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YEAR 12 *Trial Exam Paper*

Mathematical Methods

Written Examination 1

Question and Answer Book

2024 Insight Year 12 Trial Exam Paper

- **Reading time:** 15 minutes
- **Writing time:** 1 hour

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied

- Question and Answer Book of 15 pages
- Formula Sheet
- Working space is provided throughout the book.

Instructions

- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

At the end of the examination

- You may keep the Formula Sheet.

Students are **not** permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Number of questions: 9

Number of questions to be answered: 9

Number of marks: 40

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Instructions

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (3 marks)

a. Let $y = x\cos(2x)$.

Find $\frac{dy}{dx}$.

1 mark

b. Let $f(x) = \frac{\log_e(x)}{e^x - 1}$.

Find and simplify $f'(1)$.

2 marks

Question 2 (2 marks)

Solve $2\sin^2(x)+3\sin(x)-2=0$, where $x\in[0,2\pi]$.

2 marks

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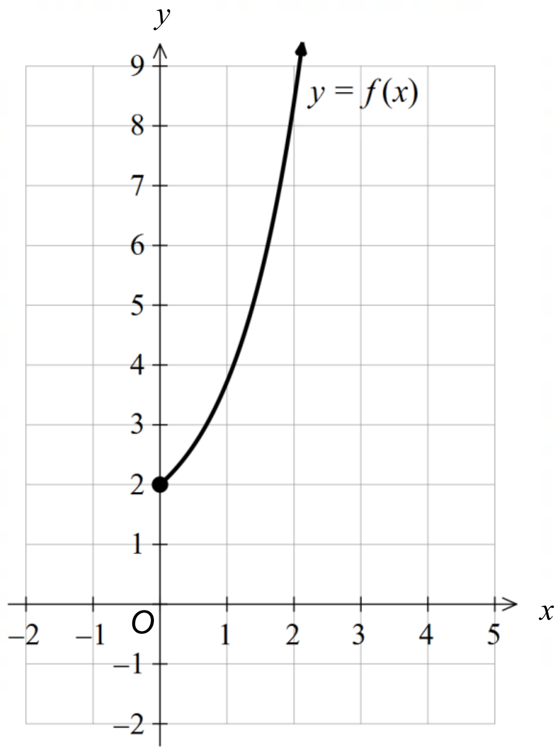
Question 3 (6 marks)

Let $f:[0,\infty)\to R,\ f(x)=e^{x}+1$.

The graph of $y=f(x)$ is shown below over part of its domain.



a. Use two trapeziums of equal width to approximate the area between the curve, the x-axis and the lines $x=0$ and $x=2$.

2 marks

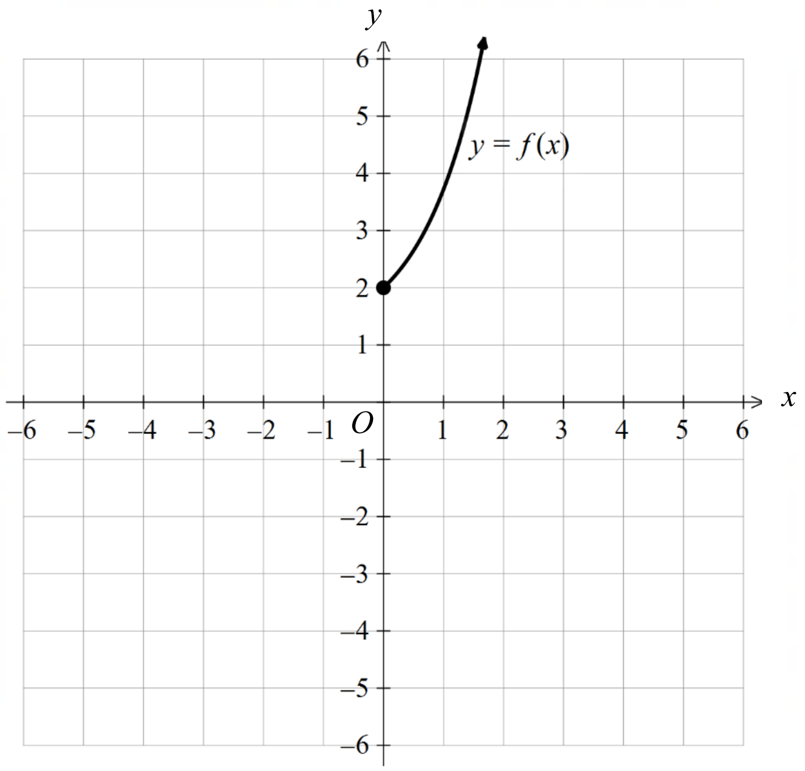
Let $g:[0,\infty)\to R,\ g(x)=4e^{-x}+1$.

b. Find the x-coordinate of the point where the graphs of $y=f(x)$ and $y=g(x)$ intersect. Express your answer in the form $x=\log_e(a)$, where $a\in R^+$.

2 marks

c. Part of the graph of $y=f(x)$ is shown below. Sketch the graph of $y=g(x)$ on the same axes. Label any asymptotes with their equation, and any end points and/or axis intercepts with their coordinates.

2 marks



Question 4 (5 marks)

Let $f : R \to R, f(x) = 2\cos(4x) + 1$.

a. State the range of f.

1 mark

Let $g : [0, a] \to R, g(x) = 2\cos(4x) + 1$ and $h : [0, \infty) \to R, h(x) = \sqrt{x}$.

b. **i.** Find the largest possible value of a, such that $(h \circ g)(x)$ exists.

2 marks

ii. If $a = \frac{\pi}{8}$, state the range of $(h \circ g)(x)$.

2 marks

Question 5 (4 marks)

Let $f: R \to R,\ f(x) = x^3 + x^2$.

a. Determine the x-coordinates of the stationary points of f and state the nature of each stationary point.

2 marks

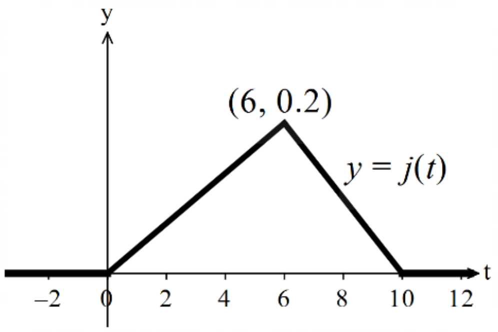
b. Find the coordinates of the point of inflection of f.

2 marks

Question 6 (4 marks)

The length of time, t hours, that Jia uses her laptop on any given day is a continuous random variable, with probability density function $j(t)$.

The graph of $y = j(t)$ is shown below.



The maximum value of $j(t)$ occurs when $x = 6$. Further, $j(t) = 0$ when $t \le 0$ and $t \ge 10$.

a. Find the probability that Jia uses her laptop for more than 6 hours on a given day.

1 mark

b. The length of time, X hours, that Khan uses his laptop on any given day is a continuous random variable. The probability density function of X is given by

$$k(x) = \begin{cases} \dfrac{x+1}{12} & 0 \le x \le 4 \\ 0 & \text{otherwise} \end{cases}$$

Find the value of w, such that $\Pr(X \le w) = \dfrac{1}{3}$.

3 marks

Question 7 (6 marks)

Let $g:\left[\frac{3}{2},\infty\right)\to R,\ g(x)=\sqrt{2x-3}$.

a. Show that $g'(x)=\dfrac{1}{\sqrt{2x-3}}$.

1 mark

b. Find the angle from the positive direction of the x-axis to the tangent of the graph of g at $x=2,$ measured in an anticlockwise direction. State your answer in degrees.

2 marks

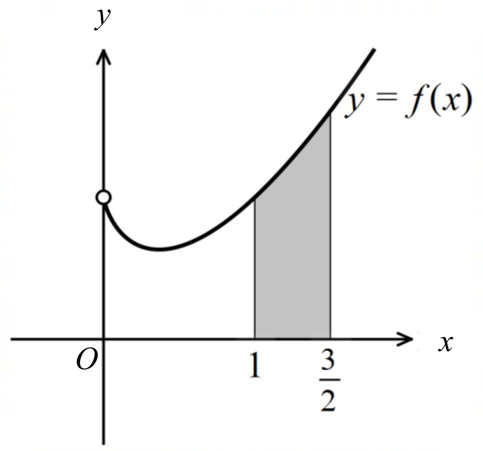
c. Consider the angle from the positive direction of the x-axis to the tangent of the graph of g at $x=k,$ measured in an anticlockwise direction. Find the set of values of k for which this angle is at least $30°$.

3 marks

Question 8 (4 marks)

Let $f : R^+ \to R,\ f(x) = x\log_e(x) + 1$.

Part of the graph of $y = f(x)$ is shown below.



a. Show that $\frac{d}{dx}\left(x^2\log_e(x)\right) = 2x\log_e(x) + x$.

1 mark

b. Hence, find the shaded area that is bound by the graph of $y = f(x)$, the x-axis and the lines $x = 1$ and $x = \frac{3}{2}$. Express your answer in the form $a\log_e(b) - c,$ where $a,\ b$ and c are real constants.

3 marks

Question 9 (6 marks)

A restaurant manager has been collecting data on customer preferences.

a. She has found that the probability that a randomly selected customer requests an outside table is $\frac{2}{3}$ and the probability that they order dessert is $\frac{1}{4}$. Whether a customer requests an outside table is independent of whether they order dessert.

Find the probability that any particular customer requests an outside table and orders dessert.

1 mark

b. The probability that a customer orders salad is $2p$, where $p > 0$. If a customer orders salad, then the probability that they order chips is p. If they don't order salad, the probability they order chips is $4p$.

Find the maximum probability that a customer orders salad or chips, but not both.

3 marks

c. The restaurant also sells jars of chilli sauce. The volume of jars of chilli sauce can be represented by the variable V, which is normally distributed with a mean of 205 g and a standard deviation of 3 g.

If Z is the standard normal random variable, $\Pr(Z < -2) = a$ and $\Pr(-2 < Z < -1) = b$, express $\Pr(V > 202 \mid V < 211)$ in terms of a and b.

2 marks

End of Question and Answer Book