

2024 VCE Mathematical Methods Year 12 Trial Examination 1



Kilbaha Education

Quality educational content

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**Victorian Certificate of Education
2024**

STUDENT NUMBER

Figures
Words

Letter

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MATHEMATICAL METHODS

Trial Written Examination 1

Reading time: 15 minutes

Total writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
10	10	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software) notes of any kind, blank sheets of paper, and/or correction fluid/tape.

Materials supplied

- Question and answer book of 16 pages.
- Detachable sheet of miscellaneous formulas at the end of this booklet.
- Working space is provided throughout the booklet.

Instructions

- Detach the formula sheet from the end of this book during reading time.
- Write your **student number** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Instructions

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (4 marks)

- a. Evaluate $f'(2)$, where $f(x) = \log_e(\sqrt{x^3+1})$.

2 marks

- b. If $\frac{d}{dx}\left(\frac{x}{\sqrt{4x+9}}\right) = \frac{px+q}{(4x+9)^n}$, find the values of p , q and n .

2 marks

Question 2 (3 marks)

a. Solve for x if $3^{x^2+6x} = \frac{1}{243}$

1 mark

b. Solve for x if $\log_2(x^2 + 4\sqrt{2}) + \log_2(x^2 - 4\sqrt{2}) = 5$

2 marks

Question 3 (3 marks)

Find the values of a and b for which the simultaneous linear equations,

$$2ax - 2by = 5$$

$$(1-3b)x+12y=2-4b$$

have an infinite number of solutions.

[illegible]

Question 5 (3 marks)

Consider the function defined by $f(x) = \begin{cases} \sqrt{5-x^2}, & x \leq 2 \\ ax^2 + bx, & x > 2 \end{cases}$ where a and b are real numbers.

If the function has a smooth join at $x = 2$, find the values of a and b .

Question 6 (3 marks)

A certain curve has its gradient given by $5\sin\left(\frac{x}{2}\right) + me^{-2x} + 4$, if the curve has a turning point at the origin, find the value of m and the equation of the curve.

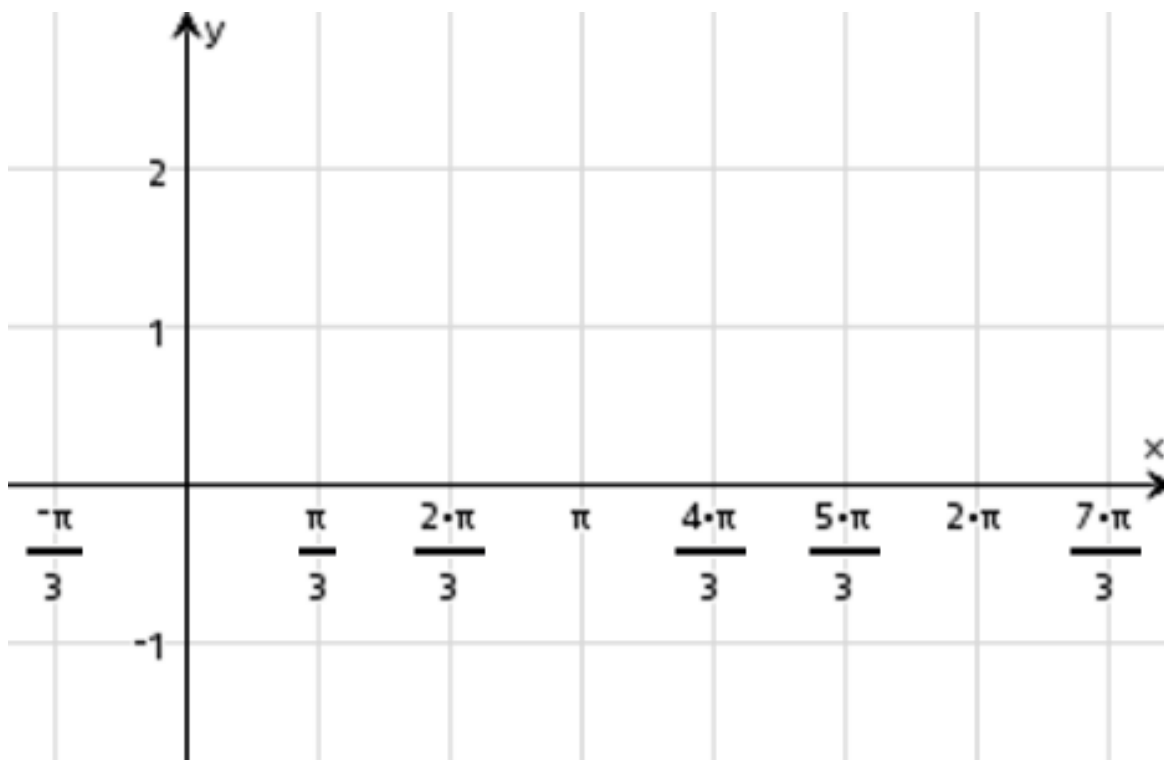
Question 7 (6 marks)

- a. Find the general solution of $2\sin^2(2x) + \cos(2x) - 1 = 0$ for $x \in \mathbb{R}$.

3 marks

- b. Consider the functions $f: [0, 2\pi] \rightarrow \mathbb{R}$, $f(x) = 2\sin^2(2x)$ and $g: [0, 2\pi] \rightarrow \mathbb{R}$, $g(x) = 1 - \cos(2x)$, on the axes below, sketch the graphs of the functions $y = f(x)$ and $y = g(x)$ and determine $2\sin^2(2x) < 1 - \cos(2x)$ for $x \in [0, 2\pi]$.

3 marks



Question 10 (7 marks)

- a. The random variable X has a probability density function f given by

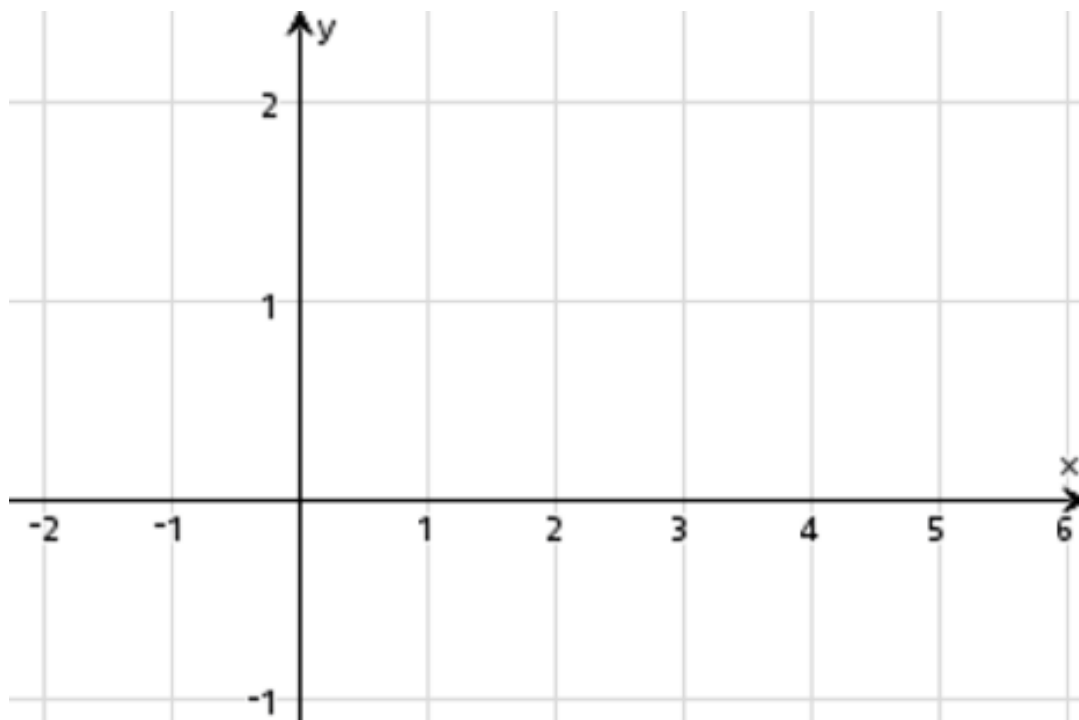
$$f(x) = \begin{cases} \frac{a}{(2x+1)^2} & 1 \leq x \leq 4 \\ 0 & \text{elsewhere} \end{cases} \quad \text{where } a \text{ is a positive real number.}$$

- i. Show that $a = 9$.

2 marks

- ii. Sketch the graph of f on the axes below, stating the coordinates of the endpoints.

1 mark



MATHEMATICAL METHODS

Written examination 1

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

Mathematical Methods formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of triangle	$\frac{1}{2}bc \sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}(x^n) = nx^{n-1}$	$\int x^n dx = \frac{1}{n+1}x^{n+1} + c, n \neq -1$		
$\frac{d}{dx}((ax+b)^n) = na(ax+b)^{n-1}$	$\int (ax+b)^n dx = \frac{1}{a(n+1)}(ax+b)^{n+1} + c, n \neq -1$		
$\frac{d}{dx}(e^{ax}) = ae^{ax}$	$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$		
$\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$	$\int \frac{1}{x} dx = \log_e(x) + c, x > 0$		
$\frac{d}{dx}(\sin(ax)) = a \cos(ax)$	$\int \sin(ax) dx = -\frac{1}{a} \cos(ax) + c$		
$\frac{d}{dx}(\cos(ax)) = -a \sin(ax)$	$\int \cos(ax) dx = \frac{1}{a} \sin(ax) + c$		
$\frac{d}{dx}(\tan(ax)) = \frac{a}{\cos^2(ax)} = a \sec^2(ax)$			
product rule	$\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$	quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$	Newton's method	$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$
trapezium rule approximation	$Area \approx \frac{x_n - x_0}{2n} [f(x_0) + 2f(x_1) + 2f(x_2) + \dots + 2f(x_{n-2}) + 2f(x_{n-1}) + f(x_n)]$		

Probability

$\Pr(A) = 1 - \Pr(A')$		$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$	
$\Pr(A B) = \frac{\Pr(A \cap B)}{\Pr(B)}$			
mean	$\mu = E(X)$	variance	$\text{var}(X) = \sigma^2 = E((X - \mu)^2) = E(X^2) - \mu^2$
binomial coefficient	$\binom{n}{x} = \frac{n!}{x!(n-x)!}$		

Probability distribution		Mean	Variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \sum x p(x)$	$\sigma^2 = \sum (x - \mu)^2 p(x)$
binomial	$\Pr(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$	$\mu = np$	$\sigma^2 = np(1-p)$
continuous	$\Pr(a < X < b) = \int_a^b f(x) dx$	$\mu = \int_{-\infty}^{\infty} x f(x) dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x - \mu)^2 f(x) dx$

Sample proportions

$\hat{p} = \frac{X}{n}$		mean	$E(\hat{p}) = p$
standard deviation	$\text{sd}(\hat{p}) = \sqrt{\frac{p(1-p)}{n}}$	approximate confidence interval	$\left(\hat{p} - z \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}, \hat{p} + z \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \right)$

END OF FORMULA SHEET