2 - 6)} | k = \sqrt{-2 \cdot (\sqrt{6} - 3)}$ 0.10102051

3.10 3.11 3.12 *EA2024MAV RAD

fMax $\left(\frac{k^2 \cdot (k^2 - 2)}{2 \cdot (k^2 - 6)}, k \right) | 0 < k < \sqrt{2}$

$k = \sqrt{-2 \cdot (\sqrt{6} - 3)}$

$\frac{k^2 \cdot (k^2 - 2)}{2 \cdot (k^2 - 6)} | k = \sqrt{-2 \cdot (\sqrt{6} - 3)}$ $5 - 2 \cdot \sqrt{6}$



Question 4

$h(x) = \sqrt{x^4 - px^2 + 1}$ and $f(x) = x^4 - px^2 + 1$, and $p \in \mathbb{R}$

a. Solve $h(x) = f(x)$ when $p = 3$

$x = \pm\sqrt{3}, 0, \frac{-\sqrt{5} \pm 1}{2}, \frac{\sqrt{5} \pm 1}{2}$ **1A**

1.1 1.2 1.3 *EA2024MAV RAD

$f(x) := x^4 - p \cdot x^2 + 1$ Done

$h(x) := \sqrt{x^4 - p \cdot x^2 + 1}$ Done

solve $(f(x) = h(x), x) | p = 3$

$x = -\sqrt{3}$ or $x = \frac{-(\sqrt{5} + 1)}{2}$ or $x = \frac{-(\sqrt{5} - 1)}{2}$ or $x = 0$

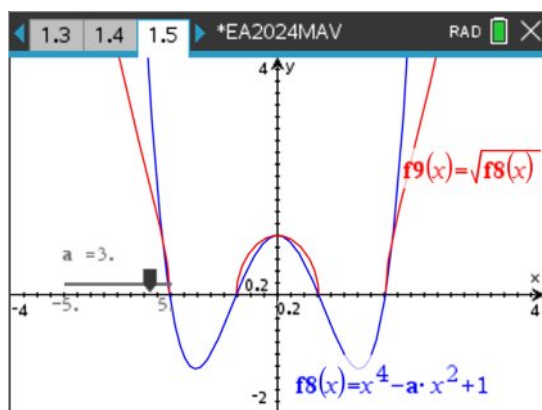
1.1 1.2 1.3 EA2024MAV RAD

$f(x) := x^4 - p \cdot x^2 + 1$ Done

$h(x) := \sqrt{x^4 - p \cdot x^2 + 1}$ Done

solve $(f(x) = h(x), x) | p = 3$

$\frac{1}{2}$ or $x = 0$ or $x = \frac{\sqrt{5} - 1}{2}$ or $x = \frac{\sqrt{5} + 1}{2}$ or $x = \sqrt{3}$

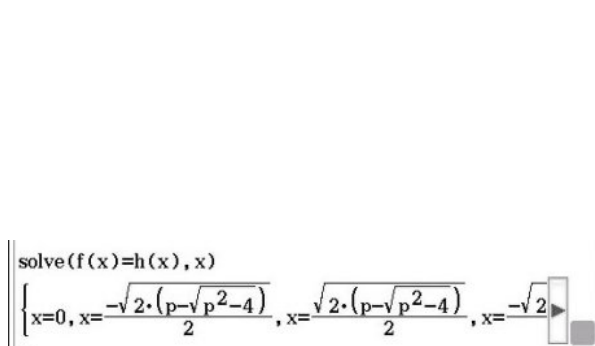
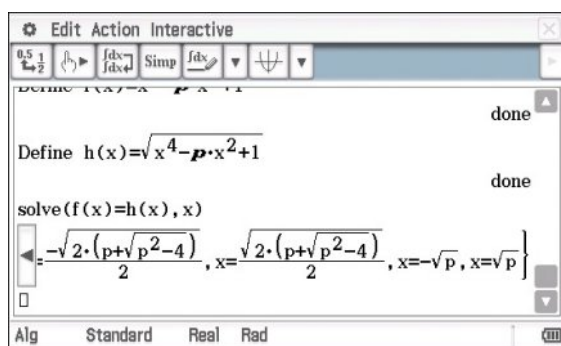
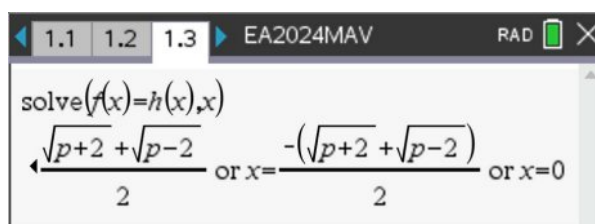
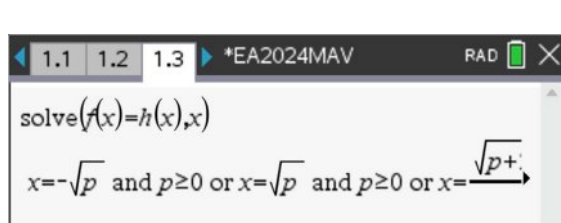


b. Solve $h(x) = f(x)$ for x .

$$x = \pm\sqrt{p}, 0, \frac{\pm\sqrt{p+2} + \sqrt{p-2}}{2}, \frac{\pm\sqrt{p+2} - \sqrt{p-2}}{2} \quad 1A$$

OR

$$x = \pm\sqrt{p}, 0, \frac{\pm\sqrt{2(p + \sqrt{p^2 - 4})}}{2}, \frac{\pm\sqrt{2(p - \sqrt{p^2 - 4})}}{2} \quad 1A$$



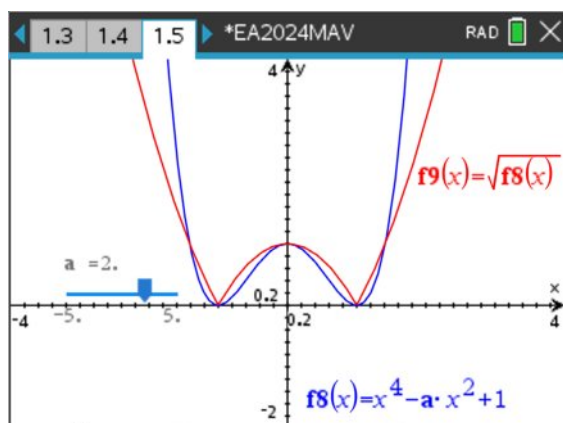
c. If $p = 2$, $\sqrt{p-2} = 0$, so $\frac{\pm\sqrt{p+2} + \sqrt{p-2}}{2} = \frac{\pm\sqrt{p+2} - \sqrt{p-2}}{2} = \frac{\pm\sqrt{p+2}}{2}$

OR

If $p = 2$, $p^2 - 4 = 0$, so $\frac{\pm\sqrt{2(p + \sqrt{p^2 - 4})}}{2} = \frac{\pm\sqrt{2(p - \sqrt{p^2 - 4})}}{2}$

ence onl five solutions.

1A

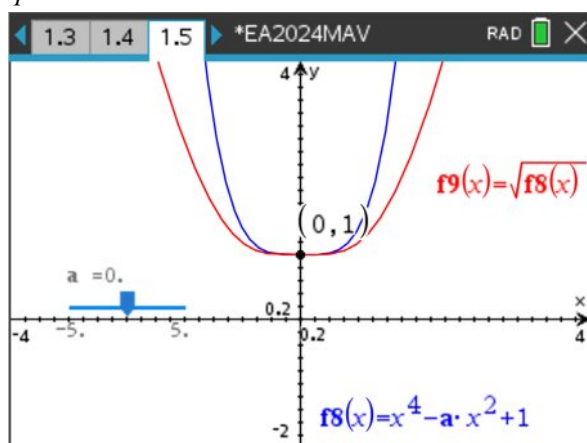


- d. 2 correct 1A
All correct 2A

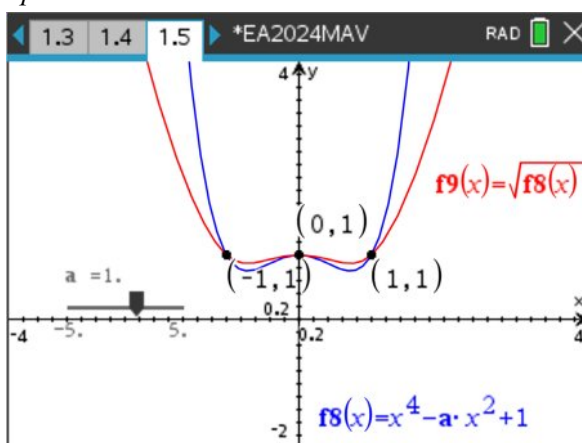
Number of points of intersection	1	3
p values	$p \leq 0$	$0 < p < 2$
x -coordinates of points of intersection	0	$0, \pm\sqrt{p}$

Examples

$p = 0$



$p = 1$



e. $g(x) = \sqrt{x}$, $f(x) = x^4 - px^2 + 1$

or $g(f(x))$ to exist the range of f has to be a subset of, or equal to, the domain of g .

The domain of g is $[0, \infty)$, the range of f will be $[0, \infty)$ for $p = 2$ and a subset of $[0, \infty)$ for $p < 2$.

The range of f , $\left[1 - \frac{p^2}{4}, \infty\right)$, is not a subset of $[0, \infty)$ for $p > 2$. 1A

Calculator screen showing the solution to the derivative equation:

$$\text{solve}\left(\frac{d}{dx}(f(x))=0, x\right)$$

$$x = \frac{\sqrt{2 \cdot p}}{2} \text{ and } p \geq 0 \text{ or } x = \frac{-\sqrt{2 \cdot p}}{2} \text{ and } p \geq 0 \text{ or } x = \frac{\sqrt{2 \cdot p}}{2}$$

$$f\left(\frac{\sqrt{2 \cdot p}}{2}\right) = 1 - \frac{p^2}{4}$$

f. Using the bounded area on the graph

$$\text{Area} = 2(0.0196238... + 0.076381...) \quad \mathbf{1M}$$

$$= 0.192 \quad \mathbf{1A}$$

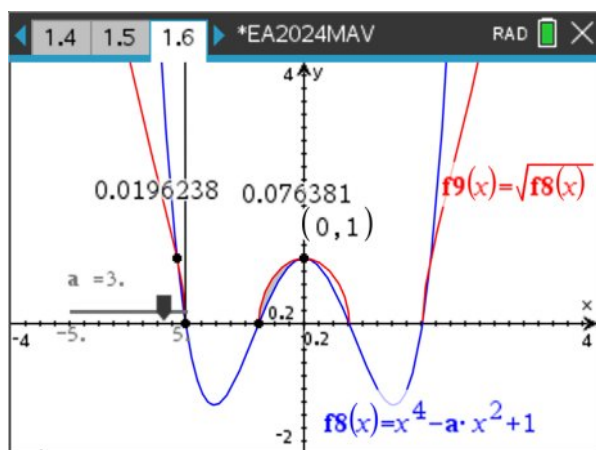
OR

Using definite integrals

$$\text{Area} = 2 \left(\int_{-\frac{\sqrt{5}-1}{2}}^{\frac{\sqrt{5}-1}{2}} (h(x) - f(x)) dx + \int_{-\frac{\sqrt{5}+1}{2}}^0 (h(x) - f(x)) dx \right) \quad \mathbf{OR}$$

$$= 2 \left(\int_0^{\frac{\sqrt{5}-1}{2}} (h(x) - f(x)) dx + \int_{\frac{\sqrt{5}+1}{2}}^{\sqrt{3}} (h(x) - f(x)) dx \right) \quad \mathbf{1M (either form)}$$

$$= 0.192 \quad \mathbf{1A}$$



Calculator screen showing the definite integral calculation:

$$2 \cdot \left(\int_{-\frac{\sqrt{5}-1}{2}}^{\frac{\sqrt{5}-1}{2}} (h(x) - f(x)) dx + \int_{-\frac{\sqrt{5}+1}{2}}^0 (h(x) - f(x)) dx \right)$$

$$= 0.19200976$$

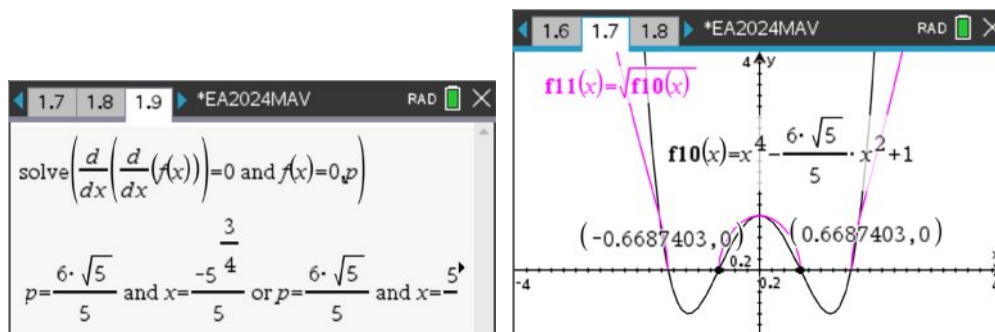
$$\mathbf{g. Area} = 2 \left(\int_{-\sqrt{p}}^{\frac{-\sqrt{p+2}-\sqrt{p-2}}{2}} (h(x) - f(x)) dx + \int_{\frac{-\sqrt{p+2}+\sqrt{p-2}}{2}}^0 (h(x) - f(x)) dx \right) \quad \mathbf{OR}$$

$$= 2 \left(\int_0^{\frac{\sqrt{p+2}-\sqrt{p-2}}{2}} (h(x) - f(x)) dx + \int_{\frac{\sqrt{p+2}+\sqrt{p-2}}{2}}^{\sqrt{p}} (h(x) - f(x)) dx \right) \quad \mathbf{OR}$$

$$= 2 \left(\int_0^{\frac{\sqrt{2(p-\sqrt{p^2-4})}}{2}} (h(x) - f(x)) dx + \int_{\frac{\sqrt{2(p+\sqrt{p^2-4})}}{2}}^{\sqrt{p}} (h(x) - f(x)) dx \right) \quad \mathbf{1A}$$

h. Solve $f''(x) = 0$ and $f(x) = 0$ for p . $\mathbf{1M}$

$$p = \frac{6\sqrt{5}}{5} \quad \mathbf{1A}$$



i. $h_v(x) = \sqrt{x^4 - 3x^2 + 1}$ and $f_v(x) = x^4 - 3x^2 + 1$

$$\text{Cross-sectional area} = \int_{-1.817...}^{1.817...} (2 - f_v(x)) dx - 0.1920... = 7.517... \quad \mathbf{1M}$$

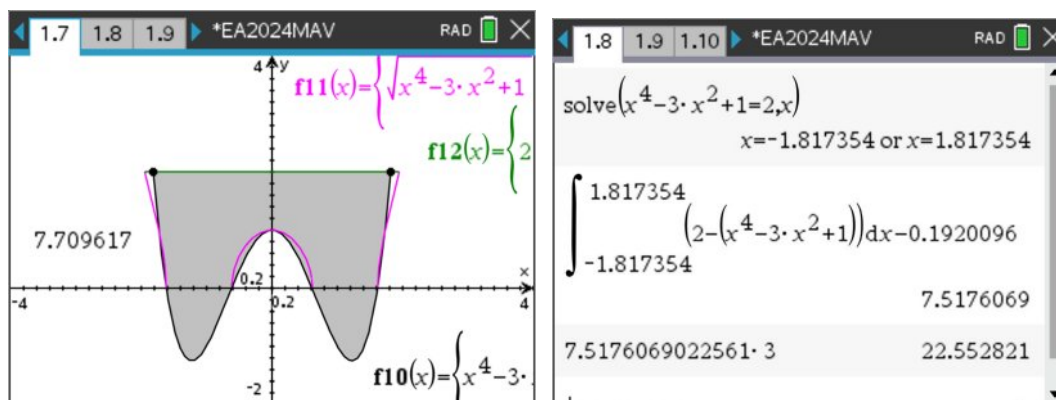
$$\text{Volume} = 7.517... \times 3 = 22.552...$$

$$\frac{dv}{dt} = 2^t$$

$$\text{Solve } \int_0^b 2^t dt = 22.552... \text{ for } b. \quad \mathbf{1H}$$

$$b = 4.06 \text{ seconds} \quad \mathbf{1A}$$

The shaded area on the graph is $\int_{-1.817...}^{1.817...} (2 - f_v(x)) dx = 7.709...$ and then you need to subtract the bound area found in **part f**.



1.7 1.8 1.9 EA2024MAV RAD

solve $\left(\int_0^b 2^t dt = 22.552820706768, b \right)$

$b = 4.0559265$

END OF SOLUTIONS